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## SMITHSONIAN

## GEOGRAPHICAL TABLES

PREPARED BY

R. S. WOODWARD

## THIRD EDITION



## CITY OF WASHINGTON

PUBLISHED BY THE SMITHSONIAN INSTITUTION
1906

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## ADVERTISEMENT TO THIRD EDITION.

The second edition of the Smithsonian Geographical Tables issued in 1897 having become exhausted, a third edition is now printed with a few necessary changes made in the plates.

Richard Rathbun, Acting Secretary.
Smithsonian Institution, Washington City, August 6, 1906 .

## ADVERTISEMENT TO SECOND EDITION.

The edition of the Smithsonian Geographical Tables issued in 1894 having become exhausted, a second edition is now printed with a few necessary changes made in the plates.

S. P. Langley,<br>Secretary.

[^0]
## ADVERTISEMENT.

In connection with the system of meteorological observations established by the Smithsonian Institution about 1850, a series of meteorological tables was compiled by Dr. Arnold Guyot, at the request of Secretary Henry, and was published in 1852 as a volume of the Miscellaneous Collections.

A second edition was published in 1857, and a third edition, with further amendments, in 1859.

Though primarily designed for meteorological observers reporting to the Smithsonian Institution, the tables were so widely used by meteorologists and physicists that, after twenty-five years of valuable service, the work was again revised, and a fourth edition was published in 1884.

In a few years the demand for the tables exhausted the edition, and it appeared to me desirable to recast the work entirely, rather than to undertake its revision again. After careful consideration I decided to publish the new work in three parts : Meteorological Tables, Geographical Tables, and Physical Tables, each representative of the latest knowledge in its field, and independent of the others ; but the three forming a homogeneous series.

Although thus historically related to Doctor Guyot's Tables, the present work is so entirely changed with respect to material, arrangement, and presentation, that it is not a fifth edition of the older tables, but essentially a new publication.

The first volume of the new series of Smithsonian Tables (the Meteorological Tables) appeared in 1893 . The present volume, forming the second of the series, the Geographical Tables, has been prepared by Professor R. S. Woodward, formerly of the United States Coast and Geodetic Survey, but now of Columbia College, New York, who has brought to the work a very wide experience both in field work and in the reduction of extensive geodetic observations.

S. P. Langley, Secretary.

## PREFACE.

In the preparation of the following work two difficulties of quite different kinds presented themselves. The first of these was to make a judicious selection of matter suited to the needs of the average geographer, and at the same time to keep the volume within prescribed limits. Of the vast amount of material available, much must be omitted from any work of limited dimensions, and it was essential to adopt some rule of discrimination. The rule adopted and adhered to, so far as practicable, was to incorporate little material already accessible in good form elsewhere, Accordingly, while numerous references are made in the volume to such accessible material, an attempt has been made wherever feasible to introduce new matter, or matter not hitherto generally available.

The second difficulty arose from the present uncertainty in the relation of the British and metric units of length, or rather from the absence of any generally adopted ratio of the British yard to the metre. The dimensions of the earth adopted for the tables are those of General Clarke, published in 1866, and now most commonly used in geodesy. These dimensions are expressed in English feet, and in order to convert them into metres it is necessary to adopt a ratio of the foot to the metre. The ratio used by General Clarke, and hitherto generally used, is now known to be erroneous by about one one hundred thousandth part. The ratio used in this volume is that adopted provisionally by the Office of Standard Weights and Measures of the United States and legalized by Act of Congress in $\mathbf{1 8 6 6}$. But inasmuch as a precise determination of this ratio is now in progress under the auspices of the International Bureau of Weights and Measures, and inasmuch as the value for the ratio found by this Bureau will doubtless be generally adopted, it has been thought best in the present edition to restrict quantities expressed in metric measures to limits which will require no change from the uncertainty in question. In conformity with this decision the dimensions of the earth are given in feet only, and, with a few unimportant exceptions, to which attention is called in the proper places, tables giving quantities in metres are limited to such a number of figures as are definitely known.

It is a matter of regret that, owing to the cause just stated, less prominence has been given in the tables to metric than to British units of length. On the other hand, it seems probable that the more general use of British units will meet the approval of the majority of those for whose use the volume is designed.

The introductory part of the volume is divided into seven sections under the heads, Useful Formulas, Mensuration, Units, Geodesy, Astronomy, Theory of Errors, and Explanation of Source and Use of Tables, respectively. In pre senting the subjects embraced under the first six of these headings an attempt was made to give only those features leading, directly to practical applications of the principles involved. It is hoped, however, that enough has been given of each subject to render the work of value in a broader sense to those who may desire to go beyond mere applications.

The most of the calculations required in the preparation of the tables were made by Mr. Charles H. Kummell and Mr. B. C. Washington, Jr. Their work was done with skill and fidelity, and it is believed that the systematic checks applied by them have rendered the tables they computed entirely trustworthy. Mention of the particular tables computed by each of them is made in the Explanation of Source and Use of Tables, where full credit is given also for data not specially prepared for the volume.

The Appendix to the present volume is that prepared by Mr. George E. Curtis for the Meteorological Tables. Its usefulness to the geographer is no less obvious and general than to the meteorologist.

The proofs have been read independently by Mr. Charles H . Kummell and the editor. The plate proofs, also, have been read by the editor ; and while it is difficult to avoid errors in a first edition of a work containing many formulas and figures, it is believed that few, if any, important errata remain in this volume.

R. S. Woodward.

Columbia College, New York, N. Y., June 15, 1894.

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## USEFUL FORMULAS.

## I. Algebraic.

a. Arithmetic and geometric means. The arithmetic mean of $n$ quantities $a, b, c, \ldots$ is

$$
\frac{1}{n}(a+b+c+\ldots) ;
$$

their geometric mean is

$$
\left(\begin{array}{lll}
a & b & c
\end{array}\right)^{\frac{1}{n}} .
$$

A case of special interest is

$$
\sqrt{a b}=\frac{1}{2}(a+b)\left\{1-\left(\frac{a-b}{a+b}\right)^{2}\right\}^{\frac{1}{2}} .
$$

b. Arithmetic progression. If $a$ is the first term, and $a+d, a+2 d$, $a+3 d, \ldots$ are the successive terms, the $n$th or last term $z$ is

$$
z=a+(n-1) d
$$

The sum $s$ of the $n$ terms of this series is

$$
\begin{aligned}
s=\frac{1}{2}(a+z) n & =\left\{a+\frac{1}{2}(n-\mathrm{I}) d\right\} n \\
& =\left\{z-\frac{1}{2}(n-\mathrm{I}) d\right\} n \\
& =\frac{1}{2}(a+z)\left(\frac{z-a}{d}+\mathrm{I}\right) .
\end{aligned}
$$

c. Geometric progression. If $a$ is the first term, and $a r, a r^{2}, \ldots$ are the successive terms, the $n$th or last term $z$ is

$$
z=a r^{n-1}
$$

The sum of the $n$ terms is

If

$$
s=\frac{a\left(r^{n}-\mathrm{I}\right)}{r-\mathrm{I}}=\frac{r z-a}{r-\mathrm{I}}=\frac{z\left(r^{n}-\mathrm{I}\right)}{(r-\mathrm{I}) r^{n-1}}
$$

$$
r<1 \text { and } n=\infty,
$$

$$
s=\frac{a}{\mathrm{I}-r}
$$

d. Sums of special series.

$$
\begin{aligned}
& \mathbf{x}+2+3+4+\cdots+n \quad=\frac{1}{2} n(n+1) \\
& 2+4+6+8+\ldots+2 n=n(n+1) \\
& 1+3+5+7+\ldots+(2 n-1)=n^{2} \\
& 1^{2}+2^{2}+3^{2}+4^{2}+\ldots+n^{2} \quad=\frac{1}{6} n(n+1)(2 n+1) \\
& 1^{8}+2^{8}+3^{9}+4^{8}+\ldots+n^{8}=\frac{1}{4} n^{2}(n+1)^{2} \text {. }
\end{aligned}
$$

## e. The binomial series and applications.

For $a>b$,

$$
\begin{aligned}
(a \pm b)^{n}=a^{n} \pm n a^{n-1} b & +\frac{n(n-1)}{1 \cdot 2} a^{n-2} b^{2} \\
& \pm \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^{n-8} b^{g}+\ldots
\end{aligned}
$$

For $x<\mathrm{r}$,

$$
\begin{aligned}
& (\mathrm{I} \pm x)^{n}=\mathrm{I} \pm n x+\frac{n(n-\mathrm{I})}{\mathrm{I} \cdot 2} x^{2} \pm \frac{n(n-\mathrm{r})(n-2)}{\mathrm{I} \cdot 2 \cdot 3} x^{8}+\ldots \\
& \frac{\mathrm{I}}{\mathrm{I}+x}=\mathrm{I}-x+x^{2}-x^{8}+x^{4}-\ldots \\
& \frac{\mathrm{I}}{\mathrm{I}-x}=\mathrm{I}+x+x^{2}+x^{8}+x^{4}+\ldots \\
& \frac{\mathrm{I}}{(\mathrm{I}-x)^{2}}=\mathrm{I}+2 x+3 x^{2}+4 x^{8}+5 x^{4}+\ldots \\
& (\mathrm{I}+x)^{\frac{1}{2}}=\mathrm{I}+\frac{1}{2} x-\frac{1}{8} x^{2}+\frac{1}{16} x^{8}-\frac{5}{28} x^{4}+\ldots \\
& (\mathrm{I}-x)^{\frac{1}{2}}=\mathrm{I}-\frac{1}{2} x-\frac{1}{8} x^{2}-\frac{1}{16} x^{8}-\mathrm{I}^{\frac{5}{2} 8} x^{4}-\ldots \\
& \frac{\mathrm{I}}{(\mathrm{I}+x)^{\frac{3}{3}}}=\mathrm{I}-\frac{1}{2} x+\frac{3}{8} x^{2}-\frac{8}{16} x^{8}+\frac{35}{128} x^{4}-\ldots \\
& \frac{\mathrm{I}}{(\mathrm{I}-x)^{\frac{3}{3}}}=\mathrm{I}+\frac{1}{2} x+\frac{3}{8} x^{2}+\frac{5}{\mathrm{I}_{6}} x^{8}+\frac{35}{128} x^{4}+\ldots .
\end{aligned}
$$

## f. Exponential and logarithmic series.

For $-\infty<x<\infty$,

$$
e^{x}=1+\frac{x}{1}+\frac{x^{2}}{1.2}+\frac{x^{8}}{1.2 .3}+\frac{x^{4}}{1.2 \cdot 3 \cdot 4}+\ldots
$$

The number $e$ is the base of the natural or "Napierian" system of logarithms. For $x=+1$, the above series gives

$$
e=2.718281828459 \ldots
$$

In the natural system the following series hold with the limitations indicated:

$$
\begin{aligned}
& a^{x}=\mathrm{I}+\frac{\log a}{\mathrm{I}} x+\frac{(\log a)^{2}}{\mathrm{I} \cdot 2} x^{2}+\frac{(\log a)^{\mathrm{8}}}{\mathrm{I} \cdot 2 \cdot 3} x^{\mathrm{s}} \ldots \\
& -\infty<x<\infty \text {; } \\
& \log (1+x)=x-\frac{x^{2}}{2}+\frac{x^{8}}{3}-\frac{x^{4}}{4}+\frac{x^{5}}{5}-\ldots \\
& x \leqq \text { I; } \\
& \log (\mathrm{r}-x)=-x-\frac{x^{2}}{2}-\frac{x^{8}}{3}-\frac{x^{4}}{4}-\frac{x^{5}}{5}-\ldots \\
& x<1 ; \\
& \log x=2\left\{\frac{x-\mathrm{I}}{x+\mathrm{r}}+\frac{1}{3}\left(\frac{x-\mathrm{r}}{x+1}\right)^{3}+\frac{1}{5}\left(\frac{x-\mathrm{r}}{x+\mathrm{I}}\right)^{5}+\frac{1}{7}\left(\frac{x-\mathrm{r}}{x+\mathrm{r}}\right)^{7}+\ldots\right\} \\
& 0<x<\infty \text {; } \\
& \log \frac{x+y}{x}=2\left\{\frac{y}{2 x+y}+\frac{1}{3}\left(\frac{y}{2 x+y}\right)^{8}+\frac{1}{8}\left(\frac{y}{2 x+y}\right)^{5}+\ldots\right\} \\
& y^{2}<(2 x+y)^{2} .
\end{aligned}
$$

g. Relations of natural logarithms to other logarithms.
$B=$ base of any system,
$N=$ any number,
$L=\log N$ to base $B=\log _{B} N$,
$l=\log N$ to base $e=\log _{e} N$.
Then

$$
\begin{aligned}
& N=e^{i}=B^{L} \\
& L=l \log _{B} e=t / \log _{e} B
\end{aligned}
$$

$\log _{B} e=\mathrm{r} / \log _{e} B=\mu$, say, which is called the modulus of the system whose base is $B$. In the common, or Briggean system,

$$
\begin{aligned}
\mu=\log _{10} e & =0.43429448 \ldots \\
\quad \log \mu & =9.6377843-10
\end{aligned}
$$

## 2. Trigonometric Formulas.

a. Signs of trigonometric functions.

| Function. |  | rst Quadrant. | 2d Quadrant. | 3d Quadrant. | 4th Quadrant. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| sine . . . . . . | + | + | - | - |  |
| cosine . . . . . | + | + | - | - | + |
| tangent . . . . | + | - | + | + | - |
| cotangent. . . . | + | - | + | - |  |

b. Values of functions for special angles.

|  | $0^{\circ}$ | $90^{\circ}$ | $180^{\circ}$ | $270^{\circ}$ | $360^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| sine | - | + I | - | - I | $\bigcirc$ | $\frac{1}{2}$ | $\frac{1}{2} \sqrt{2}$ | $\frac{1}{2} \sqrt{3}$ |
| cosine | +1 | $\bigcirc$ | - I | $\bigcirc$ | +1 | $\frac{1}{2} \sqrt{3}$ | $\frac{1}{2} \sqrt{2}$ | $\frac{1}{2}$ |
| tangent | $\bigcirc$ | $\infty$ | $\bigcirc$ | $\infty$ | $\bigcirc$ | $\frac{1}{3} \sqrt{3}$ | I | $\sqrt{3}$ |
| cotangent . . | $\infty$ | $\bigcirc$ | $\infty$ | $\bigcirc$ | $\infty$ | $\sqrt{3}$ | 1 | $\frac{1}{3} \sqrt{3}$ |

c. Fundamental formulas.

$$
\begin{aligned}
& \sin ^{2} \alpha+\cos ^{2} \alpha=1, \\
& \cos \alpha \sec \alpha=\mathrm{I}, \\
& \tan a=\frac{\sin a}{\cos \alpha}, \\
& \tan u \cot a=1, \\
& \sin u \operatorname{cosec} a=I \text {, } \\
& \cot \alpha=\frac{\cos \alpha}{\sin \alpha}, \\
& x+\tan ^{2} a=\frac{1}{\cos ^{2} \alpha}=\sec ^{2} \alpha, \quad 1+\cot ^{2} \alpha=\frac{1}{\sin ^{2} \alpha}=\operatorname{cosec}^{2} a, \\
& \text { versed } \sin a=1-\cos \alpha .
\end{aligned}
$$

d. Formulas involving two angles.
$\sin (\alpha \pm \beta)=\sin \alpha \cos \beta \pm \cos \alpha \sin \beta$, $\cos (\alpha \pm \beta)=\cos \alpha \cos \beta \mp \sin u \sin \beta$.
$\tan (\alpha \pm \beta)=(\tan \alpha \pm \tan \beta) /(\mathrm{I} \mp \tan \alpha \tan \beta)$, $\cot (\alpha \pm \beta)=(\cot a \cot \beta \mp \mathrm{I}) /(\cot a \pm \cot \beta)$.
$\sin \alpha+\sin \beta=2 \sin \frac{1}{2}(\alpha+\beta) \cos \frac{1}{2}(\alpha-\beta)$, $\sin \alpha-\sin \beta=2 \cos \frac{1}{2}(\alpha+\beta) \sin \frac{1}{2}(\alpha-\beta)$.
$\cos \alpha+\cos \beta=2 \cos \frac{1}{2}(\alpha+\beta) \cos \frac{1}{2}(\alpha-\beta)$, $\cos \alpha-\cos \beta=-2 \cdot \sin \frac{1}{2}(\alpha+\beta) \sin \frac{1}{2}(\alpha-\beta)$.
$\tan \alpha \pm \tan \beta=\frac{\sin (\alpha \pm \beta)}{\cos \alpha \cos \beta}$,
$\cot \alpha \pm \cot \beta=\frac{\sin (\beta \pm a)}{\sin a \sin \beta}$.
$2 \sin a \sin \beta=\cos (\alpha-\beta)-\cos (\alpha+\beta)$, $2 \cos \alpha \cos \beta=\cos (\alpha-\beta)+\cos (\alpha+\beta)$,
$2 \sin \alpha \cos \beta=\sin (\alpha-\beta)+\sin (\alpha+\beta)$.
$\frac{\sin \alpha+\sin \beta}{\sin \alpha-\sin \beta}=\tan \frac{1}{2}(\alpha+\beta) \cot \frac{1}{2}(\alpha-\beta)$,
$\frac{\cos \alpha+\cos \beta}{\cos \alpha-\cos \beta}=-\cot \frac{1}{2}(\alpha+\beta) \cot \frac{1}{2}(\alpha-\beta)$.

## e. Formulas involving multiple angles.

$\sin 2 a=2 \sin \alpha \cos \alpha$,
$\sin 3 \alpha=3 \sin \alpha \cos ^{2} \alpha-\sin ^{8} \alpha$.
$\cos 2 \alpha=\cos ^{2} \alpha-\sin ^{2} \alpha=1-2 \sin ^{2} \alpha=2 \cos ^{2} \alpha-1$, $\cos 3 a=\cos ^{8} \alpha-3 \sin ^{2} \alpha \cos \alpha$.
$\tan \frac{1}{2} \alpha=\frac{\sin \alpha}{1+\cos \alpha}=\frac{1-\cos \alpha}{\sin a}=\left(\frac{1-\cos \alpha}{1+\cos \alpha}\right)^{\frac{1}{3}}$,
$\tan 2 \alpha=\frac{2 \tan \alpha}{1-\tan ^{2} \alpha}, \quad \cot 2 \alpha=\frac{\cot ^{2} \alpha-1}{2 \cot \alpha}$,
$\sin \alpha=\frac{2 \tan \frac{1}{2} \alpha}{1+\tan ^{2} \frac{1}{2} a}, \quad \cos \alpha=\frac{1-\tan ^{2} \frac{1}{2} \alpha}{1+\tan ^{2} \frac{1}{2} \alpha}$
$2 \sin ^{2} \alpha=1-\cos 2 a, \quad 2 \cos ^{2} a=1+\cos 2 a$, $4 \sin ^{8} \alpha=3 \sin \alpha-\sin 3 a, \quad 4 \cos ^{8} \alpha=3 \cos \alpha+\cos 3 a_{0}$.

## f. Exponential values. Moivre's formula.

$e=$ base of natural logarithms,
$i=\sqrt{-\mathrm{I}}, i^{2}=-\mathrm{1}, i^{8}=-i, i^{4}=\mathrm{r}$, etc.
$\cos x=\frac{1}{2}\left(e^{i x}+e^{-i x}\right), \quad \sin x=\frac{1}{2 i}\left(e^{i x}-e^{-i x}\right)$,
$\cos i x=\frac{1}{2}\left(e^{-x}+e^{x}\right), \quad \sin i x=\frac{1}{2 i}\left(e^{-x}-e^{x}\right)$.
$(\cos x \pm i \sin x)^{m}=\cos m x \pm i \sin m x$.

## g. Values of functions in series.

For $x$ in arc the following series hold within the limits indicated.

$$
\begin{gathered}
\sin x=x-\frac{x^{8}}{6}+\frac{x^{6}}{120}-\frac{x^{7}}{5^{\circ} 4^{\circ}}+\ldots, \\
\cos x=\mathrm{r}-\frac{x^{2}}{2}+\frac{x^{4}}{24}-\frac{x^{6}}{7^{20}}+\ldots \\
-\infty<x<+\infty .
\end{gathered}
$$

$$
\tan x=x+\frac{1}{3} x^{8}+\frac{2}{15} x^{6}+\frac{13}{315} x^{7}+\ldots,
$$

$$
\sec x=1+\frac{1}{2} x^{2}+\frac{5}{24} x^{4}+\frac{6}{6} x^{6}+\ldots,
$$

$$
-\frac{1}{2} \pi<x<+\frac{1}{2} \pi .
$$

$$
\operatorname{arc} \sin x=x+\frac{1}{6} x^{8}+\frac{3}{40} x^{6}+\frac{8}{r_{12}^{2}} x^{7}+\ldots,
$$

$$
\arctan x=x-\frac{x^{8}}{3}+\frac{x^{6}}{5}-\frac{x^{7}}{7}+\frac{x^{9}}{9}-\ldots,
$$

$$
-\mathrm{x}<x<+\mathrm{r}
$$

$$
x=\sin x+\frac{1}{6} \sin ^{3} x+\frac{3}{40} \sin ^{5} x+\frac{1}{15} \sin ^{7} x+\ldots,
$$

$$
-\frac{1}{2} \pi<x<+\frac{1}{2} \pi .
$$

$$
x=\tan x-\frac{1}{3} \tan ^{8} x+\frac{1}{3} \tan ^{5} x-\frac{1}{4} \tan ^{7} x+\ldots,
$$

$$
-\frac{1}{4} \pi<x<+\frac{1}{4} \pi .
$$

$\log \sin x=\log x-\mu\left(\frac{1}{6} x^{2}+{ }^{\frac{1}{80}} x^{4}+\frac{1}{2835} x^{6}+\ldots\right)$,

$$
x \text { positive and }<\pi,
$$

$$
\mu=\text { modulus of common logarithms. See p. xv. }
$$

$$
\log \tan x=\log x+\mu\left(\frac{1}{3} x^{2}+\frac{7}{30} x^{4}+\frac{69}{28} \frac{2}{85} x^{6}+\ldots\right),
$$

$$
x \text { positive and }<\frac{1}{2} \pi
$$

h. Conversion of arcs into angles and angles into arcs.

Denote by $x^{\circ}, x^{\prime}$, and $x^{\prime \prime}$ respectively the angle (in degrees, minutes, or seconds) corresponding to the $\operatorname{arc} x$. Then by equality of ratios

$$
\frac{360^{\circ}}{x^{\circ}}=\frac{360 \times 60^{\prime}}{x^{\prime}}=\frac{360 \times 60 \times 60^{\prime \prime}}{x^{\prime \prime}}=\frac{2 \pi}{x},
$$

whence

$$
\begin{aligned}
& x^{\circ}=x \frac{180^{\circ}}{\pi} \\
& x^{\prime}=x \frac{180 \times 60^{\prime}}{\pi} \\
& x^{\prime \prime}=x \frac{180 \times 60 \times 60^{\prime \prime}}{\pi}
\end{aligned}
$$

$$
\begin{aligned}
& \cot x=\frac{1}{x}\left(\mathrm{x}-\frac{1}{3} x^{2}-\frac{1}{45} x^{4}-{ }_{9}^{2} \frac{2}{5} x^{6}-\ldots\right), \\
& \operatorname{cosec} x=\frac{1}{x}\left(\mathrm{r}+\frac{1}{6} x^{2}+{ }_{3}^{57}{ }^{7} x^{4}+\operatorname{TBI}^{3120} x^{6}+\ldots\right), \\
& -\pi<x<+\pi .
\end{aligned}
$$

Put

$$
\frac{\mathbf{1} 80^{\circ}}{\pi}=\rho^{\circ}=\text { number of degrees in the radius, }
$$

$$
\frac{180 \times 60^{\prime}}{\pi}=\rho^{\prime}=\text { number of minutes in the radius, }
$$

$$
\frac{180 \times 60 \times 60^{\prime \prime}}{\pi}=\rho^{\prime \prime}=\text { number of seconds in the radius. }
$$

Then

$$
\begin{gathered}
x^{\circ}=x \rho^{\circ}, \quad x^{\prime}=x \rho^{\prime}, \quad x^{\prime \prime}=x \rho^{\prime \prime} . \\
\rho^{\circ}=57 .^{\circ} 2957795, \quad \log \rho^{\circ}=1.75^{812263}, \\
\rho^{\prime}=3437 .^{\prime} 74677, \quad \log \rho^{\prime}=3.53627388, \\
\rho^{\prime \prime}=206264 .^{\prime \prime} 806, \quad \quad \log \rho^{\prime \prime}=5.3144^{\prime 2} 5 \mathrm{I} 3 .
\end{gathered}
$$

3. Formulas for Solution of Plane Triangles.

$$
\begin{aligned}
a, b, c & =\text { sides of triangle } \\
a, \beta, \gamma & =\text { angles opposite to } a, b, c, \text { respectively } \\
A & =\text { area of triangle, } \\
r & =\text { radius of inscribed circle } \\
R & =\text { radius of circumscribed circle, } \\
s & =\frac{1}{2}(a+b+c) \\
& \quad \frac{a}{\sin \alpha}=\frac{b}{\sin \beta}=\frac{c}{\sin \gamma}=2 R .
\end{aligned}
$$

$a=b \cos \gamma+c \cos \beta, \quad b=c \cos \alpha+a \cos \gamma, \quad c=a \cos \beta+b \cos \alpha_{0}$

$$
r=4 R \sin \frac{1}{2} \alpha \sin \frac{1}{2} \beta \sin \frac{1}{2} \gamma=\frac{a b c}{4 R s}
$$

$$
\begin{aligned}
& (a+b) \cos \frac{1}{2}(a+\beta)=c \cos \frac{1}{2}(\alpha-\beta) \\
& (a-b) \sin \frac{1}{2}(\alpha+\beta)=c \sin \frac{1}{2}(\alpha-\beta)
\end{aligned}
$$

$$
\frac{a+b}{a-b}=\frac{\tan \frac{1}{2}(\alpha+\beta)}{\tan \frac{1}{2}(\alpha-\beta)}=\frac{\tan \frac{1}{2} \gamma}{\tan \frac{1}{2}(\alpha-\beta)}
$$

$$
a^{2}=b^{2}+c^{2}-2 b c \cos \alpha=(b+c)^{2}-4 b c \cos ^{2} \frac{1}{2} a
$$

$$
\sin \frac{1}{2} a=\sqrt{\frac{(s-b)(s-c)}{b c}}, \quad \cos \frac{1}{2} a=\sqrt{\frac{s(s-a)}{b c}}
$$

$$
\tan \frac{1}{2} a=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{r}{s-a}
$$

$$
r=\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}
$$

$$
\begin{aligned}
A & =\frac{1}{2} a b \sin \gamma=\frac{a^{2} \sin \beta \sin \gamma}{2 \sin a}=2 R^{2} \sin a \sin \beta \sin \gamma \\
& =r^{2} \cot \frac{1}{2} a \cot \frac{1}{2} \beta \cot \frac{1}{2} \gamma=\sqrt{s(s-a)(s-b)(s-c)} \\
& =r s=\frac{1}{4} a b c / R .
\end{aligned}
$$

In right angled triangles let

$$
\begin{aligned}
& a=\text { altitude } \\
& b=\text { base } \\
& c=\text { hypothenuse } \\
& \gamma=90^{\circ}
\end{aligned}
$$

Then

$$
\begin{gathered}
a=c \sin \alpha=c \cos \beta=b \tan \alpha=b \cot \beta \\
b=c \sin \beta=c \cos \alpha=a \tan \beta=a \cot \alpha \\
A=\frac{1}{2} a b=\frac{1}{2} a^{2} \cot \alpha=\frac{1}{2} b^{2} \tan \alpha=\frac{1}{4} c^{2} \sin 2 a
\end{gathered}
$$

Table for solution of oblique triangles.

\begin{tabular}{|c|c|c|}
\hline Given. \& Sought. \& Formula. <br>
\hline $a, b, c$ \& $a$

$A$ \& $$
\begin{aligned}
\sin \frac{1}{2} a & =\sqrt{\frac{(s-b)(s-c)}{b c}}, \quad s=\frac{1}{2}(a+b+c) \\
\cos \frac{1}{2} a & =\sqrt{\frac{s(s-a)}{b c}} \\
\tan \frac{1}{2} a & =\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\
A & =\sqrt{s(s-a)(s-b)(s-c)}
\end{aligned}
$$ <br>

\hline $a, b, a$ \& \[
$$
\begin{gathered}
\beta \\
\gamma \\
c \\
A
\end{gathered}
$$

\] \& | $\sin \beta=b \sin a / a$. |
| :--- |
| When $a>b, \beta<90^{\circ}$ and but one value results. When $b>a$, $\beta$ has two values. $\begin{aligned} & \gamma=180^{\circ}-(\alpha+\beta) . \\ & c=a \sin \gamma / \sin a . \\ & A=\frac{1}{2} a b \sin \gamma . \end{aligned}$ | <br>

\hline $a, a, \beta$ \& \[
$$
\begin{gathered}
b \\
\gamma \\
c \\
A
\end{gathered}
$$

\] \& \[

$$
\begin{aligned}
& b=a \sin \beta / \sin a . \\
& \gamma=\mathrm{r} 80^{\circ}-(\alpha+\beta) . \\
& c=a \sin \gamma / \sin a=a \sin (\alpha+\beta) / \sin a . \\
& A=\frac{1}{2} a b \sin \gamma=\frac{1}{2} a^{2} \sin \beta \sin \gamma / \sin a .
\end{aligned}
$$
\] <br>

\hline $a, b, \gamma$ \& $a$

$$
a, \beta
$$ \& \[

$$
\begin{aligned}
& \tan a=\frac{a \sin \gamma}{b-a \cos \gamma} \\
& \frac{1}{2}(a+\beta)=90^{\circ}-\frac{1}{2} \gamma, \\
& \tan \frac{1}{2}(a-\beta)=\frac{a-b}{a+b} \cot \frac{1}{2} \gamma . \\
& c=\left(a^{2}+b^{2}-2 a b \cos \gamma\right)^{\frac{1}{2}}, \\
& =\left\{(a+b)^{2}-4 a b \cos ^{2} \frac{1}{2} \gamma\right\}^{\frac{1}{2}}, \\
& =\left\{(a-b)^{2}+4 a b \sin ^{2} \frac{1}{2} \gamma\right\}^{\frac{1}{2}}, \\
& =(a-b) / \cos \phi, \text { where } \tan \phi=2 \sqrt{a b} \sin \frac{1}{2} \gamma /(a-b), \\
& =a \sin \gamma / \sin a . \\
& A=\frac{1}{2} a b \sin \gamma .
\end{aligned}
$$
\] <br>

\hline
\end{tabular}

## 4. Formulas for Solution of Spherical Triangles.

a. Right angled spherical triangles.
$a, b, c=$ sides of triangle, $c$ being the hypotenuse, $a, \beta, \gamma=$ angles opposite to $a, b, c$, respectively,
$\gamma=90^{\circ}$.
$\sin a=\sin c \sin \alpha, \quad \sin b=\sin c \sin \beta$, $\tan a=\tan c \cos \beta, \quad \tan b=\tan c \cos \alpha$,
$=\sin b \tan u, \quad=\sin a \tan \beta ;$
$\cos a=\cos a \sin \beta, \quad \cos \beta=\cos b \sin a ;$
$\cos c=\cos a \cos b=\cot a \cot \beta$.
b. Oblique angled triangles.
$a, b, c=$ sides of triangle,
$a, \beta, \gamma=$ angles opposite to $a, b, c$, respectively,
$s=\frac{1}{2}(a+b+c)$,
$\sigma=\frac{1}{2}(a+\beta+\gamma)$,
$\epsilon=a+\beta+\gamma-180^{\circ}=$ spherical excess,
$S=$ surface of triangle on sphere of radius $r$.

$$
\frac{\sin a}{\sin a}=\frac{\sin b}{\sin \beta}=\frac{\sin c}{\sin \gamma}
$$

$\cos a=\cos b \cos c+\sin b \sin c \cos u$,
$\sin ^{2} \frac{1}{2} a=\frac{-\cos \sigma \cos (\sigma-a)}{\sin \beta \sin \gamma}, \quad \cos ^{2} \frac{1}{2} a=\frac{\cos (\sigma-\beta) \cos (\sigma-\gamma)}{\sin \beta \sin \gamma}$,

$$
\tan ^{2} \frac{1}{2} a=\frac{-\cos \sigma \cos (\sigma-a)}{\cos (\sigma-\beta) \cos (\sigma-\gamma)} .
$$

$\sin ^{2} \frac{1}{2} \alpha=\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}, \quad \cos ^{2} \frac{1}{2} \alpha=\frac{\sin s \sin (s-a)}{\sin b \sin c}$,
$\tan ^{2} \frac{1}{2} a=\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}$.
$\cot \frac{1}{2} \epsilon=\frac{\cot \frac{1}{2} a \cot \frac{1}{2} b+\cos \gamma}{\sin \gamma}$,
$\tan ^{2} \frac{1}{4} \epsilon=\tan \frac{1}{2} s \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)$.

$$
S=\frac{\epsilon}{180^{\circ}} \pi r^{2} .
$$

## Napier's analogies.

$\tan \frac{1}{2}(\alpha+b)=\frac{\cos \frac{1}{2}(\alpha-\beta)}{\cos \frac{1}{2}(\alpha+\beta)} \tan \frac{1}{2} c, \quad \tan \frac{1}{2}(\alpha-b)=\frac{\sin \frac{1}{2}(\alpha-\beta)}{\sin \frac{1}{2}(\alpha+\beta)} \tan \frac{1}{2} c$, $\tan \frac{1}{2}(\alpha+\beta)=\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{1}{2} \gamma, \quad \tan \frac{1}{2}(\alpha-\beta)=\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{1}{2} \gamma$.

Gauss's formulas.
$\cos \frac{1}{2}(a+\beta) \cos \frac{1}{2} c=\cos \frac{1}{2}(a+b) \sin \frac{1}{2} \gamma$, $\sin \frac{1}{2}(a+\beta) \cos \frac{1}{2} c=\cos \frac{1}{2}(a-b) \cos \frac{1}{2} \gamma$,
$\cos \frac{1}{2}(a-\beta) \sin \frac{1}{2} c=\sin \frac{1}{2}(a+b) \sin \frac{1}{2} \gamma$, $\sin \frac{1}{2}(a-\beta) \sin \frac{1}{2} c=\sin \frac{1}{2}(a-b) \cos \frac{1}{2} \gamma$.

## 5. Elementary Differential Formulas.

## a. Algebraic.

$u, v, v, \ldots=$ variables subject to differentiation, $a, b, c, \ldots=$ constants.

$$
\begin{gathered}
d(a+u)=d u, \quad d(a u)=a d u, \\
d(u+v+w+\ldots)=d u+d v+d w+\ldots, \\
d(u v)=u d v+v d u, \\
d(u v w \ldots)=\left(\frac{d u}{u}+\frac{d v}{v}+\frac{d w}{w}+\ldots\right) u v w \ldots, \\
d\left(\frac{u}{v}\right)=\frac{v d u-u d v}{v^{2}}=\frac{d u}{v}-\frac{u d v}{v^{2}}, \\
d\left(\frac{a+b u}{h+g u}\right)=\frac{b h-a g}{\left(h+g^{u}\right)^{2}} d u . \\
d v^{n}=n v^{n-1} d v, \quad d \sqrt{v}=\frac{d v}{2 \sqrt{v}} \\
d a^{v}=a^{v} \log a d v, \quad d e^{v}=e^{v} d v \\
(e=\text { base of natural logarithms }), \\
d \log v=d v / v . \\
d F(u, v, w \ldots)=\frac{\partial F}{\partial u} d u+\frac{\partial F}{\partial v} d v+\frac{\partial F}{\partial w} d w+\ldots .
\end{gathered}
$$

b. Trigonometric and inverse trigonometric.
$d \sin x=\cos x d x, \quad d \cos x=-\sin x d x$, $d \tan x=\sec ^{2} x d x, \quad d \cot x=-\operatorname{cosec}^{2} x d x$, $d \sec x=\sec ^{2} x \sin x d x, \quad d \operatorname{cosec} x=-\operatorname{cosec}^{2} x \cos x d x$.
$d \log \sin x=\cot x d x, \quad d \log \cos x=-\tan x d x$. $d \operatorname{arc} \sin x= \pm \frac{d x}{\sqrt{1-x^{2}}}, \quad d \arccos x=\mp \frac{d x}{\sqrt{1-x^{2}}}$,
$d \arctan x=\frac{d x}{1+x^{2}}, \quad d \operatorname{arccot} x=-\frac{d x}{1+x^{2}}$.

## 6. Taylor's and Maclaurin's Series.

## a. Taylor's series.

If $u=f(x+h)$, any finite and continuous function of $x+h, h$ being an arbitrary increment to $x$; and if $d u / d x, d^{2} u / d x^{2}$, . . are finite and determinate,

$$
u=f(x+h)=f(x)+f^{\prime}(x) h+f^{\prime \prime}(x) \frac{h^{2}}{2}+f^{\prime \prime \prime}(x) \frac{h^{8}}{1.2 .3}+\cdots,
$$

where $f(x), f^{\prime}(x), f^{\prime \prime}(x), \ldots$ are the values of $f(x+h), d u / d x, d^{2} u / d x^{2}, \ldots$ when $h=0$. This is Taylor's series or theorem. The remainder after the first $n$ terms in $h$ is expressed by the definite integral

$$
\frac{1}{\text { 1. } 2 \cdot 3 \cdots n} \int_{0}^{h} f^{n+1}(x+h-z) z^{n} d z
$$

## b. Maclaurin's series.

If in Taylor's series we make $x=0$, and $h=x$, the result is

$$
u=f(x)=f(0)+f^{\prime}(0) x+f^{\prime \prime}(0) \frac{x^{2}}{1.2}+f^{\prime \prime \prime}(0) \frac{x^{8}}{1.2 .3}+\ldots
$$

where $f(0), f^{\prime}(0), f^{\prime \prime}(0), \ldots$ are the values of $f(x), d u / d x, d^{2} u / d x^{2}, \ldots$ when $x=0$. This is Maclaurin's series or theorem. The remainder after the first $n$ terms in $x$ is expressed by the definite integral

$$
\begin{gathered}
\frac{\mathbf{1}}{\mathbf{1} \cdot 2 \cdot 3 \cdots n} \int_{0}^{x} f^{n+1}(x-z) z^{n} d z \\
\text { c. Example of Taylor's series. } \\
u=f(x+h)=\log (x+h) \\
f(x)=\log x \\
\frac{d u}{d x}=\frac{\mathbf{1}}{x+h}, \quad f^{\prime}(x)=+x^{-1} \\
\frac{d^{2} u}{d x^{2}}=-\frac{1}{(x+h)^{2}}, \quad f^{\prime \prime}(x)=-x^{-2} \\
\frac{d^{8} u}{d x^{8}}=+\frac{2}{(x+h)^{3}}, \quad f^{\prime \prime \prime}(x)=+2 x^{-8}
\end{gathered}
$$

Hence for common logarithms, $\mu$ being the modulus,

$$
\log (x+h)=\log x+\mu\left(x^{-1} h-\frac{1}{2} x^{-2} h^{2}+\frac{1}{3} x^{-3} h^{8}-\ldots\right)
$$

and the sum of the remaining terms is

$$
-\frac{\mu}{1 \cdot 2 \cdot 3} \int_{0}^{h} \frac{2 \cdot 3}{(x+h-z)^{4}} z^{8} d z
$$

Since $x$ is the least value of $(x+h-z)$ within the limits of this integral, the sum of the remaining terms is negative, and numerically

$$
<\frac{1}{4} \mu\left(\frac{h}{x}\right)^{4} .
$$

If, for example, $(h / x)=1 / 100$, the remainder in question is less than $4 \times 0.434 \times 10^{-8}$, or about one unit in the ninth place of decimals.
d. Example of Maclaurin's series.

$$
\begin{array}{cl}
u=f(x)=\sin x . \\
& f(0)=0 \\
\frac{d u}{d x}=\cos x, & f^{\prime}(0)=+\mathrm{I}, \\
\frac{d^{2} u}{d x^{2}}=-\sin x, & f^{\prime \prime}(0)=0 \\
\frac{d^{8} u}{d x^{3}}=-\cos x, & f^{\prime \prime \prime}(0)=-\mathrm{I}
\end{array}
$$

Hence

$$
f(x)=\sin x=x-\frac{x^{8}}{1 \cdot 2 \cdot 3}+\frac{x^{6}}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}-\ldots
$$

and the sum of the remaining terms is

$$
-\frac{1}{5!} \int_{0}^{x} \sin (x-z) z^{3} d z
$$

If $g$ is the greatest value of $\sin (x-z)$ within the limits of this integral the remainder in question is negative and numerically

$$
<\frac{g}{6} \times \frac{\mathrm{x}}{5!} x^{6}
$$

If, for example, $x=\pi / 6$ (the arc of $30^{\circ}$ ), $g=\frac{1}{2}$, and the remainder is numerically less than $0.0000{ }^{4} 43$.
7. Elementary Formulas for Integration.

$$
\begin{gathered}
\text { a. Indefinite integrals. } \\
\int a d x=a \int d x=a x+C . \\
\int f(x) d x+\int \phi(x) d x=\int\{f(x)+\phi(x)\} d x
\end{gathered}
$$

If $x=\phi(y)$, and $d x=\phi^{\prime}(y) d y$,

$$
\begin{aligned}
& \int f(x) d x=\int f\{\phi(y)\} \phi^{\prime}(y) d y \\
& \frac{d}{d y} \int f(x ; y) d x=\int \frac{d f(x, y)}{d y} d x .
\end{aligned}
$$

Since $d(u v)=u d v+v d u$,

$$
\int u d v=u v-\int v d u ; \text { and }
$$

if $u=f(x)$ and $v=\phi(x)$,

$$
\begin{aligned}
& \int f(x) \frac{d \phi(x)}{d x} d x=f(x) \phi(x)-\int \phi(x) \frac{d f(x)}{d x} d x \text {. }^{*} \\
& \int d x \int f(x, y) d y=\int d y \int f(x, y) d x . \\
& \int d x \int f(x) d x=x \int f(x) d x-\int x f(x) d x . \\
& \int x^{n} d x=\frac{1}{n+1} x^{n-1}+C . \\
& \int \frac{d x}{x^{n}}=-\frac{1}{n-1} x^{-(n-1)}+C, \quad n>1 . \\
& \int(a+b x)^{n} d x=\frac{(a+b x)^{n+1}}{(n+1) b}+C . \\
& \int \frac{d x}{x}=\log x \dagger+C, \quad \int \frac{d x}{a+b x}=b^{-1} \log (a+b x) \text {. } \\
& \int \frac{d x}{x^{2}}=-\frac{\mathbf{1}}{x}+C, \quad \int \frac{d x}{(a+b x)^{2}}=-\frac{\mathbf{1}}{b(a+b x)}+C . \\
& \int \frac{d x}{1+x^{2}}=\arctan x+C, \quad \int \frac{-d x}{1+x^{2}}=\operatorname{arccot} x+C . \\
& \int \frac{d x}{1-x^{2}}=\frac{1}{2} \log \frac{\mathrm{I}+x}{\mathrm{I}-x}+C, \quad \int \frac{d x}{x^{2}-\mathrm{I}}=\frac{1}{2} \log \frac{x-\mathrm{I}}{x+\mathrm{I}}+C . \\
& \int \frac{d x}{a+b x^{2}}=(a b)^{-\frac{1}{2}} \arctan (b / a)^{\frac{1}{2}} x+C \text {, for } a \text { and } b \text { both positive, } \\
& =(a b)^{-\frac{1}{3}} \operatorname{arccot}(b / a)^{\frac{1}{2}} x+C \text {, for } a \text { and } b \text { both negative, } \\
& =\frac{1}{2}(-a b)^{-\frac{1}{2}} \log \frac{(-a b)^{\frac{1}{2}}-b x}{(-a b)^{\frac{1}{2}}+b x}+C \text {, for } a b \text { negative. } \\
& \int \frac{d x}{a+2 b x+c x^{2}}=\left(a c-b^{2}\right)^{-\frac{2}{2}} \arctan \frac{b+c x}{\left(a c-b^{2}\right)^{\frac{1}{2}}}+C, \text { for } a c-b^{2}>0, \\
& =\frac{1}{2}\left(b^{2}-a c\right)^{-\frac{1}{2}} \log \frac{\left(b^{2}-a c\right)^{\frac{3}{2}}-b-c x}{\left(b^{2}-a c\right)^{\frac{3}{3}}+b+c x}+C \text {, for } b^{2}-a c>0 . \\
& \int\left(a+x^{2}\right)^{\frac{3}{2}} d x=\frac{1}{2} x\left(a+x^{2}\right)^{\frac{1}{2}}+\frac{1}{2} a \log \left\{x+\left(a+x^{2}\right)^{3}\right\}+C . \\
& \int\left(a^{2}-x^{2}\right)^{\frac{1}{2}} d x=\frac{1}{2} x\left(a^{2}-x^{2}\right)^{\frac{1}{2}}+\frac{1}{2} a^{2} \operatorname{arc} \sin \frac{x}{a}+C . \\
& \int(a+b x)^{\frac{3}{3}} d x=\frac{2}{3}(a+b x)^{3} / b+C .
\end{aligned}
$$

* This is the formula for integration by parts.
$\dagger$ Natural logarithms are used in this and the following integrals. For relation of natural to common logarithms see section $\mathbf{I}, \mathrm{g}$.

$$
\begin{aligned}
& \int\left(a+2 b x+c x^{2}\right)^{\frac{1}{2}} d x=\frac{1}{2}(b+c x)\left(a+2 b x+c x^{2}\right)^{3} / c \\
& +\frac{1}{2}\left(a c-b^{2}\right) / c \int\left(a+2 b x+c x^{2}\right)^{-1} d x+C . \\
& \int(a+b x)^{-\frac{3}{2}} d x=2(a+b x)^{3} / b+C . \\
& \int(a+\beta x)(a+b x)^{-\frac{1}{2}} d x=\frac{2}{3}(3 a b-2 a \beta+\beta b x)(a+b x)^{\frac{1}{2}} / b^{a}+C . \\
& \int\left(a^{2}-x^{2}\right)^{-1} d x= \pm \arcsin \frac{x}{a}+C, \\
& =\mp \arccos \frac{x}{a}+C \text {, } \\
& =2 \arctan \left(\frac{a+x}{a-x}\right)^{\frac{1}{2}}+C . \\
& \int\left(a+x^{2}\right)^{-1} d x=\log \left\{x+\left(a+x^{2}\right)^{3}\right\}+C, \\
& =\frac{1}{2} \log \frac{x+\left(a+x^{2}\right)^{\frac{1}{2}}}{x-\left(a+x^{2}\right)^{\frac{1}{2}}}+C . \\
& \int\left(a+2 b x+c x^{2}\right)^{-\frac{1}{2}} d x=\frac{1}{\sqrt{c}} \log \left\{b+c x+\left(a c+b c x+c^{2} x^{2}\right)^{3}\right\}+C \text {, for } c>0, \\
& =-\frac{1}{\sqrt{-c}} \operatorname{arc} \sin \frac{b+c x}{\left(\bar{b}^{2}-a c\right)^{\frac{7}{2}}}+C \text {, for } c<0 . \\
& \int a^{x} d x=a^{x} / \log a+C, \quad \int e^{x} d x=e^{x}+C . \\
& \int \log x d x=x \log x-x+C . \\
& \int(\log x)^{n} x^{-1} d x=\frac{1}{n+1}(\log x)^{n+1}+C . \\
& \int \sin x d x=-\cos x+C, \quad \int \cos x d x=\sin x+C . \\
& \int \sin ^{2} x d x=\frac{1}{2} x-\frac{1}{4} \sin 2 x+C, \quad \int \cos ^{2} x d x=\frac{1}{2} x+\frac{1}{4} \sin 2 x+C \text {. } \\
& \int \tan x d x=-\log \cos x+C, \quad \int \cot x d x=\log \sin x+C . \\
& \int \frac{d x}{\sin x}=\log \tan \frac{1}{2} x+C, \quad \int \frac{d x}{\cos x}=\log \tan \frac{1}{2}\left(x+\frac{1}{2} \pi\right)+C . \\
& \int \frac{d x}{\sin ^{2} x}=-\cot x+C, \quad \int \frac{d x}{\cos ^{2} x}=\tan x+C . \\
& \int e^{a x} \sin b x d x=\frac{a \sin b x-b \cos b x}{a^{2}+b^{2}} e^{a x}+C . \\
& \int e^{a x} \cos b x d x=\frac{a \cos b x+b \sin b x}{a^{2}+b^{2}} e^{a x}+C . \\
& \int \arcsin x d x=x \arcsin x \pm\left(1-x^{2}\right)^{\frac{1}{2}}+C \text {. } \\
& \int \arccos x d x=x \arccos x \mp\left(\mathrm{I}-x^{2}\right)^{2}+C . \\
& \int \arctan x d x=x \arctan x-\frac{1}{2} \log \left(\mathrm{r}+x^{2}\right)+C . \\
& \int \operatorname{arccot} x d x=x \operatorname{arccot} x+\frac{1}{2} \log \left(\mathrm{x}+x^{2}\right)+C \text {. }
\end{aligned}
$$

## b. Definite Integration.

$$
\begin{aligned}
\int_{a}^{n} \phi(x) d x= & \int_{a}^{b} \phi(x) d x+\int_{b}^{c} \phi(x) d x+\ldots \int_{m}^{n} \phi(x) d x \\
& \int_{a}^{b} \phi(x) d x=-\int_{b}^{a} \phi(x) d x . \\
& \int_{0}^{a} \phi(x) d x=
\end{aligned}
$$

If $\phi(x)=\phi(-x)$, an " even function" of $x$,

$$
\int_{0}^{a} \phi(x) d x=\int_{-a}^{0} \phi(x) d x=\frac{1}{2} \int_{-a}^{a} \phi(x) d x
$$

If $\phi(x)=-\phi(-x)$, an " odd function" of $x$,

$$
\int_{-a}^{0} \phi(x) d x=\int_{0}^{a} \phi(x) d x, \text { and } \int_{-a}^{+a} \phi(x) d x=0
$$

If $A$ be the greatest and $B$ the least value of $\phi(x)$ within the limits $a$ and $b$,

$$
A(b-a)>\int_{a}^{b} \phi(x) d x>B(b-a)
$$

a formula useful in determining approximate values of integrals. See, e. g., section 6, d.

$$
\begin{gathered}
\text { If } u=\int_{a}^{b} \phi(x) d x, \\
\frac{d u}{d a}=-\phi(a), \quad \frac{d u}{d b}=\phi(b) . \\
\int_{0}^{\infty} \frac{d x}{\mathbf{I}+x^{2}}=\frac{1}{2} \pi . \\
\int_{0}^{\infty} \frac{d x}{a+x^{2}}=\int_{\mathbf{I}}^{\infty} \frac{d x}{\mathbf{I}+x^{2}}=\frac{1}{4} \pi . \\
\int_{0}^{\infty}=\frac{1}{2} \pi / \sqrt{ }(a b), \quad \int_{0}^{a} \frac{d x}{\sqrt{a^{2}-x^{2}}}=\frac{1}{2} \pi .
\end{gathered}
$$

$$
\begin{gathered}
\int_{0}^{\infty} e^{-x^{2}} d x=\frac{1}{2} \sqrt{\pi}, \quad \int_{0}^{\infty} e^{-a^{2} x^{2}} d x=\frac{1}{2} \sqrt{ }\left(\pi / a^{2}\right) \\
\int_{0}^{\infty} e^{-a^{2} x^{2}} x^{2 n} d x=1 \cdot 3 \cdot 5 \cdots(2 n-1) a^{-n}(2 a)^{-(n+1)} \sqrt{n} x_{1} \\
\int_{0}^{\infty} e^{-a x} x^{-1} d x=\sqrt{ }(\pi / a) \\
\int_{0}^{\pi} \sin m x \sin n x d x=\int_{0}^{\pi} \cos m x \cos n x d x=0
\end{gathered}
$$

when $m$ and $n$ are unequal integers.
$\int_{0}^{\pi} \sin m x \cos n x d x=\frac{2 m}{m^{2}-n^{2}}$, for $m$ and $n$ integers and $m-n$ odd,
$=0$, for $m$ and $n$ integers and $m-n$ even.

$$
\begin{gathered}
\int_{0}^{\pi} \sin ^{2} m x d x=\int_{0}^{\pi} \cos ^{2} m x d x=\frac{1}{2} \pi, \text { for } m \text { an integer. } \\
\int_{0}^{\frac{1}{2} \pi} \sin ^{n} x d x=\int_{0}^{\frac{1}{2} \pi} \cos ^{n} x d x=\int_{0}^{1}\left(1-x^{2}\right)^{\frac{1}{2}(n-1)} d x \\
\int_{0}^{\infty} \frac{\sin x}{\sqrt{x}} d x=\int_{0}^{\infty} \frac{\cos x}{\sqrt{x}} d x=\sqrt{ }(\pi / 2) \\
\int_{0}^{\infty} \sin x^{2} d x=\int_{0}^{\infty} \cos x^{2} d x=\frac{1}{2} \sqrt{ }(\pi / 2) \\
\int_{0}^{\infty} e^{-a^{2} x^{2}} \cos 2 b x d x=\frac{1}{2} e^{-(b / a)^{2}} \sqrt{ }(\pi / a) \\
\int_{0}^{\infty} e^{-a^{2} x^{2}} \sin 2 b x d x=0 .
\end{gathered}
$$

## MENSURATION.

## r. Lines.

a. In a circle.
$r=$ radius of circle,
$c=$ length of any chord,
$s=$ arc subtended by $c$,
$a=$ angle corresponding to $s$,
$h=$ height of arc $s$ above $c$, or perpendicular distance from middle point of are to chord.

$$
\begin{aligned}
& \text { Circumference }=2 \pi r \text {, } \\
& \pi=3.14159265, \quad \log \pi=0.49714987, \\
& 2 \pi=6.28318531, \quad \log 2 \pi=0.79817987 . \\
& c=2 r \sin \frac{1}{2} a, \quad s=r a .
\end{aligned}
$$

Length of perpendicular from center on chord

$$
\begin{aligned}
& =r \cos \frac{1}{2} a \\
& =\left(r^{2}-\frac{1}{4} c^{2}\right)^{\frac{3}{2}} \\
& =r\left\{1-\frac{1}{2}\left(\frac{c}{2 r}\right)^{2}-\frac{1}{8}\left(\frac{c}{2 r}\right)^{4}-\frac{1}{16}\left(\frac{c}{2 r}\right)^{0}-\ldots\right\} \text {. } \\
& h=r\left(1-\cos \frac{1}{2} a\right) \\
& =2 r \sin ^{2} \frac{1}{4} a \\
& =r-\left(r^{2}-\frac{1}{4} c^{2}\right)^{\frac{1}{2}} \\
& =\frac{1}{8} r\left\{\left(\frac{c}{r}\right)^{2}+\frac{1}{16}\left(\frac{c}{r}\right)^{4}+{ }^{\frac{1}{2} \frac{18}{8}}\left(\frac{c}{r}\right)^{8}+\ldots\right\} . \\
& s-c=\frac{1}{24} s\left(a^{2}-\frac{1}{80} a^{4}+\ldots\right) \\
& =\frac{8}{3} \frac{h^{2}}{s}\left\{\mathrm{I}+\frac{28}{1}\left(\frac{h}{s}\right)^{2}+\ldots\right\} . \\
& a=8\left\{\frac{h}{s}+\frac{4}{3}\left(\frac{h}{s}\right)^{8}+\ldots\right\} \text {. } \\
& \text { b. In regular polygon. } \\
& r=\text { radius of inscribed circle, } \\
& R=\text { radius of circumscribed circle, } \\
& n=\text { number of sides, } \\
& s=\text { length of any side, } \\
& \beta=\text { angle subtended by } s \text {, } \\
& p=\text { perimeter of polygon. }
\end{aligned}
$$

$$
\begin{aligned}
& \beta=360^{\circ} / n, \\
& s=2 r \tan \frac{1}{2} \beta=2 R \sin \frac{1}{2} \beta, \\
& p=n s=2 n r \tan \frac{1}{2} \beta=2 n R \sin \frac{1}{2} \beta .
\end{aligned}
$$

See table under c, below.
c. In ellipse.

$$
\begin{aligned}
a & =\text { semi-axis major }, \\
b & =\text { semi-axis minor, } \\
e & =\text { eccentricity }=\left(1-b^{2} / z^{2}\right)^{\frac{1}{2}}, \\
P & =\text { perimeter of ellipse }, \\
n & =(a-b) /(a+b) \\
& =\frac{1-\sqrt{1-e^{2}}}{1+\sqrt{1-e^{2}}}=\frac{e^{2}}{4}+\frac{e^{4}}{8}+\frac{5 e^{6}}{64}+\cdots \cdots
\end{aligned}
$$

Distance from centre to focus $=a e$,
Distance from focus to extremity of major axis $=a(\mathrm{x}-e)$,
Distance from focus to extremity of minor axis $=a$.
$P=\pi(a+b)\left(1+\frac{1}{4} n^{2}+\frac{1}{64} n^{4}+\frac{1}{\frac{1}{56}} n^{6}+\ldots\right)$
$=\pi(a+b) q$, say, where $q$ stands for the series in $n$. The values of $q$ corresponding to a few values of $n$ are :-

| $n$ | $q$ | $n$ | $q$ |
| :---: | :---: | :---: | :---: |
| 0 | 1.0000 | 0.5 | 1.0635 |
| 0.1 | 1.0025 | 0.6 | 1.0922 |
| 0.2 | 1.0100 | 0.7 | 1.1267 |
| 0.3 | 1.0226 | 0.8 | 1.1677 |
| 0.4 | 1.0404 | 0.9 | 1.2155 |
|  |  | 1.0 | 1.2732 |

## 2. Areas.

a. Area of plane triangle.
(See table on p. xix.)
b. Area of Trapezoid.
$b_{1}=$ upper base of trapezoid,
$b_{2}=$ lower base of trapezoid,
$a=$ altitude of trapezoid, or perpendicular distance between bases.

$$
\text { Area }=\frac{1}{2}\left(b_{1}+b_{2}\right) a .
$$

## c. Area of regular polygon.

$$
\begin{aligned}
& A=\text { area, } \\
& r, R=\text { radii of inscribed and circumscribed circles, } \\
& s=\text { length of any side } \\
& n=\text { number of sides, } \\
& \beta=\text { angle subtended by } s=360^{\circ} / n . \\
& A=n r^{2} \tan \frac{1}{2} \beta=\frac{1}{2} n R^{2} \sin \beta=\frac{1}{4} n s^{2} \cot \frac{1}{2} \beta . \\
& \quad \text { TaBLE OF VALUES. }
\end{aligned}
$$

| $n$ | $\beta$ | A | A | $R$ | $s$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $120^{\circ}$ | $0.4330 s^{2}$ | $1.2990 R^{2}$ | 0.5774 S | 1.7321 $R$ |
| 4 | 90 | 1.0000 | 2.0000 | 0.7071 | 1.4142 |
| 5 | $7{ }^{2}$ | 1.7205 | 2.3776 | 0.8507 | 1.1756 |
| 6 | 60 | 2.5981 | 2.5981 | 1.0000 | 1.0000 |
| 7 | $51 \frac{3}{7}$ | 3.6339 | 2.7364 | 1.1524 | 0.8678 |
| 8 | 45 | 5.8284 | 2.8284 | 1.3066 | 0.7654 |
| 9 | 40 | 6.1818 | 2.8925 | 1.4619 | 0.6840 |
| 10 | 36 | 7.6942 | 2.9389 | 1.6180 | 0.6180 |
| II | $32^{1} \mathrm{I}^{8}$ | 9.3656 | 2.9735 | 1.7747 | 0.5635 |
| 12 | 30 | 11.1962 | 3.0000 | 1.9319 | 0.5176 |
| 13 | $288{ }_{13}$ | 13.1858 | 3.0207 | 2.0893 | 0.4786 |
| 14 | 25 岩 | 15.3345 | 3.0372 | 2.2470 | 0.4450 |
| 15 | 24 | 17.6424 | 3.0505 | 2.4049 | 0.4158 |
| 16 | 22 $\frac{1}{2}$ | 20.1094 | 3.0615 | 2.5629 | 0.3902 |

d. Area of circle, circular annulus, etc.
$r=$ radius of circle,
$d=$ diameter,
$a=$ angle of any sector,
$r_{1}, r_{2}=$ smaller and greater radii of an annulus.
Area of circle $=\pi r^{2}=\frac{1}{4} \pi d^{2}$,
$\pi=3.14159265, \quad \log \pi=0.49714987$.
Area of sector $=a r^{2}$, for $\alpha$ in arc,
$=\pi r^{2}(\alpha / 360)$, for $a$ in degrees.
Area of annulus $=\pi\left(r_{2}^{2}-r_{1}^{2}\right)$.
e. Area of ellipse.
$a, b=$ semi axes respectively
$e=$ eccentricity $=\left(a^{2}-b^{2}\right)^{\frac{1}{2}} / a$
$=\{(a+b)(a-b)\}^{\frac{1}{2}} / a$.

$$
\begin{aligned}
\text { Area of ellipse } & =\pi a b \\
& =\pi a^{2} \sqrt{1-e^{2}} \\
& =\pi a^{2} \cos \phi, \text { if } e=\sin \phi
\end{aligned}
$$

## f. Surface of sphere, etc.

$r=$ radius of sphere,
$\phi_{1}, \phi_{2}=$ latitudes of parallels bounding a zone,
$\epsilon=$ spherical excess of a spherical triangle
$=$ sum of spherical angles less $180^{\circ}$,

$$
\text { Total surface }=4 \pi r^{2}
$$

Surface of zone $=2 \pi r^{2}\left(\sin \phi_{2}-\sin \phi_{1}\right)$,
$=4 \pi r^{2} \cos \frac{1}{2}\left(\phi_{2}+\phi_{1}\right) \sin \frac{1}{2}\left(\phi_{2}-\phi_{1}\right)$.
Surface of spherical triangle $=\gamma^{2} \epsilon$, for $\epsilon$ in arc,

$$
=r^{2} \epsilon / \rho^{\prime \prime}, \text { for } \epsilon \text { in seconas, }
$$

$\rho^{\prime \prime}=206264.8^{\prime \prime}, \quad \log \rho^{\prime \prime}=5 \cdot 3 \mathrm{r} 4425 \mathrm{r} 3$.
g. Surface of right cylinder.
$r=$ radius of bases of cylinder,
$h=$ altitude of cylinder.
Area cylindrical surface $=2 \pi r h$.
Total surface $=2 \pi r(r+h)$.
h. Surface of right cone.
$r=$ radius of base, $h=$ altitude, $s=$ slant height.

Conical surface $=\pi r s=\pi r\left(h^{2}+r^{2}\right)^{\frac{1}{2}}$,
Total surface $=\pi r(s+r)$.
i. Surface of spheroid.
$a, b=$ semi axes,
$e=$ eccentricity $=\{(a+b)(a-b)\}^{\frac{1}{2}} / a$.
Surface of oblate spheroid $=2 \pi a^{2}\left\{\mathrm{I}+\frac{\mathrm{I}-e^{2}}{2 e} \log \left(\frac{\mathrm{I}+e}{1-e}\right)\right\}^{*}$

$$
=4 \pi a^{2}\left(\mathrm{x}-\frac{1}{3} e^{2}-\frac{1}{15} e^{4}-\frac{1}{35} e^{6}-\ldots\right) .
$$

Surface of prolate spheroid $=2 \pi a b\left\{\left(\mathrm{r}-e^{2}\right)^{\frac{1}{2}}+\frac{\arcsin e}{e}\right\}$

$$
=4 \pi a b\left(\mathrm{I}-\frac{1}{6} e^{2}-\frac{1}{40} e^{4}-\mathrm{Tt} e^{6}-\ldots\right)
$$

* The logarithm in this formula refers to the natural or "Napierian" system. For areas of zones and quadrilaterals of an oblate spheroid, see pp. 1-lii.


## 3. Volumes.

a. Volume of prism.

$$
\begin{gathered}
A=\text { area of base, } \quad h=\text { altitude, } \quad V=\text { volume. } \\
V=A h .
\end{gathered}
$$

For an oblique triangular prism whose edges $a, b, c$ are inclined at an angle $a$. to the base,

$$
V=\frac{1}{3}(a+b+c) A \sin a .
$$

## b. Volume of pyramid.

$$
\begin{gathered}
A=\text { area of base, } \begin{array}{c}
h=\text { altitude, } \quad V=\text { volume } \\
V=\frac{1}{3} A h .
\end{array}
\end{gathered}
$$

For a truncated pyramid whose parallel upper and lower bases nave areas $\boldsymbol{A}_{1}$ and $\boldsymbol{A}_{2}$ respectively and whose distance apart is $h$,

$$
V=\frac{\frac{1}{3}}{3} h\left(A_{2}+\sqrt{A_{2} A_{1}}+A_{1}\right) .
$$

The volume of a wedge and obelisk may be expressed by means of the volumes of pyramids and prisms.

$$
\begin{aligned}
& \text { c. Volume of right circular cylinder. } \\
& r=\text { radius of base, } \quad h=\text { altitude, } \quad V=\text { volume. } \\
& \qquad V=\pi r^{2} h . \\
& \pi=3.14559265, \quad \log \pi=0.49714987 .
\end{aligned}
$$

For an obliquely truncated cylinder (having a circular base) whose shortest and longest elements are $h_{1}$ and $h_{2}$ respectively,

$$
V=\frac{1}{2} \pi r^{2}\left(h_{2}+h_{1}\right) .
$$

For a hollow cylinder the radii of whose inner and outer surfaces are $r_{1}$ and $r_{2}$ respectively, and whose altitude is $h$,

$$
V=\pi h\left(r_{2}^{\frac{2}{2}}-r_{1}^{2}\right)
$$

d. Volume of right cone with circular base.
$r=$ radius of base,$\quad h=$ altitude, $\quad V=$ volume.

$$
V=\frac{1}{3} \pi r^{2} h
$$

For a right truncated cone the radii of whose upper and lower parallel bases are $r_{1}$ and $r_{2}$ respectively, and whose altitude is $h$,

$$
V=\frac{1}{3} \pi h\left(r_{2}^{2}+r_{2} r_{1}+r_{4}^{2}\right) .
$$

e. Volume of sphere and spherical segments.

$$
r=\text { radius of sphere, } \quad h=\text { altitude of segment, } \quad V=\text { volume }
$$

For the entire sphere

$$
\begin{aligned}
& V=\frac{S_{3}}{3} \pi r^{8}=4.1888 r^{8} \text { approximately. } \\
& \text { (For } \pi \text { and } \log \pi \text { see } c \text { above.) }
\end{aligned}
$$

For a spherical segment of height $h$

$$
V=\pi h^{2}\left(r-\frac{1}{3} h\right) .
$$

For a zone, or difference in volume of two segments whose altitudes are $h_{1}$ and $h_{2}$ respectively

$$
\begin{aligned}
V & =\pi r\left(h_{2}^{2}-h_{1}^{2}\right)-\frac{1}{3} \pi\left(h_{2}^{3}-h_{1}^{3}\right) \\
& =\frac{1}{6} \pi \Delta h\left(3 r_{2}^{2}+3 r_{1}^{2}+\Delta h^{2}\right),
\end{aligned}
$$

where $r_{1}$ and $r_{2}$ are the radii of the bases of the zone and $\Delta h=h_{2}-h_{b}$

$$
\begin{gathered}
\text { f. Volume of ellipsoid. } \\
a, b, c=\text { semi axes, } V=\text { volume. } \\
V=\frac{4}{3} \pi a b c .
\end{gathered}
$$

For an ellipsoid of revolution about
the $a$-axis, $V=\frac{4}{3} \pi a b^{2}$, the $b$-axis, $V=\frac{4}{3} \pi a^{2} b_{0}$

## UNITS.

## i. Standards of Length and Mass.

The only systems of units used extensively at the present day are the British and metric. The fundamental units in these systems are those of time, length, and mass. From these all other units are derived. The unit of time, the mean solar second, is common to both systems.

The standard unit of length in the British system is the Imperial Yard, which is defined to be the distance between two marks on a metallic bar, kept in the Tower of London, when the temperature of the bar is $62^{\circ} \mathrm{F}$.

The standard unit of mass in the British system is the Imperial Pound Avoirdupois. It is a cylindrical mass of platinum marked "P. S. 1844, I lb.," preserved in the office of the Exchequer at Westminster.

In the metric system the standard unit of length is the Metre, now represented by numerous platinum iridium Prototypes prepared by the International Bureau of Weights and Measures.

The standard of mass in the metric system is the Kilogramme, now represented by numerous platinum iridium Prototypes prepared by the International Bureau of Weights and Measures.

Both systems of units have been legalized by the United States. Virtually, however, the material standards of length and mass of the United States are cerain Prototype Metres and certain Prototype Kilogrammes. The present status of the two systems of units so far as it relates to the United States is set forth in the following statement from the Superintendent of Standard Weights and Measures, bearing the date April 5, 1893.

## Fundamental Standards of Length and Mass.*

"While the Constitution of the United States authorizes Congress to 'fix the itandard of weights and measures,' this power has never been definitely exercised, and but little legislation has been enacted upon the subject. Washington regarded the matter of sufficient importance to justify a special reference to it in his first annual message to Congress (January, i790), and Jefferson, while Secretary of State, prepared a report at the request of the House of Representatives, in which he proposed (July, i790) 'to reduce every branch to the decimal ratio already established for coins, and thus bring the calculation of the principal affairs of life within the arithmetic of every man who can multiply and divide.' The consideration of the subject being again urged by Washington, a committee

[^1]of Congress reported in favor of Jefferson's plan, but no legislation followed. In the mean time the executive branch of the Government found it necessary to procure standards for use in the collection of revenue and other operations in which weights and measures were required, and the Troughton 82 -inch brass scale was obtained for the Coast and Geodetic Survey in 1814, a platinum kilogramme and metre, by Gallatin, in 1821, and a Troy pound from London in 1827 , also by Gallatin. In 1828 the latter was, by act of Congress, made the standard of mass for the Mint of the United States, and although totally unfit for such purpose it has since remained the standard for coinage purposes.
"In 1830 the Secretary of the Treasury was directed to cause a comparison to be made of the standards of weight and measure used at the principal customhouses, as a result of which large discrepancies were disclosed in the weights and measures in use. The Treasury Department, being obliged to execute the constitutional provision that all duties, imposts, and excises shall be uniform throughout the United States, adopted the Troughton scale as the standard of length; the avoirdupois pound to be derived from the Troy pound of the Mint as the unit of mass. At the same time the Department adopted the wine gallon of 231 cubic inches for liquid measure and the Winchester bushel of 2150.42 cubic inches for dry measure. In 1836 the Secretary of the Treasury was authorized to cause a complete set of all weights and measures, adopted as standards by the Department for the use of custom-houses and for other purposes, to be delivered to the Governor of each State in the Union for the use of the States respectively, the object being to encourage uniformity of weights and measures throughout the Union. At this time several States had adopted standards differing from those used in the Treasury Department, but after a time these were rejected, and finally nearly all the States formally adopted by act of legislature the standards which had been put in their hands by the National Government. Thus a good degree of uniformity was secured, although Congress had not adopted a standard of mass or of length other than for coinage purposes as already described.
"The next and in many respects the most important legislation upon the subject was the Act of July 28, 1866, making the use of the metric system lawful throughout the United States, and defining the weights and measures in common use in terms of the units of this system. This was the first general legislation upon the subject, and the metric system was thus the first, and thus far the only system made generally legal throughout the country.
"In 1875 an International Metric Convention was agreed upon by seventeeri governments, including the United States, at which it was undertaken to establish and maintain at common expense a permanent International Bureau of Weights and Measures, the first object of which should be the preparation of a new international standard metre and a new international standard kilogramme, copies of which should be made for distribution among the contributing governments. Since the organization of the Bureau, the United States has regularly contributed to its support, and in 1889 the copies of the new international prototypes were ready for distribution. This was effected by lot, and the United States received metres Nos. 21 and 27 , and kilogrammes Nos. 4 and 20. The metres and kilogrammes are made from the same material, which is an alloy of platinum with ten per cent of iridium.
"On January 2, 1890, the seals which had been placed on metre No. 27 and kilogramme No. 20, at the International Bureau of Weights and Measures near Paris, were broken in the Cabinet room of the Executive Mansion by the President of the United States, in the presence of the Secretary of State and the Secretary of the Treasury, together with a number of invited guests. They were thus adopted as the National Prototype Metre and Kilogramme.
"The Troughton scale, which in the early part of the century had been tentatively adopted as a standard of length, has long been recognized as quite unsuitable for such use, owing to its faulty construction and the inferiority of its graduation. For many years, in standardizing length measures, recourse to copies of the imperial yard of Great Britain had been necessary, and to the copies of the metre of the archives in the Office of Weights and Measures. The standard of mass originally selected was likewise unfit for use for similar reasons, and had been practically ignored.
"The recent receipt of the very accurate copies of the International Metric Standards, which are constructed in accord with the most advanced conceptions of modern metrology, enables comparisons to be made directly with those standards, as the equations of the National Prototypes are accurately known. It has seemed, therefore, that greater stability in weights and measures, as well as much higher accuracy in their comparison, can be secured by accepting the international prototypes as the fundamental standards of length and mass. It was doubtless the intention of Congress that this should be done when the International Metric Convention was entered into in 1875 ; otherwise there would be nothing gained from the annual contributions to its support which the Government has constantly made. Such action will also have the great advantage of putting us in direct relation in our weights and measures with all civilized nations, most of which have adopted the metric system for exclusive use. The practical effect upon our customary weights and measures is, of course, nothing. The most careful study of the relation of the yard and the metre has failed thus far to show that the relation as defined by Congress in the Act of 1866 is in error. The pound as there defined, in its relation to the kilogramme, differs from the imperial pound of Great Britain by not more than one part in one hundred thousand, an error, if it be so called, which utterly vanishes in comparison with the allowances in all ordinary transactions. Only the most refined scientific research will demand a closer approximation, and in scientific work the kilogramme itself is now universally used, both in this country and in England.*

[^2]Equations.
I yard $=\frac{3^{600}}{3937}$ metre.
I pound avoirdupois $=\frac{1}{2 \cdot 2046}$ kilo.
A more precise value of the English pound avoirdupois is $\frac{1}{2 \cdot 20462}$ kilo., differing from the above by about one part in one hundred thousand, but the equation established by law is sufficiently accurate for all ordinary conversions.
As already stated, in work of high precision the kilogramme is now all but universally used, and no conversion is required.
"In view of these facts, and the absence of any material normal standards of customary weights and measures, the Office of Weights and Measures, with the approval of the Secretary of the Treasury, will in the future regard the International Prototype Metre and Kilogramme as fundamental standards, and the customary units, the yard and the pound, will be derived therefrom in accordance with the Act of July 28, r866. Indeed, this course has been practically forced upon this office for several years, but it is considered desirable to make this formal announcement for the information of all interested in the science of metro'ogy or in measurements of precision.

T. C. Mendenhall, Superintendent of Standard Weights and Measures.

```
"Approved:
    J. G. Carlisle,
        Secretary of the Treasury.
April 5, 1893."
```

No ratios of the yard to the metre and of the pound to the kilogramme have as yet been adopted by international agreement; but precise values of these ratios will doubtless be determined and adopted within a few years by the International Bureau of Weights and Measures. In the mean time, it will suffice for most purposes to use the values of the ratios adopted provisionally by the Office of Standard Weights and Measures of the United States. These values are -

$$
\mathrm{I} \text { yard }=\frac{3600}{89} \text { metres, or } \mathrm{I} \text { merre }=\frac{39}{38} 87 \text { yards, }
$$

1 pound $=\frac{1080}{2000} 0$
These ratios were legalized by Act of Congress in r866. Expressed decimally these values are * -

$$
\begin{gathered}
\mathrm{r} \text { yard }=0.914402 \text { metres, } \quad \mathrm{r} \text { metre }=1.09361 \mathrm{r} \text { yards, } \\
\mathrm{r} \text { pound }=0.45359 \text { kilogrammes, } \quad \mathrm{r} \text { kilogramme }=2.20462 \text { pounds. }
\end{gathered}
$$

The above values of the relations of the standards of the British and Metric systems of units are adopted in this work. Tables I and 2 give the equivalents of multiples of the standard units and also equivalents of multiples of the derived units of surface and volume. These tables are published by the Office of Standard Weights and Measures of the United States, and are here republished by permission of the Superintendent of that Office.

## 2. British Measures and Weights.

## a. Linear measures.

The unit of linear measure is the yard. Its principal sub-multiples and multiples are the inch ; the foot ; the rod, perch, or pole ; the furlong; and the mile. The following table exhibits the relations among these measures:-

[^3]| Inches. | Feet. | Yards. | Rods. | Furlongs. | Miles. |
| ---: | :---: | :---: | :---: | :---: | :--- |
| 1 | 0.083 | 0.028 | 0.00505 | 0.00012626 | 0.0000157828 |
| 12 | 1. | 0.333 | 0.06060 | 0.00151515 | 0.00018939 |
| 36 | 3. | 1. | 0.1818 | 0.004545 | 0.00056818 |
| 198 | 16.5 | 5.5 | 1. | 0.025 | 0.003125 |
| 7920 | 660. | 220. | 40. | 1. | 0.125 |
| 63360 | 5280. | 1760. | 320. | 8. | 1. |

Other measures are the -
Surveyor's or Gunter's chain $=4$ rods $=66$ feet $=100$ links of 7.92 inches each.

Fathom $=6$ feet; Cable length $=120$ fathoms.
Hand $=4$ inches ; Palm $=3$ inches; Span $=9$ inches.

## b. Surface or square measures.

The unit of square measure is the square yard. Its relations to the principal derived units in use are shown in the following table:-

| Sq. feet. | Sq. yards. | Sq. rods. | Roods. | Acres. | Sq. miles. |
| :---: | :---: | :---: | :---: | :--- | :--- |
|  | 1. | 0.1 III | 0.00367309 | 0.000091827 | 0.000022957 |
| 9. | I. | 0.0330579 | 0.000826448 | 0.000206612 |  |
| 27.25 | 30.25 | I. | 0.025 | 0.00625 |  |
| 10890. | 1210. | 40. | I. | 0.25 |  |
| 43560 | 4840. | 160. | 4. | I. |  |
| 27878400 | 3097600. | 102400. | 2560. | 640. | I. |

## c. Measures of capacity.

The unit of capacity for dry measure is the bushel ( 2150.4 cubic inches about). The units of capacity for liquid measure are the British gallon (of 277.3 cubic inches about) and the wine gallon (of 23 r cubic inches, nominally). The latter gallon is most commonly used in the United States. The following table shows the relations of the sub-multiples and multiples of the bushel and gallon :-

| Dry Measures. |  |  | Liquids. |  |
| :---: | :---: | :---: | :---: | :---: |
| Pint | $=\frac{1}{64}$ | shel. | Gill | $=\frac{1}{32}$ gall. |
| Quart $=2$ pints | $=\frac{1}{32}$ | " | Pint $=4$ gills | $=\frac{1}{8}$ " |
| Peck $=8$ quarts | $=\frac{1}{4}$ | " | Quart $=2$ pints | = 4 " |
| Bushel $=4$ pecks | $=1$ |  | Gallon $=4$ quarts | $=1$ |
|  |  |  | Barrel $=31 \frac{1}{2}$ gallons | $=31 \frac{1}{2}$ |
|  |  |  | Hhd. $=2$ barrels | $=63$ " |

Besides the above measures of capacity the following volumetric units are used: —

Cubic foot $=1728$ cubic inches.
Cubic yard $=27$ cubic feet $=46656$ cubic inches.
Board-measure foot $=1$ square foot $X \mathrm{r}$ inch thickness $=144$ cubic inches.
Perch (of masonry) $=$ r perch ( 16.5 feet) length $X$ I foot height $X$ r. 5 feet thickness $=\mathbf{2 4 . 7 5}$ cubic feet; $\mathbf{2 5}$ cubic feet are commonly called a perch for convenience.

Cord (of wood) $=8$ feet length $\times 4$ feet breadth $\times 4$ feet height. $=128$ cubic feet.

## d. Measures of weight.

The unit of weight is the avoirdupois pound. One 7000th part of this is called a grain, and 5760 such grains make the troy pound. The sub-multiples and multiples of these two pounds are exhibited in the following table:-

| Avoirdupois. | Troy. |
| :---: | :---: |
| Dram $\quad={ }_{2}^{\frac{1}{5} 6} \mathrm{lb}$. | Grain $\quad={ }_{57}{ }^{\frac{1}{6} 6 \mathrm{l}} \mathrm{l}$. |
| Ounce $\quad=16 \mathrm{drs} .=\frac{1}{16}{ }^{\prime}$ | Pennyweight $=24 \mathrm{grs} .=\frac{1}{24}{ }^{\frac{1}{0}}{ }^{\prime}$ |
| Pound $=16 \mathrm{ozs} .=1{ }^{\prime}$ | Ounce $\quad=20 \mathrm{dwt}={ }_{1}^{12}{ }^{\frac{1}{2}}$ |
| Quarter $=28 \mathrm{lbs} .=28{ }^{\prime}$ | Pound $=12$ ozs. $=1{ }^{\prime}$ |
| Hundred-wt. $=4$ qrs. $=112{ }^{\text {a }}$ |  |
| Longton $=20 \mathrm{cwt}$. $=2240{ }^{\circ}$ |  |
| Short ton $=2000{ }^{\prime}$ |  |

## 3. Metric Measures and Weights.

As explained in section r above, the standards of length and mass in the metric system are the metre and the kilogramme. Two material representatives of each of these standards are possessed by the United States and preserved at the Office of Standard Weights and Measures at Washington, D. C.
The standards of length are Prototype Metres Nos. 21 and 27. These are platinum iridium bars of X cross section, and their lengths are defined by lines ruled on their neutral surfaces. Their lengths at any temperature $t$ Centigrade are given by the following equations:-

$$
\begin{aligned}
& \text { Prototype No. } 2 \mathrm{I}=\mathrm{r}^{m}+{ }^{2} \mu_{5}+8{ }^{\mu} 665 t+0{ }^{\mu} 00100 t^{2}, \\
& \text { Prototype No. } 27=\mathrm{r}^{m}-1^{\mu} 6+8 . \mu^{\mu} 67 t+0{ }^{\mu} 00100 t^{2},
\end{aligned}
$$

where the symbol $\mu$ stands for one micron, or one millionth of a metre. The probable errors of these Prototypes may be taken as not exceeding $\pm 0 . \mu_{\mu_{2}}$, or $1 / 5000$ oooth of a metre for temperatures between $0^{\circ}$ and $30^{\circ} \mathrm{C}$.

The standards of mass are Prototype Kilogrammes Nos. 4 and 20. They are cylindrical masses of platinum iridium. Their masses and volumes are given by the following equations :-

where the -
Symbol kg stands for one kilogramme, Symbol $m g$ stands for one milligramme $=0 .{ }^{k j 000001}$, Symbol $m l$ stands for one millilitre $=$ one cubic centimetre.

The definitive probable error assigned to the Prototype Kilogrammes by the International Bureau is $\pm 0.0^{m 002}$, or $1 / 500000000$ th of a kilogramme.

The act of Congress approved July 28, 1866, authorizing the use of the metric system in the United States, provides that the tables in a schedule annexed shall be recognized "as establishing, in terms of the weights and measures now in use in the United States, the equivalents of the weights and measures expressed therein in terms of the metric system; and said tables may be lawfully used for computing, determining, and expressing, in customary weights and measures, the weights and measures of the metric system." The following copy of that schedule gives the denominations of the multiples and sub-multiples of the measures of length, surface, capacity, and weight in the metric system as well as their legalized equivalents in British units.

| Schedule annexed to Act of July 28, 1866. Measures of Length. |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Metric Denominations. |  |  |  Values in Metres. <br> $\vdots$ <br> $\vdots$ <br> 10000. <br> 1000. <br> 100. <br> 10. <br> 0 <br> 0.1 <br> 0.1 <br> 0.01 <br> 0.001 <br>   |  | Equivalents in Denominatioas in Use. |  |
| Myriametre <br> Kilometre <br> Dectometre <br> Metre <br> Decimetre <br> Centimetre <br> Millimetre . |  |  |  |  | 6.2137 miles. <br> 0.62137 mile, or 3280 feet and 10 inches. <br> 328 feet and 1 inch. <br> 393.7 inches. <br> 39.37 inches. <br> 3.937 inches. <br> 0.3937 inch. <br> 0.0394 inch. |  |
| Measures of Surface. |  |  |  |  |  |  |
| Metric Denominations. |  |  | Values in Square Metres. |  | Equivalents in Denomioations in Use. |  |
|  |  |  | $\begin{gathered} 10000 \\ 100 \\ 1 \end{gathered}$ |  | 2.475 acres. <br> 119.6 square yards. <br> r550 square inches |  |
| Measures of Capacity. |  |  |  |  |  |  |
| Metric Deaominations and Values. |  |  |  | Equivalents in Denominations in Use. |  |  |
| Names. | $\xrightarrow{\text { No. of }}$ Litres. |  | c Measure. |  | Measure. | Liquid or Wine Measure. |
|  | $\begin{gathered} \text { rooo. } \\ \text { 100. } \\ \text { ro. } \\ \text { 1. } \\ 0.1 \\ 0.01 \\ 0.01 \\ 0.001 \end{gathered}$ | I cubic metre . o.r cubic metre ro cubic decimetres y cubic decimetre. io cubic centimetres. I cubic centimetre |  | 1. 308 cubic yards <br> 2 bus. and 3.35 pks. <br> 9.08 quarts. <br> 0.908 quart. . 6.1022 cubic inches 0.6102 cubic inch o.06I cubic inch |  | $\therefore \quad:$264.17 gallons. <br> 26.417 gallons. <br> 2.647 gallons. <br> r.0567 quarts. <br> 0.845 gill. <br> 0.338 <br> 0.27 <br> 0.27 <br> fluid-ounce. |
| Measures of Weight. |  |  |  |  |  |  |
| Metric Denominations and Values. |  |  |  |  |  | Equivalents in Denominations in Use. |
| Names. | Number of Grammes. |  | Weight of what Quantity of Water at Maximum Density. |  |  | Avoirdupois Weight. |
| Millier or tomeau <br> Quintal <br> Myriagramme <br> Kilogramme, or kilo <br> Hectogramme <br> Decagramme <br> Gramme <br> Decigramme <br> Centigramme <br> Milligramme |  |  | I cubic metre <br> 7 hectolitre <br> to litres <br> 1 litre <br> I decilitre <br> to cubic centimetres <br> I cubic centimetre <br> o.I cubic centimetre <br> I cubic millimetre |  |  | 2204.6 pounds. 220.46 pounds 22.046 pounds. 2.2046 pounds 3.5274 ounces. 0.3527 ounce. 15.432 grains. 0.1543 grain. 0.0154 grain. |

## 4. The C. G. S. System of Units.

The C. G. S. system of units is a metric system in which the fundamental units are the centimetre, the gramme, and the mean solar second. It is the system now generally used for the expression of physical quantities.

The most important of the derived units in the C. G. S. system, their equivalents in terms of ordinary units, and their dimensions in terms of the fundamental units of length, mass, and time, are given in the Appendix to this volume.

For an elaborate consideration of the subject of units and their interrelations the reader may be referred to "Units and Physical Constants," by J. D. Everett, London, Macmillan \& Co., 12 mo , $4^{\text {th }}$ ed., 189 I.

## GEODESY.

## I. Form of the Earth. The Earth's Spherotd. The Geotd.

The shape of the earth is defined essentially by the sea surface, which embraces about three fourths of the entire surface. The sea surface is an equipotential surface due to the attraction of the earth's mass and to the centrifugal force of its rotation. We may imagine this surface to extend through the continents, and thus to be continuous. Its position at any continental point is the height at which water would stand if a canal connected the point with the ocean.
Geodetic measurements show that this surface is represented very closely by an oblate spheroid, whose shorter axis coincides with the rotation axis of the earth. This is called the earth's spheroid. The actual sea surface, on the other hand, is called the geoid. With respect to the spheroid the geoid is a wavy surface lying partly above and partly below; but the extent of the divergence of the two surfaces is probably confined to a few hundred feet.

## 2. Adopted Dimensions of Earth's Spheroid.

The dimensions of the earth's spheroid here adopted are those of General A. R. Clarke, published in 1866, to wit:

Semi major axis, $a=20926062$ English feet.
Semi minor axis, $b=2085512 \mathrm{I}$ " "

## 3. Auxiliary Quantities.

The following quantities are of frequent use in geodetic formulas:-

$$
\begin{gathered}
e=\sqrt{\frac{a^{2}-b^{2}}{a^{2}}}, \text { the eccentricity of generating ellipse, } \\
f=\frac{a-b}{a}, \text { the flattening, ellipticity, or compression, } \\
n=\frac{a-b}{a+b} \\
b=a \sqrt{1-e^{2}}=a(1-f)=a \frac{1-n}{1+n} \\
e^{2}=2 f-f^{2} \\
f=1-\sqrt{1-e^{2}}=\frac{e^{2}}{2}+\frac{e^{4}}{8}+\frac{e^{6}}{16}+\frac{5 e^{8}}{128}+\ldots \\
=\frac{2 n}{1+n}=2\left(n-n^{2}+n^{8}-n^{4}+\ldots\right)
\end{gathered}
$$

$$
\begin{gathered}
n=\frac{f}{2-f}=\left(\frac{1}{2} f\right)+\left(\frac{1}{2} f\right)^{2}+\left(\frac{1}{2} f\right)^{8}+\left(\frac{1}{2} f\right)^{4}+\ldots \\
e^{2}=\frac{4 n}{(1+n)^{2}}=4\left(n-2 n^{2}+3 n^{8}-4 n^{4}+\ldots\right) \\
m=\frac{e^{2}}{2-e^{2}}=\frac{e^{2}}{2}+\frac{e^{4}}{4}+\frac{e^{6}}{8}+\frac{e^{8}}{16}+\ldots \\
n=\frac{1-\sqrt{1-e^{2}}}{1+\sqrt{1-e^{2}}}=\frac{e^{2}}{4}+\frac{e^{4}}{8}+\frac{5 e^{6}}{64}+\frac{7 e^{8}}{128}+\ldots
\end{gathered}
$$

The numerical values of the most useful of these quantities and their logarithms are -

$$
\begin{aligned}
& a=20926062 \text { feet, } \\
& b=20855 \mathrm{I21} \text { feet, } \\
& e^{2}=0.00676866, \\
& m=0.00339583 \\
& n=0.00169792
\end{aligned}
$$

$$
\begin{aligned}
& \log \\
& 7.3206875, \\
& 7.3192127, \\
& 7.8305030-10, \\
& 7.5309454-10, \\
& 7.2299162-10 .
\end{aligned}
$$

## 4. Equations to Generating Ellipse of Spheroid.

With the origin at the centre of the ellipse, and with its axes as coördinate axes, the equation in Cartesian co-ordinates is

$$
\begin{equation*}
\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=\mathbf{x} \tag{I}
\end{equation*}
$$

$a$ and $\delta$ being the major and minor axes respectively, and $x$ and $y$ being parallel to those axes respectively.

For many purposes it is useful to replace equation (r) by the two following : -

$$
\begin{align*}
& x=a \cos \theta \\
& y=b \sin \theta \tag{2}
\end{align*}
$$

which give ( I ) by the elimination of $\theta$. This angle is called the reduced latitude. See section 5 .

## 5. Latitudes used in Geodesy.

Three different latitudes are used in geodesy, namely: (1) Astronomical or geographical latitude; (2) geocentric latitude; (3) reduced latitude. The astronomical latitude of a place is the angle between the normal (or plumb line) at that place and the plane of the earth's equator ; or when the plumb line at the place coincides with the normal to the generating ellipse, it is the angle between that normal and the major axis of the ellipse. The geocentric latitude of a place is the angle between the equator and a line drawn from the place to the earth's centre; or it is the angle between the radius-vector of the place and the equator. The reduced latitude is defined by equations (2) in section 4 above. The geometrical relations of these different latitudes are shown in Fig. 1 by the notation given below.

In order to express the analytical relations between the different İatitudes let


Fig. 1.
$\phi=$ the astronomical latitude,
$\psi=$ the geocentric latitude,
$\theta=$ the reduced latitude.
Then, referring to equations ( 1 ) and (2) under section 4 above, and to Fig. I, it appears that

$$
\tan \phi=-\frac{d x}{d y}=+\frac{a^{2} y}{b^{2} x}
$$

$$
\tan \psi=\frac{y}{x}, \quad \tan \theta=\frac{a y}{b x}
$$

Hence

$$
\begin{aligned}
& \tan \psi=\frac{b^{2}}{a^{2}} \tan \phi=\left(1-e^{2}\right) \tan \phi \\
& \tan \theta=\left(1-e^{2}\right)^{\frac{1}{2}} \tan \phi=\left(1-e^{2}\right)^{-\frac{1}{2}} \tan \psi \\
& \phi-\psi=m \sin 2 \phi-\frac{1}{2} m^{2} \sin 4 \phi+\ldots, \\
& \phi-\theta=n \sin 2 \phi-\frac{1}{2} n^{2} \sin 4 \phi+\ldots .
\end{aligned}
$$

For the adopted spheroid

$$
\log \left(1-e^{2}\right)=9.9970504
$$

and

$$
\begin{aligned}
& \phi-\psi(\text { in seconds })=700 .^{\prime \prime} 44 \sin 2 \phi-1 . .^{\prime \prime} 19 \sin 4 \phi, \\
& \phi-\theta \text { (in seconds) }=350^{\prime \prime} 22 \sin 2 \phi-0 .^{.1} 30 \sin 4 \phi .
\end{aligned}
$$

## 6. Radil of Curvature.

$\rho_{m}=$ radius of curvature of meridian section of spheroid at any point whose latitude is $\phi=P O$, Fig. m ,
$\rho_{n}=$ radius of curvature of normal section perpendicular to the meridian at the same point $=P Q$, Fig. r,
$\rho_{\alpha}=$ radius of curvature of normal section making angle $\alpha$ with the meridian at same point.

$$
\begin{aligned}
\rho_{m} & =a\left(\mathrm{I}-e^{2}\right)\left(\mathrm{I}-e^{2} \sin ^{2} \phi\right)^{-\frac{3}{2}}, \\
\rho_{n} & =a\left(\mathrm{I}-e^{2} \sin ^{2} \phi\right)^{-\frac{1}{2}} \\
\frac{\mathrm{I}}{\rho_{a}} & =\frac{\cos ^{2} a}{\rho_{m}}+\frac{\sin ^{2} \alpha}{\rho_{n}} \\
& =\frac{\mathrm{I}}{a}\left(\mathrm{I}+\frac{e^{2}}{\mathrm{I}-e^{2}} \cos ^{2} \phi \cos ^{2} a\right)\left(\mathrm{I}-e^{2} \sin ^{2} \phi\right)^{\frac{1}{2}} \\
\log \left(\mathrm{I}-e^{2} \sin ^{2} \phi\right)^{-3}= & +\log (\mathrm{I}+n) \\
& \quad-\mu n \cos 2 \phi \\
& +\frac{1}{2} \mu n^{2} \cos 4 \phi \\
& \quad+\frac{1}{3} \mu n^{3} \cos 6 \phi \\
& +\cdots
\end{aligned}
$$

$\mu=$ modulus of common logarithms and $n$ is same as in section 3. For the adopted spheroid -

Radius of curvature of meridian section $\rho_{m}$ in feet.

$$
\begin{aligned}
\log \rho_{m}= & +7.3199482 \\
& -[4.34482] \cos 2 \phi \\
& +[1.274] \cos 4 \phi \\
& -\ldots .
\end{aligned}
$$

Radius of curvature of normal section $p_{n}$ in feet.

$$
\begin{aligned}
\log \rho_{n}= & \text { 十 } 7.32 \text { I } 4243 \\
& -[3.86770] \cos 2 \phi \\
& +[0.797] \cos 4 \phi \\
& -\ldots .
\end{aligned}
$$

The numbers in brackets in these formulas are logarithms to be added to the logarithms of $\cos 2 \phi$ and $\cos 4 \phi$. The numbers corresponding to the sums of these logarithms will be in units of the seventh decimal place of the first constant. Thus, for $\phi=0$,

## 7. Length of Arcs of Meridians and Parallels of Latitude.

## a. Arcs of Meridian.

For the computation of short meridional arcs lying between given parallels of latitude the following simple formulas suffice :

$$
\begin{align*}
\Delta \phi & =\phi_{2}-\phi_{1}, \\
\phi & =\frac{1}{2}\left(\phi_{2}+\phi_{1}\right)_{v}  \tag{I}\\
\Delta M & =\rho_{m} \Delta \phi .
\end{align*}
$$

In these, $\phi_{1}$ and $\phi_{2}$ are the latitudes of the ends of the arc, $\Delta M$ is the required length, and $\rho_{m}$ is the meridian radius of curvature for the latitude $\phi$ of the middle point of the arc. The formula for $\Delta M$ implies that $\Delta \phi$ is expressed in parts of the radius. If $\Delta \phi$ is expressed in seconds, minutes, or degrees of arc, the formula becomes -

Meridional distance $\Delta M$ in feet.

$$
\begin{align*}
\Delta M & =\frac{\rho_{m} \Delta \phi \text { (in seconds) }}{206264.8} \\
& =\frac{\rho_{m} \Delta \phi \text { (in minutes) }}{3437.747}, \\
& =\frac{\rho_{m} \Delta \phi \text { (in degrees) }}{57.2957^{8}} ; \tag{2}
\end{align*}
$$

$$
\log (1 / 206264.8)=4.6855749-10,
$$

$$
\log (1 / 3437.747)=6.463726 I-10,
$$

$$
\log (1 / 57.29578)=8.2418774-10 .
$$

$\phi_{1}, \phi_{2}=$ end latitudes of arc, $\quad \Delta \phi=\phi_{2}-\phi_{1}$,

$$
\begin{aligned}
& \log \rho_{n}=7.3214243 \\
& -7373.9 \\
& +\quad 6.3 \\
& =\overline{7.3206875}=\log a \text {. }
\end{aligned}
$$

The relations (2) will answer most practical purposes when $\Delta \phi$ does not exceed $5^{\circ}$. A comparison with the precise formula (3) below shows in fact that the error of (2) is very nearly

$$
\frac{1}{8} e^{2} \Delta \phi^{2} \cos 2 \phi \cdot \Delta M
$$

which vanishes for $\phi=45^{\circ}$, and which for $\Delta \phi=5^{\circ}$ is at most ${ }_{\text {I }}{ }^{\frac{1}{0} 0 \delta ण} \Delta M$, or about in feet.

Numerical example. Suppose -

$$
\begin{aligned}
& \phi_{2}=37^{\circ} 29^{\prime} 48 .^{\prime \prime}{ }^{\prime} 7 \\
& \phi_{1}=35^{\circ} 48^{\prime} 29 . " 89 .
\end{aligned}
$$

Then

$$
\begin{array}{rlrl}
\phi=\frac{1}{2}\left(\phi_{2}+\phi_{1}\right) & =36^{\circ} 39^{\prime} & 09 .^{\prime \prime} 03 \\
\Delta \phi & =\phi_{2}-\phi_{1} & =\mathrm{I}^{\circ} 4 \mathbf{1}^{\prime} & 18 .^{\prime \prime 2} 2, \\
& = & 6078 .^{\prime \prime} 28 .
\end{array}
$$

From the first of (2)

$$
\begin{array}{cc}
\text { cons't. } \log & 4.6855749-\text { 1о } \\
\text { Table 1o, } \log \rho_{m} & 7.3193112 \\
\log \Delta \phi & 3.7837807 \\
\hline \Delta M=6147 \circ 5 \text { feet, } \log \Delta M & 5.7886668
\end{array}
$$

The values of $\Delta M$ for intervals of $10^{\prime \prime \prime}, 20^{\prime \prime} \ldots 60^{\prime \prime}$, and for $10^{\prime}, 20^{\prime} \ldots 60^{\prime}$ are given in Table 17 for each degree of latitude from $0^{\circ}$ to $90^{\circ}$.

For precise computation of long meridional arcs the following formula is ade-quate:-

$$
\begin{align*}
\Delta M=A_{0} \Delta \phi & -A_{1} \cos 2 \phi \sin \Delta \phi \\
& +A_{2} \cos 4 \phi \sin 2 \Delta \phi \\
& -A_{3} \cos 6 \phi \sin 3 \Delta \phi  \tag{3}\\
& +A_{4} \cos 8 \phi \sin 4 \Delta \phi
\end{align*}
$$

In this, $\Delta M, \phi$, and $\Delta \phi$ have the same meanings as above, and $A_{0}, A_{1}, \ldots$ are functions of $a$ and $e$ or of $a$ and $n$.

Thus, in terms of $a$ and $n$,

$$
\begin{aligned}
& A_{0}=a(\mathrm{I}+n)^{-1}\left(\mathrm{I}+\frac{1}{4} n^{2}+\frac{1}{64} n^{4}+\ldots\right) \\
& A_{1}=3 a(\mathrm{I}+n)^{-1}\left(n-\frac{1}{8} n^{8}-\cdots\right) \\
& A_{2}=\frac{18}{8} a(\mathrm{I}+n)^{-1}\left(n^{2}-\frac{1}{4} n^{4}-\cdots\right) \\
& A_{3}=\frac{35}{24} a(\mathrm{I}+n)^{-1}\left(n^{8}-\ldots\right) \\
& A_{4}=\frac{315}{25} a(\mathrm{I}+n)^{-1}\left(n^{4}-\ldots\right)
\end{aligned}
$$

Introducing the adopted values of $a$ and $n$, these constants become -
$\log$.

$$
\begin{array}{rrl}
A_{0}=20890606 \text { feet, } & 7.3199510 \\
A_{1}= & 1064 \text { II feet, } & 5.0269880, \\
A_{2}= & \text { II3 feet, } & 2.0528, \\
A_{3}= & 0.15 \text { feet, } 9.174-\text { го. }
\end{array}
$$

It appears, therefore, that the first three terms of (3) will give $\Delta M$ with an accuracy considerably surpassing that of the constant $\boldsymbol{A}_{0}$. In the use of (3) it will generally be most convenient to express $\Delta \phi$ in degrees, and in this case $A_{0}$ must be divided by the number of degrees in the radius, viz.: 57.2957795 [r.7581226]. Applying this value and writing the logarithms of $A_{0}, A_{1}$, etc., in rectangular brackets in place of $A_{0}, A_{1}$, etc., (3) becomes

Meridional distance $\Delta M$ in feet.

$$
\begin{align*}
\Delta M & =[5.5618284] \Delta \phi \text { (in degrees) } \\
& -[5.0269880] \cos 2 \phi \sin \Delta \phi  \tag{4}\\
& +[2.0528] \cos 4 \phi \sin 2 \Delta \phi \\
& -\ldots \cdots \\
2 \phi=\phi_{2}+\phi_{1}, \quad & \Delta \phi=\phi_{2}-\phi_{1}, \quad \phi_{1}, \phi_{2}=\text { end latitudes of arc. }
\end{align*}
$$

Formula (4) will suffice for the calculation of any portion or the whole of a quadrant. The length of a quadrant is the value of the first term of (4) when $\phi=45^{\circ}$ and $\Delta \phi=90^{\circ}$, since all of the remaining terms vanish.

Numerical examples. - $\mathrm{r}^{\circ}$. Suppose

$$
\phi_{1}=0^{\circ} \text { and } \phi_{2}=45^{\circ} .
$$

Then

$$
\begin{aligned}
2 \phi & =45^{\circ}, \\
\Delta \phi & =45^{\circ} .
\end{aligned}
$$



The third term of the series vanishes by reason of the factor $\cos 4 \phi=\cos 90^{\circ}$ $=0$. The sum of the first two terms, or length of a meridional arc from the equator to the parallel of $45^{\circ}$, is 16354237 feet.
$2^{\circ}$. Suppose $\quad \phi_{1}=45^{\circ}$ and $\phi_{2}=90^{\circ}$.
Then

$$
\begin{aligned}
2 \phi & =135^{\circ}, \\
\Delta \phi & =45^{\circ} .
\end{aligned}
$$

The numerical values of the terms will be the same as in the previous example, but the sign of the second term will be plus. Hence the length of the meridional arc between the parallel of $45^{\circ}$ and the adjacent pole is 16460649 feet. The sum of these two computed distances, or the length of a quadrant, is 32814886 feet.

This agrees as it should with the length given by (4) when $2 \phi=90^{\circ}$ and $\Delta \phi$ $=90^{\circ}$.*

## b. Arcs of parallel.

The radius of any parallel of latitude is equal to the product of the radius of curvature of the normal section for the same latitude by the cosine of that latitude. That is, see Fig. r, $r$ being the radius of the parallel -

$$
r=\rho_{n} \cos \phi
$$

and the entire length of the parallel is -

$$
2 \pi r=2 \pi \rho_{n} \cos \phi
$$

Designate the portion of a parallel lying between meridians whose longitudes are $\lambda_{1}$ and $\lambda_{2}$ by $\Delta P$, and call the difference of longitude $\lambda_{2}-\lambda_{1}, \Delta \lambda$.

Then -
Arc of parallel $\Delta P$ in feet.

$$
\begin{align*}
\Delta P & =\frac{2 \pi \rho_{n} \cos \phi}{1296000} \Delta \lambda \text { (in seconds) } \\
& =\frac{2 \pi \rho_{n} \cos \phi}{21600} \Delta \lambda \text { (in minutes) }  \tag{I}\\
& =\frac{2 \pi \rho_{n} \cos \phi}{360} \Delta \lambda \text { (in degrees) }
\end{align*}
$$

$\log (2 \pi / 1296000)=4.6855749-10$,
$\log (2 \pi / 21600)=6.4637261-10$,
$\log (2 \pi / 360)=8.2418774-10$.
$\lambda_{1}, \lambda_{2},=$ end longitudes of arc, $\Delta \lambda=\lambda_{2}-\lambda_{1}$,
$\rho_{n}=$ radins of curvature of normal section for latitude of parallel; for $\log \rho_{n}$ see Table in.
Numerical Example. - Suppose $\phi=35^{\circ}$, and $\Delta \lambda=72^{\circ}$. Then from the third of (9)

$$
\begin{array}{rrr} 
& \text { log. } \\
\text { cons't } & 8.24187 \\
\text { Table II, } & \rho_{n} 7.321 \mathrm{I} 7 \\
& \cos \phi 9.91336 \\
\Delta \lambda & \mathrm{I} .85733 \\
\Delta P=2 \mathrm{I} 564827 \text { feet, } & \Delta P & \begin{aligned}
7.33374
\end{aligned} \\
&
\end{array}
$$

* The best formula for computing the entire length of a meridian curve is this:

$$
\pi(a+b)\left(1+\frac{1}{2} n^{2}+\frac{d}{d} n^{4}+\ldots\right),
$$

in which $a, b$, and $n$ are the same as defined in section $z$. For the values here adopted -

|  | log. |
| :--- | :---: |
| $\left(\mathrm{I}+\frac{1}{2} n^{2}+\ldots\right)$ | 0.000003 |
| $(a+b)$ | 7.6209807 |
| $\pi$ | 0.4971499 |
| length | $\mathbf{8 . 1 1 8 1 3 0 9}$ |

The length of the perimeter of the generating ellipse, or the meridian circumference of the earth, is, therefore -

$$
131259550 \text { feet }=24859.76 \text { miles. }
$$

The values of $\Delta P$ for intervals of $10^{\prime \prime}, 20^{\prime \prime} \ldots 60^{\prime \prime}$, and for $10^{\prime}, 20^{\prime} \ldots 60^{\prime}$ are given in Table 18 for each degree of latitude from $0^{\circ}$ to $90^{\circ}$.

## 8. Radius-Vector of Earth's Spheroid.

$$
\begin{aligned}
& \rho=\text { radius-vector } \\
& =\sqrt{x^{2}+y^{2}} \\
& =a\left(\mathrm{r}-2 e^{2} \sin ^{2} \phi+e^{4} \sin ^{2} \phi\right)^{4}\left(\mathrm{r}-e^{2} \sin ^{2} \phi\right)^{-4} . \\
& \log \rho=\log \frac{a\left(2-e^{2}\right)}{1+\sqrt{1-e^{2}}}+\mu(m-n) \cos 2 \phi \\
& -\frac{1}{2} \mu\left(m^{2}-n^{2}\right) \cos 4 \phi \\
& +\frac{1}{3} \mu\left(m^{8}-n^{2}\right) \cos 6 \phi
\end{aligned}
$$

For the adopted spheroid

$$
\begin{aligned}
\log (\rho \text { in feet })=7.3199520 & +[3.86769] \cos 2 \phi \\
& -[1.2737] \cos 4 \phi,
\end{aligned}
$$

the logarithms for the terms in $\phi$ corresponding to units of the seventh decimal place. Thus, for $\phi=0$,

$$
\begin{aligned}
\log \rho= & 7.3199520 \\
& +\quad 7373.8 \\
& = \\
= & 18.8206875=\log a .
\end{aligned}
$$

9. Areas of Zones and Quadrilaterals of the Earth's Surface.

An expression for the area of a zone of the earth's surface or of a quadrilateral bounded by meridians and parallels may be found in the following manner :-

The area of an elementary zone $d Z$, whose middle latitude is $\phi$ and whose width is $\rho_{m} d \phi$, is (see Fig. I),

$$
\begin{aligned}
d Z & =2 \pi r \rho_{m} d \phi \\
& =2 \pi \rho_{m} \rho_{n} \cos \phi d \phi
\end{aligned}
$$

By means of the relations in section 6 this becomes

$$
\begin{align*}
d Z & =2 \pi a^{2}\left(\mathrm{I}-e^{2}\right) \frac{\cos \phi d \phi}{\left(1-e^{2} \sin ^{2} \phi\right)^{2}} \\
& =2 \pi a^{2} \frac{1-e^{2}}{e} \frac{d(e \sin \phi)}{\left(1-e^{2} \sin ^{2} \phi\right)^{2}} \tag{I}
\end{align*}
$$

The integral of this between limits corresponding to $\phi_{1}$ and $\phi_{2}$, or the area of a zone bounded by parallels whose latitudes are $\phi_{1}$ and $\phi_{2}$ respectively, is

$$
Z=\pi a^{2} \frac{\mathrm{I}-e^{2}}{e}\left\{\begin{array}{c}
\frac{e \sin \phi_{2}}{\mathrm{I}-e^{2} \sin ^{2} \phi_{2}}-\frac{e \sin \phi_{1}}{\mathrm{I}-e^{2} \sin ^{2} \phi_{1}}  \tag{2}\\
+\frac{1}{2} \text { Nap. } \log \frac{\left(\mathrm{I}+e \sin \phi_{2}\right)\left(\mathrm{I}-e \sin \phi_{1}\right)}{\left(\mathrm{I}-e \sin \phi_{2}\right)\left(\mathrm{I}+e \sin \phi_{1}\right)}
\end{array}\right\} .
$$

To get the area of the entire surface of the spheroid, make $\phi_{1}=-\frac{1}{2} \pi$ and $\phi_{9}$ $=十 \frac{1}{2} \pi$ in (2). The result is

$$
\begin{equation*}
\text { Surface of spheroid }=2 \pi a^{2}\left[\mathrm{I}+\frac{\mathrm{I}-e^{2}}{2 e} \text { Nap. } \log \left(\frac{\mathrm{I}+e}{\mathrm{I}-e}\right)\right] . \tag{3}
\end{equation*}
$$

For numerical applications it is most advantageous to express (3) in a series of powers of $e$. Thus, by Maclaurin's theorem,

$$
\begin{equation*}
\text { Surface of spheroid }=4 \pi a^{2}\left(1-\frac{e^{2}}{3}-\frac{e^{4}}{15}-\frac{e^{8}}{35}-\ldots\right) . \tag{4}
\end{equation*}
$$

For the calculation of areas of zones and quadrilaterals it is also most advantageous to expand (2) in a series of powers of $e \sin \phi_{1}$ and $e \sin \phi_{2}$ and express the result in terms of multiples of the half sum and half difference of $\phi_{1}$ and $\phi_{2}$. Thus, (2) readily assumes the form

$$
Z=2 \pi a^{2}\left(\mathrm{I}-e^{2}\right)\left[\left(\sin \phi_{2}-\sin \phi_{1}\right)+\frac{2}{3} e^{2}\left(\sin ^{8} \phi_{2}-\sin ^{8} \phi_{1}\right)+\ldots\right] .
$$

From this, by substitution and reduction, there results

$$
Z=2 \pi\left\{\begin{array}{c}
C_{1} \cos \phi \sin \frac{1}{2} \Delta \phi-C_{2} \cos 3 \phi \sin \frac{3}{2} \Delta \phi  \tag{5}\\
+C_{3} \cos 5 \phi \sin \frac{\beta}{2} \Delta \phi-.
\end{array}\right\},
$$

wherein

$$
\begin{gather*}
\phi=\frac{1}{2}\left(\phi_{2}+\phi_{1}\right), \\
\phi_{2}-\phi_{1}, \\
C_{1}=2 a^{2}\left(\mathrm{I}-\frac{e^{2}}{2}-\frac{e^{4}}{8}-\frac{e^{6}}{16}-\ldots\right), \\
C_{2}=2 a^{2}\left(\frac{e^{2}}{6}+\frac{e^{4}}{48}+\circ+\ldots\right),  \tag{6}\\
C_{3}=2 a^{2}\left(\frac{3 e^{4}}{80}+\frac{e^{6}}{40}+\ldots\right) .
\end{gather*}
$$

If $Q$ be the area of a quadrilateral bounded by the parallels whose latitudes are $\phi_{1}$ and $\phi_{2}$ and by meridians whose difference of longitude is $\Delta \lambda$,

$$
Q=\frac{\Delta \lambda}{2 \pi} Z .
$$

Hence, using the English mile as unit of length, (5) and (6) give for the adopted spheroid -

Area of quadrilateral in square miles.

$$
\begin{gathered}
Q=\Delta \lambda \text { (in degrees) }\left\{\begin{array}{l}
c_{1} \cos \phi \sin \frac{1}{2} \Delta \phi-\epsilon_{2} \cos 3 \phi \sin \frac{3}{2} \Delta \phi \\
+\epsilon_{3} \cos 5 \phi \sin \frac{5}{2} \Delta \phi-\ldots
\end{array}\right\}, \\
\log c_{1}^{*}=5.7375398, \\
\log c_{2}=2.79173, \\
\log \epsilon_{3}=9.976-10 .
\end{gathered}
$$

[^4]Numerical examples. - $1^{\circ}$. Suppose $\phi_{1}=0, \phi_{2}=90^{\circ}$ and $\Delta \lambda=360^{\circ}$. Then (7) should give the area of a hemispheroid. The calculation runs thus :

| log. | log. | log. |
| :---: | :---: | :---: |
| $c_{1} 5.7375398$ | $\mathrm{c}_{2} 2.79173$ | $c_{3} 9.976-10$ |
| $\cos \phi 9.8494850$ - 10 | $\cos 3 \boldsymbol{\phi} 9.84948_{n}$ - 10 | $\cos 5$ ¢ 9.849n - ro |
| $\sin \frac{1}{2} \Delta \phi 9.8494850-10$ | $\sin \frac{3}{2} \Delta \phi 9.84949$ - 10 | $\sin \frac{5}{2} \Delta \phi 9.848_{n}-10$ |
| 3602.5563025 | 3602.55630 | 3602.556 |
| Sum 7.9928123 | $5.04700_{n}$ | 2.229 |

Hence -

$$
Q \begin{aligned}
& \begin{array}{l}
\text { rst term } \\
\text { 2d term } \\
3^{\text {d }} \text { term }
\end{array}=+\begin{array}{r}
9835^{8} 59 \mathrm{r} \\
\text { III429 } \\
169
\end{array} \\
& Q=\quad \text { sum }=\begin{array}{l}
98470189
\end{array}
\end{aligned}
$$

Twice this is the area of the spheroidal surface of the earth; i.e., $19694037^{8}$ square miles.
$2^{\circ}$. The last result may be checked by (4). Thus,

$$
\begin{aligned}
&\left(\frac{e^{2}}{3}+\frac{e^{4}}{15}+\ldots\right)=0.00225928 \\
& \log \left(1-\frac{e^{2}}{3}-\ldots\right)=9.9990177 \\
&=7.1961072 \\
& \log a^{2}=\underline{1.0992099} \\
& \log 4 \pi \\
& \log (196940407) \\
& \hline 8.2943348
\end{aligned}
$$

This number agrees with the number derived above as closely as 7 -place logarithms will permit, the discrepancy between the two values being about
 precision of the elements of the adopted spheroid warrants,

Area earth's surface $=196940400$ square miles.
The areas of quadrilaterals of the earth's surface bounded by meridians and parallels of $\mathrm{I}^{\circ}, 30^{\prime}, 15^{\prime}$, and $10^{\prime}$ extent respectively, in latitude and longitude, are given in Tables 25 to 29 .
10. Spheres of Equal Volume and Equal Surface with Earth's Spheroid.
$r_{1}=$ radius of sphere having same volume as the earth's spheroid,
$r_{2}=$ radius of sphere having same surface as that spheroid.

$$
\begin{aligned}
r_{1} & =\sqrt[3]{a^{2} b} \\
& =a\left(\mathrm{I}-\frac{1}{6} e^{2}-\frac{A^{2}}{7^{4}} e^{4}-\mathrm{T}_{2}^{286} e^{6}-\ldots\right) .
\end{aligned}
$$

$$
\begin{aligned}
r_{2} & =a\left(\mathrm{I}-\frac{e^{2}}{3}-\frac{e^{4}}{15}-\frac{e^{6}}{35}-\ldots\right)^{\frac{1}{2}} \\
& =a\left(\mathrm{I}-\frac{1}{8} e^{2}-\frac{117}{360} e^{4}-{ }_{3}^{67} \frac{7}{24} e^{6}-\ldots\right), \\
a-r_{1} & =\frac{1}{8} a e^{2}\left(\mathrm{I}+\frac{5}{12} e^{2}+\ldots\right)=0.00113 a, \text { about. } \\
r_{2}-r_{1} & =\frac{1}{45} a e^{4}+\ldots=0.000001 a, \text { about. }
\end{aligned}
$$

## i1. Co-ordinates for the Polyconic Projection of Maps.

In the polyconic system of map projection every parallel of latitude appears on the map as the developed circumference of the base of a right cone tangent to the spheroid along that parallel. Thus the parallel $E F$ (Fig. 2) will appear in projection as the arc of a circle $E O F$ (Fig. 3) whose radius $O G=l$ is equal to the slant height of the tangent cone $E F G$ (Fig. 2). Evidently one meridian and only one will appear as a straight line. This meridian is generally made the central meridian of the area to be projected. The distances along this central meridian between consecutive parallels are made equal (on the scale of the map) to the real distances along the surface of the spheroid. The circles in which the parallels are developed are not concentric, but their centres all lie on the central meridian. The meridians are concave
 toward the central meridian, and, except near the corners of maps showing large
 central meridian, and let the rectangular axes of $Y(O G)$ and $X(O Q)$ be re-
spectively coincident with and perpendicular to this meridian. Call the interval
in longitude between the central meridian and the next adjacent one $\Delta \lambda$, and central meridian, and let the rectangular axes of $Y(O G)$ and $X(O Q)$ be re-
spectively coincident with and perpendicular to this meridian. Call the interval
in longitude between the central meridian and the next adjacent one $\Delta \lambda$, and central meridian, and let the rectangular axes of $Y(O G)$ and $X(O Q)$ be re-
spectively coincident with and perpendicular to this meridian. Call the interval
in longitude between the central meridian and the next adjacent one $\Delta \lambda$, and denote the angle at the centre $G$ subtended by the developed arc $O P$ by $\alpha$. lels at angles differing little from right angles.

In the practical work of map making, the meridians and parallels are most advantageously defined by the co-ordinates of their points of intersection. These coordinates may be expressed in the following manner: For any parallel, as EOF (Fig. 3), take the origin $O$ at the intersection with the

Then from Fig. 3 it appears that

$$
\begin{aligned}
& x=l \sin \alpha \\
& y=2 l \sin ^{2} \frac{1}{2} \alpha
\end{aligned}
$$

But from Figs. 2 and 3,

$$
\begin{gathered}
l=\rho_{n} \cot \phi \\
l a=r \Delta \lambda=\rho_{n} \Delta \lambda \cos \phi
\end{gathered}
$$

whence

$$
a=\Delta \lambda \sin \phi
$$

Hence, in terms of known quantities there result

$$
\begin{gather*}
x=\rho_{n} \cot \phi \sin (\Delta \lambda \sin \phi)  \tag{I}\\
y=2 \rho_{n} \cot \phi \sin ^{2} \frac{1}{2}(\Delta \lambda \sin \phi) .
\end{gather*}
$$

Numerical example. - Suppose $\phi=40^{\circ}$ and $\Delta \lambda=25^{\circ}=90000^{\prime \prime}$.
Then

$$
\begin{array}{ll}
\log 90000^{\prime \prime} & =4.9542425 \\
\log \sin 40^{\circ} & =9.8080675-10 \\
\log 57850 . .^{\prime \prime} 88 & =4.7623100 ; \\
\Delta \lambda \sin \phi & =16^{\circ} 04^{\prime} 10 .^{\prime \prime} 88 \\
\frac{1}{2}(\Delta \lambda \sin \phi) & =8^{\circ} 02^{\prime} \circ 5^{\prime \prime} 44 .
\end{array}
$$

$\log$.

| $\sin (\Delta \lambda \sin \phi)$ | $9.4421760-10$ |
| :--- | :--- |
| $\cot \phi$ | 0.0761865 |
| $\rho_{n j}$ Table II | 7.3212956 |

$x$
6.8396581
$x=6912865$ feet
log.
$\sin \frac{1}{2}(\Delta \lambda \sin \phi) 9.1454305-10$
$\sin \frac{1}{2}(\Delta \lambda \sin \phi) 9.1454305-10$ $\cot \phi \quad 0.0761865$
$\rho_{n}$, Table II $\quad 7.32$ I 2956
20.3010300
$y \quad 5.9893731$
$y=975828$ feet.

The equations ( I ) are exact expressions for the co-ordinates. But when $\Delta \lambda$ is small, one may use the first terms in the expansions of $\sin (\Delta \lambda \sin \phi)$ and $\sin ^{2} \frac{1}{2}(\Delta \lambda \sin \phi)$ and reach results of a much simpler form.

Thus,

$$
\begin{aligned}
& \sin (\Delta \lambda \sin \phi)=\Delta \lambda \sin \phi-\frac{1}{6}(\Delta \lambda \sin \phi)^{8}+\ldots, \\
& \sin ^{2} \frac{1}{2}(\Delta \phi \sin \phi)=\frac{1}{4}(\Delta \lambda \sin \phi)^{2}-\frac{1}{48}(\Delta \lambda \sin \phi)^{4}+\ldots ;
\end{aligned}
$$

whence, to terms of the second order,

$$
\begin{align*}
& x=\rho_{n} \Delta \lambda \cos \phi\left[1-\frac{1}{6}(\Delta \lambda \sin \phi)^{2}\right],  \tag{2}\\
& y=\frac{1}{4} \rho_{n}(\Delta \lambda)^{2} \sin 2 \phi\left[1-\frac{1}{12}(\Delta \lambda \sin \phi)^{2}\right]
\end{align*}
$$

If the terms of the second order in these equations be neglected, the value of $x$ will be too great by an amount somewhat less than $\frac{1}{6}(\Delta \lambda \sin \phi)^{2} \cdot x$, and the value of $y$ will be too great by an amount somewhat less than $\frac{1}{2}(\Delta \lambda \sin \phi)^{2} . y$. An idea of the magnitudes of these fractions of $x$ and $y$ may be gained from the following table, which gives the values of $\frac{1}{6}(\Delta \lambda \sin \phi)^{2}$ for a few values of the arguments $\Delta \lambda$ and $\phi$.

Values of $\frac{1}{6}(\Delta \lambda \sin \phi)^{2}$.

| $\Delta \lambda$ | $\phi$ |  |  |
| :---: | :---: | :---: | :---: |
|  | $20^{\circ}$ | $40^{\circ}$ | $60^{\circ}$ |
| - |  |  |  |
| 1 | r/168000 | 1/47700 | 1/26260 |
| 2 | 1/42000 | 1/11900 | 1/6560 |
| 3 | 1/18700 | 1/5300 | 1/2920 |

It appears from this table that the first terms of (2) will suffice in computing the co-ordinates for projection of all maps on ordinary scales, and of less extent in longitude than $2^{\circ}$ from the middle meridian. For example, the value of $x$ for $\Delta \lambda=2^{\circ}$, and $\phi=40^{\circ}$, and for a scale of two miles to one inch ( $\mathrm{I} / \mathrm{I} 26{ }^{2} 20$ ), is 53.063 inches less $\mathrm{r} / \mathrm{m} 1900$ part, or about 0.004 inch, which may properly be regarded as a vanishing quantity in map construction. For the computation of the co-ordinates given in the tables 19 to 24 , where $\Delta \lambda$ does not exceed $\mathrm{r}^{\circ}$, it is amply sufficient, therefore, to use

$$
\begin{align*}
& x=\rho_{n} \Delta \lambda \cos \phi, \\
& y=\frac{1}{4} \rho_{n}(\Delta \lambda)^{2} \sin 2 \phi . \tag{3}
\end{align*}
$$

In these formulas and in (2), if $\Delta \lambda$ is expressed in seconds, minutes, or degrees, it must be divided by the number of seconds, minutes, or degrees in the radius. The logarithms of the reciprocals of these numbers are given on p . xlvi. In the construction of tables like 19 to 24 , it is most convenient, when English units are used, to express $\Delta \lambda$ in minutes and $x$ and $y$ in inches. For this purpose, supposing $\log \rho_{n}$ to be taken from Table ir, if $s$ be the scale of the map, or scale factor, equations (3) become -

Co-ordinates $x$ and $y$ in inches for scale s.

$$
\begin{align*}
& x=\frac{12}{3437.747} \rho_{n} s \Delta \lambda \cos \phi \\
& y=\frac{3}{(3437.747)^{2}} \rho_{n} s(\Delta \lambda)^{2} \sin 2 \phi \tag{4}
\end{align*}
$$

$\Delta \lambda$ in minutes ;

$$
\begin{aligned}
& \log (12 / 3437.747)=7.5429 \mathrm{r}-10, \\
& \log \left(3 /(3437.747)^{2}\right)=3.4046-10 .
\end{aligned}
$$

Tables 19 to 24 give the values of $x$ and $y$ for various scales and for the zone of the earth's surface lying between $0^{\circ}$ and $80^{\circ}$.

Numerical example. - Suppose $\phi=40^{\circ}$ and $\Delta \lambda={ }^{1} 5^{\prime}$; and let the scale of the map be one mile to the inch, or $s=1 / 63360$. Then the calculation by (4) runs thus:


These values of $x$ and $y$, it will be observed, agree with those corresponding to the same arguments in Table 22.

When many values for the same scale are to be computed, $\log s$ should, of course, be combined with the constant logarithms of (4). Moreover, since in (4) $x$ varies as $\Delta \lambda$ and $y$ as $(\Delta \lambda)^{2}$, when several pairs of co-ordinates are to be computed for the same latitude, it will be most advantageous to compute the pair corresponding to the greatest common divisor of the several values of $\Delta \lambda$ and derive the other pairs by direct multiplication.

## 12. Lines on a Spheroid.

The most important lines on a spheroid used in geodesy are (a) the curve of a vertical section; (b) the geodesic line; and (c) the alignment curve. Imagine two points in the surface of a spheroid, and denote them by $P_{1}$ and $P_{2}$ respectively. The vertical plane at $P_{1}$ containing $P_{2}$ and the vertical plane at $P_{2}$ containing $P_{1}$ give vertical section curves or lines. The curves cut out by these two planes coincide only when $P_{1}$ and $P_{2}$ are in a meridian plane. The geodesic line is the shortest line joining $P_{1}$ and $P_{2}$, and lying in the surface of the spheroid. The alignment curve on a spheroid is a curve whose vertical tangent plane at every point of its length contains the terminal points $P_{1}$ and $P_{2}$. The curve (a) lies wholly in one plane, while (b) and (c) are curves of double curvature. In the case of a triangle formed by joining three points on a spheroid by lines lying in its surface, the curves of class (a) give two distinct sets of triangle sides, while the curves of classes (b) and (c) give but one set of sides each. For all intervisible points on the surface of the earth, these different lines differ immaterially in length; the only appreciable differences they present are in their azimuths (see formula under b below). Of the three classes of curves the first two only are of special importance.

## a. Characteristic property of curves of vertical section.

Let $\quad a_{1,2}=$ azimuth of vertical section at $P_{1}$ through $P_{2}$,
$\alpha_{2.1}=$ azimuth of vertical section at $P_{2}$ through $P_{1}$,
$\theta_{1}, \theta_{2}=$ reduced latitudes of $P_{1}$ and $P_{2}$ respectively,
$\delta_{1}, \delta_{2}=$ angles of depression at $P_{1}$ and $P_{2}$ respectively of the chord joining these points.

Then the characteristic property of the vertical section curve joining $P_{1}$ and $P_{2}$ is

$$
\sin a_{1,2} \cos \theta_{1} \cos \delta_{1}=\sin \left(\alpha_{2,1}-180^{\circ}\right) \cos \theta_{2} \cos \delta_{2}
$$

The azimuths $a_{1.2}$ and $a_{2.1}$, it will be observed, are the astronomical azimuths or the azimuths which would be determined astronomically by means of an altitude and azimuth instrument.

## b. Characteristic property of geodesic line.

Let

$$
\begin{aligned}
a_{1.2}^{\prime} & =\text { azimuth of geodesic line at } P_{1}, \\
a_{2,1}^{\prime} & =\text { azimuth of geodesic line at } P_{2,}, \\
\theta_{1}, \theta_{2} & =\text { reduced latitudes of } P_{1} \text { and } P_{2} \text { respectively. }
\end{aligned}
$$

Then the characteristic property of the geodesic line is

$$
\sin a_{1.2} \cos \theta_{1}=\sin \left(180^{\circ}-a_{2.1}\right) \cos \theta_{2}=\cos \theta_{0}
$$

where $\theta_{0}$ is the reduced latitude of the point where the geodesic through $P_{1}$ and $P_{2}$ is at right angles to a meridian plane.
The difference between the astronomical azimuth $a_{1.2}$ and the geodesic azimuth $a_{1,2}^{\prime}$ is expressed by the following formula:

$$
a_{1.2}-a_{1.2}^{\prime} \text { (in seconds) }=\frac{1}{12} \rho^{\prime \prime} e^{2}\left(\frac{s}{a}\right)^{2} \cos ^{2} \phi \sin 2 a_{1.2}
$$

where

$$
\begin{aligned}
& s=\text { length of geodesic line } P_{1} P_{2}, \\
& a=\text { major semi-axis of spheroid, } \\
& e=\text { eccentricity of spheroid, } \\
& \rho^{\prime \prime}=206264 .^{\prime \prime} 8, \\
& \phi=\text { astronomical latitude of } P_{1}, \\
& a_{1.2}=\text { azimuth (astronomical or geodesic) of } P_{1} P_{2,}, \\
& \log \frac{1}{12} \rho^{\prime \prime}\left(\frac{e}{a}\right)^{2}=7.4244-20, \text { for } a \text { in feet. }
\end{aligned}
$$

Thus, for $\phi=0$ and $a_{1.2}=45^{\circ}$, for which $\cos ^{2} \phi \sin 2 a_{1.2}=1$, the above formula gives

$$
\begin{aligned}
a_{1.2}-a_{1.2}^{\prime} & =0 . " \circ 74, \text { for } s=100 \text { miles } \\
& =0.296, \text { for } s=200 \text { miles } \\
& =\ldots ;
\end{aligned}
$$

so that for most geodetic work this difference is of little if any importance.

## 13. Solution of Spheroidal Triangles.

The data for solution of a spheroidal triangle ordinarily presented are the measured angles and the length of one side. This latter may be either a geodesic line or a vertical section curve, since their lengths are in general sensibly equal. Such triangles are most conveniently solved in accordance with the rule afforded by Legendre's theorem, which asserts that the sides of a spheroidal triangle (of any measurable size on the earth) are sensibly equal to the sides of a plane triangle having a base of the same length and angles equal respectively to the spheroidal angles diminished each by one third of the excess of the spheroidal triangle. In other words, the computation of spheroidal triangles is thus made to depend on the computation of plane triangles.

## a. Spherical or spheroidal excess.

The excess of a spheroidal triangle of ordinary extent on the earth is given by

$$
\epsilon(\text { in seconds })=\rho^{\prime \prime} \frac{S}{\rho_{m} \rho_{n}}
$$

where $S$ is the area of the spheroidal or corresponding plane triangle; $\rho_{m}, \rho_{n}$ are the principal radii of curvature for the mean latitude of the vertices of the triangle; and $\rho^{\prime \prime}=206264 .{ }^{\prime \prime} 8$. For a sphere, $\rho_{m}=\rho_{n}=$ radius of the sphere.

Denote the angles of the spheroidal triangle by $A, B, C$, respectively; the corresponding angles of the plane triangle by $a, \beta, \gamma$ (as on p. xviii) ; and the sides common to the two triangles by $a, b, c$. Then

$$
\begin{gathered}
S=\frac{1}{2} a b \sin \gamma=\frac{1}{2} b c \sin \alpha=\frac{1}{2} c a \sin \beta \\
a=A-\frac{1}{3} \epsilon, \quad \beta=B-\frac{1}{3} \epsilon, \quad \gamma=C-\frac{1}{3} \epsilon
\end{gathered}
$$

Tables 13 and 14 give the values of $\log \left(\rho^{\prime \prime} / 2 \rho_{m} \rho_{n}\right)$ for intervals of $1^{\circ}$ of astronomical or geographical latitude.*

## 14. Geodetic Differences of Latitude, Longitude, and Azimuth.

## a. Primary triangulation.

Denote two points on the surface of the earth's spheroid by $P_{1}$ and $P_{2}$ respectively. Let

$$
\begin{aligned}
s & =\text { length of geodesic line joining } P_{1} \text { and } P_{2}, \\
\phi_{1}, \phi_{2} & =\text { astronomical latitudes of } P_{1} \text { and } P_{2}, \\
\lambda_{1}, \lambda_{2} & =\text { longitudes of } P_{1} \text { and } P_{2}, \\
\Delta \lambda & =\lambda_{2}-\lambda_{1}, \\
a_{1.2} & =\text { azimuth of } P_{1} P_{2}(s) \text { at } P_{1}, \\
a_{2.1} & =\text { azimuth of } P_{2} P_{1}(s) \text { at } P_{2}, \\
e & =\text { eccentricity of spheroid, } \\
\rho_{m,} \rho_{n} & =\text { principal (meridian and normal) radii of curvature at the point } P_{1} .
\end{aligned}
$$

Then for the longest sides of measurable triangles on the earth the following formulas will give $\phi_{2}, \lambda_{2}$, and $\alpha_{2.1}$ in terms of $\phi_{1}, \lambda_{1}, a_{1.2}$ and $s$. The azimuths are astronomical, and are reckoned from the south by way of the west through $360^{\circ}$.

$$
\begin{gather*}
a^{\prime}=180^{\circ}-\alpha_{1.2,}, \quad \text { and } \alpha_{2.1}=180^{\circ}+\alpha^{\prime \prime}, \quad \text { for } a_{1,2}<180^{\circ} \\
\alpha^{\prime}=\alpha_{1.2}-180^{\circ}, \quad \text { and } \alpha_{2.1}=180^{\circ}-\alpha^{\prime \prime}, \quad \text { for } \alpha_{1,2}>180^{\circ}  \tag{I}\\
\eta=\frac{s}{\rho_{n}}\left\{1+\frac{1}{8} \frac{e^{2}}{1-e^{2}}\left(\frac{s}{\rho_{n}}\right)^{2} \cos ^{2} \phi_{1} \cos ^{2} \alpha^{\prime}\right\}  \tag{2}\\
\zeta=\frac{1}{4} \frac{e^{2} \eta^{2}}{1-e^{2}} \cos ^{2} \phi_{1} \sin 2 \alpha^{\prime} \tag{3}
\end{gather*}
$$

[^5]\[

$$
\begin{gather*}
\tan \frac{1}{2}\left(a^{\prime \prime}+\Delta \lambda+\zeta\right)=\frac{\cos \frac{1}{2}\left(90^{\circ}-\phi_{1}-\eta\right)}{\cos \frac{1}{2}\left(90^{\circ}-\phi_{1}+\eta\right)} \cot \frac{1}{2} a^{\prime}  \tag{4}\\
\tan \frac{1}{2}\left(a^{\prime \prime}-\Delta \lambda+\zeta\right)=\frac{\sin \frac{1}{2}\left(90^{\circ}-\phi_{1}-\eta\right)}{\sin \frac{1}{2}\left(90^{\circ}-\phi_{1}+\eta\right)} \cot \frac{1}{2} a^{\prime} \\
\phi_{2}-\phi_{1}=\frac{s}{\rho_{m}} \frac{\sin \frac{1}{2}\left(a^{\prime \prime}-a^{\prime}+\zeta\right)}{\sin \frac{1}{2}\left(a^{\prime \prime}+a^{\prime}+\zeta\right)}\left\{1+\frac{1}{12} \eta^{2} \cos ^{2} \frac{1}{2}\left(a^{\prime \prime}-a^{\prime}\right)\right\} . \tag{5}
\end{gather*}
$$
\]

To express $\eta, \zeta$, and $\phi_{2}-\phi_{1}$ in seconds of arc we must multiply the right hand sides of (2), (3), and (5) by $\rho^{\prime \prime}=206264 .^{\prime \prime} 8$. For logarithmic compution of $\eta^{\prime \prime}$ and $\zeta^{\prime \prime}$, or $\eta$ and $\zeta$ in seconds, we may write with an accuracy generally sufficient

$$
\begin{align*}
& \log \eta^{\prime \prime}=\log \left(\rho^{\prime \prime} s / \rho_{n}\right)+\frac{1}{6} \frac{\mu e^{2}}{1-e^{2}}\left(\frac{s}{\rho_{n}}\right)^{2} \cos ^{2} \phi_{1} \cos ^{2} \alpha^{\prime}  \tag{6}\\
& \log \zeta^{\prime \prime}=\log \frac{1}{4} \frac{e^{2}}{\left(\mathrm{I}-e^{2}\right) \rho^{\prime \prime}}+\log \left\{\left(\eta^{\prime \prime}\right)^{2} \cos ^{2} \phi_{1} \sin 2 a^{\prime}\right\} \tag{7}
\end{align*}
$$

where $\mu$ in (6) is the modulus of common logarithms. For units of the 7 th decimal place of $\log \eta^{\prime \prime}$ we have for the adopted spheroid

$$
\log \frac{1}{6} \frac{\mu e^{2}}{1-e^{2}}=3.69309
$$

Also

$$
\log \frac{1}{4} \frac{e^{2}}{\left(1-e^{2}\right) \rho^{\prime}}=1.91697-10
$$

Similarly, for the computation of the logarithm of the last factor in (5) we have

$$
\log \left\{1+\frac{1}{12} \eta^{2} \cos ^{2} \frac{1}{2}\left(a^{\prime \prime}-a^{\prime}\right)\right\}=\log \left\{1+\frac{1}{12\left(\rho^{\prime \prime}\right)^{2}}\left(\eta^{\prime \prime}\right)^{2} \cos ^{2} \frac{1}{2}\left(a^{\prime \prime}-a^{\prime}\right)\right\}
$$

Putting for brevity

$$
q=\frac{\mathrm{r}}{\mathrm{I2}\left(\rho^{\prime \prime}\right)^{2}}\left(\eta^{\prime \prime}\right)^{2} \cos ^{2} \frac{1}{2}\left(a^{\prime \prime}-a^{\prime}\right)
$$

the logarithm of the desired logarithm is given to terms of the second order inclusive in $q$ by

$$
\log \log (1+q)=\log \mu q-\frac{1}{2} \mu q
$$

For the adopted spheroid

$$
\log \frac{\mu}{12\left(\rho^{\prime \prime}\right)^{2}}=4.92975-10
$$

for units of the seventh decimal place.
For a line 200 miles (about 320 kilometres) long, the maximum value of the second term in (6) is but 12.6 units in the 7 th place of $\log \eta^{\prime \prime}$. For the same length of line, the maximum value of $\zeta_{\ell}^{\prime \prime}$ is $0 .{ }^{\prime \prime} 895$, and the maximum value of the $\operatorname{logarithm}$ of the last factor in (5), or $\log (1+q)$, is less than 922 units in the seventh place of decimals.

For computing differences of latitude, longitude, and azimuth in primary triangulation whose sides are $1^{\circ}$ (about 70 miles, or 100 kilometres) or less in length, the most convenient means are formulas giving $\phi_{2}-\phi_{1}, \lambda_{2}-\lambda_{1}$, and
$a_{2.1}-\left(180^{\circ}-a_{1.2}\right)$, in series proceeding according to powers of the distance $s$. Formulas of this kind with convenient tables for facilitating the computations are given in the Reports of the U. S. Coast and Geodetic Survey.*

## b. Secondary triangulation.

For secondary triangulation, wherein the sides are 12 miles ( 20 kilometres) or less in length, and wherein differences of latitude and longitude are needed to the nearest hundredth of a second only, the following formulas may suffice. Using the same notation as in the preceding section, the formulas are : -

$$
\begin{gather*}
\phi_{2}=\phi_{1}+\Delta \phi, \\
\lambda_{2}=\lambda_{1}+\Delta \lambda,  \tag{I}\\
a_{2.1}=180^{\circ}+a_{1.2}+\Delta a \\
\Delta \phi=-\quad a_{1} s \cos a_{1.2}-a_{2} s^{2} \sin ^{2} a_{1.2} \\
\Delta \lambda=+b_{1} \sec \phi_{1} s \sin a_{1.2}-b_{2} s^{2} \sin a_{1.2} \cos a_{1.2}  \tag{2}\\
\Delta a=-c_{1} \tan \phi_{1} s \sin a_{1.2}+c_{2} s^{2} \sin a_{1.2} \cos a_{1.2}
\end{gather*}
$$

The constants entering the latter equations are defined by the following expressions, wherein $\rho_{m}$ and $\rho_{n}$ are the principal radii of curvature of the spheroid at the point whose latitude is $\phi_{1}$ and $\rho^{\prime \prime}=206264 .{ }^{\prime \prime} 8$ :

$$
\begin{gathered}
a_{1}=\frac{\rho^{\prime \prime}}{\rho_{m}}, \quad b_{1}=c_{1}=\frac{\rho^{\prime \prime}}{\rho_{n}} \\
a_{2}=\frac{\rho^{\prime \prime} \tan \phi_{1}}{2 \rho_{m} \rho_{n}}, \quad b_{2}=\frac{\rho^{\prime \prime} \sec \phi_{1} \tan \phi_{1}}{\rho_{n}{ }^{2}}, \quad c_{2}=\frac{\rho^{\prime \prime}\left(\mathrm{I}+2 \tan ^{2} \phi_{1}\right)}{2 \rho_{n}{ }^{2}} .
\end{gathered}
$$

The logarithms of the factors $a_{1}, b_{1}, c_{1}, a_{2}, b_{2}, c_{2}$, are given in Table 15 for the English foot as unit, and in Table 16 for the metre as unit, the argument being the initial latitude $\phi_{1}$ for all of them.

When all of the differences given by (2) are computed, they may be checked by the formula

$$
\begin{equation*}
\sin \frac{1}{2}\left(\phi_{2}+\phi_{1}\right)=\frac{\Delta a}{\Delta \dot{\lambda}} \tag{3}
\end{equation*}
$$

For convenience of reference in numerical applications of the above formulas, (2) may be written thus:

$$
\begin{aligned}
& \Delta \phi=A_{1}+A_{2} \\
& \Delta \lambda=B_{1}+B_{2} \\
& \Delta a=C_{1}+C_{2}
\end{aligned}
$$

in which, for example, $A_{1}$ and $A_{2}$ are the first and second terms respectively of $\Delta \phi$, due regard being paid to the signs of the functions of $a_{1.2}$.

Numerical example. The following example will serve to illustrate the use of formulas (1) to (3). The value of $\log s$ is for $s$ in English feet, $s$ being in this case about 12.3 miles.

$$
\begin{aligned}
& \Delta \phi \quad \text {-07' } 50.121 \quad \Delta \lambda \quad+09^{\prime} 20 .{ }^{\prime \prime} 22 \quad \Delta a \quad \text {-05 } 51 .^{\prime \prime} 32 \\
& \phi_{2} \quad 38^{\circ} 46^{\prime} 18 .^{\prime \prime \prime} 17 \quad \lambda_{2} \quad 88^{\circ} 12^{\prime} 44^{\prime \prime} 37 \quad a_{2.1} 222^{\circ} 55^{\prime} 54 .^{\prime \prime} 97 \\
& \frac{1}{2}\left(\phi_{2}+\phi_{1}\right) \quad 3^{8^{\circ}} 5^{\prime}{ }^{\prime} 13 . .^{\prime \prime} 27
\end{aligned}
$$

* See Appendix 7, Report of 1884 , for latest edition of these tables.

| $\log$ | $\log$ | $\log$ | $\log$ |
| :---: | :---: | :---: | :---: |
| s 4.81308 | s 4.81308 | $s \sin a_{1.2} 4.647$ | $s \sin a_{1.2} 4.647$ |
| $\cos a_{1.2} 9.86392$ | $\sin a_{1.2} 9.83402$ | $s \sin a_{1.2} 4.647$ | $s \cos a_{1.2} 4.677$ |
| $a_{1} 7.99495$ | $\sec \phi_{1} 0.10890$ | $a_{2} 0.279$ | $b_{2} 0.688$ |
|  | $b_{1} 7.99316$ |  | $c_{2} 0.733$ |
| $A_{1} 2.67195$ | $B_{1} 2.74916$ | $A_{2} 9.573$ | $B_{2} 0.012$ |
|  | $\sin \phi_{1} 9.79795$ |  | $C_{2} 0.057$ |
|  | $C_{1} 2.547 \mathrm{II}$ |  |  |
| $A_{1}-469 .{ }^{\prime \prime} 84$ | $B_{1}+56 \mathrm{r} .{ }^{\prime \prime} 25$ | $C_{1}-352.146$ | $\Delta \mathrm{a} 2.54570$ |
| $A_{2}-\quad 0.137$ | $B_{2}-$ r." ${ }^{\prime \prime}$ | $C_{2}+1{ }^{\prime \prime} 14$ | $\Delta \lambda 2.74836$ |
| $\Delta \phi-470.121$ | $\Delta \lambda+560.122$ | $\Delta \alpha-351 .{ }^{\prime \prime}{ }^{\prime \prime}$ | $\left(\phi_{2}+\phi_{1}\right) 9.79734$ |

## 15. Trigonometric Leveling.

a. Computation of heights from observed zenith distances.

Let $\quad s=$ sea level distance between two points $P_{1}$ and $P_{2}$,
$H_{1}, H_{2}=$ heights above sea level of $P_{1}$ and $P_{2}$,
$z_{1}=$ observed zenith distance of $P_{2}$ from $P_{1}$,
$z_{2}=$ observed zenith distance of $P_{1}$ from $P_{2}$,
$\rho=$ radius of curvature of vertical section at $P_{1}$ through $P_{2}$, or at $P_{2}$ through $P_{1}$, the curvature being sensibly the same for both for this purpose,
$C=$ angle at centre of curvature subtended by $s$,
$m_{1}, m_{2}=$ coefficients of refraction at $P_{1}$ and $P_{2}$,
$\Delta z_{1}, \Delta z_{2}=$ angles of refraction at $P_{1}$ and $P_{2}$.
Then, the fundamental relations are

$$
\begin{gather*}
C=\frac{s}{\rho}, \quad \Delta z_{1}=m_{1} C, \quad \Delta z_{2}=m_{2} C  \tag{I}\\
z_{1}+z_{2}+\Delta z_{1}+\Delta z_{2}=180^{\circ}+C \\
H_{2}-H_{1}=s \tan \frac{1}{2}\left(z_{2}+\Delta z_{2}-z_{1}-\Delta z_{1}\right)\left(\mathrm{I}+\frac{H_{2}+H_{1}}{2 \rho}+\frac{s^{2}}{12 \rho^{2}}+\ldots\right) \tag{2}
\end{gather*}
$$

When the zenith distances $z_{1}$ and $z_{2}$ are simultaneous, or when $\Delta z_{1}$ and $\Delta z_{2}$ are assumed to be equal, (2) becomes

$$
\begin{equation*}
H_{2}-H_{1}=s \tan \frac{1}{2}\left(z_{2}-z_{1}\right)\left(1+\frac{H_{2}+H_{\mathrm{I}}}{2 \rho}+\frac{s^{2}}{12 \rho^{2}}+\ldots\right) \tag{3}
\end{equation*}
$$

For the case of a single observed zenith distance $z_{1}$, say, and a known or assumed value of $m=m_{1}=m_{2}$, the following formula may be applied :

$$
\begin{equation*}
H_{2}-H_{1}=s \cot z_{1}+\frac{\mathbf{1}-2 m}{2 \rho} s^{2}+\frac{\mathbf{1}-m}{\rho} s^{2} \cot ^{2} z_{1} \tag{4}
\end{equation*}
$$

The coefficient of refraction $m$ varies very greatly under different atmospheric conditions. Its average value for land lines is about 0.07 . The following table gives the values of $\log \frac{1}{2}(1-2 m)$ and $\log (1-m)$ for values of $m$ ranging from 0.05 to 0.10. It is taken from Appendix 18, Report of U. S. Coast and Geodetic

Survey for 1876 . Table 12 taken from the same source gives values of $\log \mathrm{s}$ needed for use in (3) and (4).

Table of values of $\log \frac{1}{2}(1-2 m)$ and $\log (1-m)$.

| $m$ | $\log \frac{1}{2}(\mathrm{I}-2 m)$. | $\log (\mathrm{I}-m)$. | $m$ | $\log \frac{1}{2}(\mathrm{I}-2 m)$. | $\log (1-m)$. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.050 | 9.6532 I | 9.978 | 0.075 | 9.62839 | 9.966 |
| 51 | 65225 | 77 | 76 | 62737 | 66 |
| 52 | 65128 | 77 | 77 | 62634 | 65 |
| 53 | 65031 | 76 | 78 | 62531 | 65 |
| 54 | 64933 | 76 | 79 | 62428 | 64 |
| 0.055 | 9.64836 | 9.975 | 0.080 | 9.62325 | 9.964 |
| 56 | 64738 | 75 | 81 | 62221 | 63 |
| 57 | 64640 | 75 | 82 | 62118 | 63 |
| 58 | 64542 | 74 | 83 | 62014 | 62 |
| 59 | 64444 | 74 | 84 | 61910 | 62 |
| 0.060 | 9.64345 | 9.973 | 0.085 | 9.61805 | 9.961 |
| 61 | 64246 | 73 | 86 | 61700 | 61 |
| 62 | 64147 | 72 | 87 | 61595 | 60 |
| 63 | 64048 | 72 | 88 | 61490 | 60 |
| 64 | 63949 | 71 | 89 | 61384 | 60 |
| 0.065 | 9.63849 | $9 \cdot 971$ | 0.090 | 9.61278 | 9.959 |
| 66 | 63749 | 70 | 91 | 61172 | 59 |
| 67 | 63649 | 70 | 92 | 61066 | 58 |
| 68 | 63548 | 69 | 93 | 60959 | 58 |
| 69 | 63448 | 69 | 94 | 60853 | 57 |
| 0.070 | 9.63347 | 9.968 | 0.095 | 9.60746 | 9.957 |
| 71 | 63246 | 68 | 96 | 60638 | 56 |
| 72 | 63144 | 68 | 97 | 60531 | 56 |
| 73 | 63043 | 67 | 98 | 60423 | 55 |
| 74 | 62941 | 67 | 99 | 60315 | 55 |
|  |  |  | 0.100 | 9.60206 | 9.954 |

For less precise work one may use equation (4) in the form

$$
\begin{equation*}
H_{2}-H_{1}=s \cot z_{1}+c s^{2} \tag{5}
\end{equation*}
$$

wherein, if we make $m=0.07$ and use for $\rho$ its average value, or $\sqrt{\rho_{m} \rho_{n}}$ for latitude $45^{\circ}$,

$$
\begin{aligned}
\log c & =2.313-10 \text { for } s \text { in feet, } \\
& =2.829-10 \text { for } s \text { in metres. }
\end{aligned}
$$

Thus, for a distance ( $s$ ) of 10 miles the value of the term $c s^{2}$ in (5) is 57.3 feet.
If altitudes $a_{1}$, say, are observed in the place of zenith distances $z_{1}$, it is most convenient to write (5) thus:-

$$
\begin{equation*}
H_{2}-H_{1}= \pm s \tan \alpha_{1}+c s^{2} \tag{6}
\end{equation*}
$$

where the upper sign is used when $a_{1}$ is an angle of elevation and the lower sign when $a_{1}$ is an angle of depression.

## b. Coefficients of refraction.

When $z_{1}$ and $z_{2}$ are both observed for a given line, a coefficient of refraction may be computed from the assumption of equality of coefficients at the two ends of the line. Thus, equations (i) give

$$
\Delta z_{1}+\Delta z_{2}=180^{\circ}+C-\left(z_{1}+z_{2}\right)
$$

or

$$
\left(m_{1}+m_{2}\right) \frac{s}{\rho}=180^{\circ}+\frac{s}{\rho}-\left(z_{1}+z_{2}\right)
$$

whence

$$
m_{1}+m_{2}=\mathrm{I}-\frac{\rho}{s}\left(z_{1}+z_{2}-180^{\circ}\right)
$$

Assuming $m_{1}=m_{2}=m$, and supposing $z_{1}+z_{2}-180^{\circ}$ expressed in seconds of arc,

$$
\begin{aligned}
& \quad m=\frac{1}{2}\left\{1-\frac{\rho}{s \rho^{\prime \prime}}\left(z_{1}+z_{2}-180^{\circ}\right)\right\} . \\
& \rho^{\prime \prime}=206264^{\prime \prime} 8, \quad \log \rho^{\prime \prime}=5.314425^{1} \\
& \text { c. Dip and distance of sea horizon. }
\end{aligned}
$$

Let
$h=$ height of eye above sea level,
$\delta=$ dip or angle of depression of horizon,
$s=$ distance of horizon from observer.

Then

$$
\begin{aligned}
\delta \text { (in seconds) } & =58.82 \sqrt{h \text { in feet }} \\
& =106.54 \sqrt{h \text { in metres. }} \\
s \text { (in miles) } & =1.317 \sqrt{h \text { in feet, }} \\
s \text { (in kilometres) } & =3.839 \sqrt{h \text { in metres. }}
\end{aligned}
$$

The above formulas take account of curvature and refraction. They depend on the value 0.0784 for the coefficient of refraction, and are quite as accurate as the uncertainties in such data justify. For convenience of memory, and for an accuracy amply sufficient in most cases, the coefficients of the radicals in the last two formulas may be written $\frac{4}{3}$ and $\frac{1}{5}$ respectively.

## i6. Miscellaneous Formulas.

a. Correction to observed angle for eccentric position of instrument

Let $C^{\prime}$ be the eccentric position of the instrument, and $C_{0}$ the observed value of the angle at that point between two other points $A$ and $B$. Let $C$ denote the central point as well as the angle $A C B$ desired. Call the distance $C C^{\prime} r$ and denote the angle $A C C^{\prime}$ by $\theta$. Denote the lines $B C$ and $A C$, which are assumed to be sensibly the same as $B C^{\prime}$ and $A C^{\prime}$, by $a$ and $b$ respectively. Then

$$
\begin{gathered}
C-C_{0}(\text { in seconds })=\frac{\rho^{\prime \prime} r \sin \left(\theta-C_{0}\right)}{a}-\frac{\rho^{\prime \prime} r \sin \theta}{b}, \\
\rho^{\prime \prime}=206264 .^{\prime \prime} 8, \quad \log \rho^{\prime \prime}=5.3144251 .
\end{gathered}
$$

Attention must be paid to the signs of $\sin \left(\theta-C_{0}\right)$ and $\sin \theta$, and to the fact that angles are counted from $A$ towards $B$ through $360^{\circ}$. A diagram drawn in accordance with the above specifications will elucidate any special case.

## b. Reduction of measured base to sea level.

Let $l$ be the length of the bar, tape or other unit used in measuring the base. Let $l_{0}$ be the corresponding length reduced to sea level for a height $h$, this latter being the observed height of $l$. Then if $\rho$ denote the radius of curvature of the earth's surface in the direction of the base,

$$
l_{0}=\frac{\rho l}{\rho+h}=\left(1-\frac{h}{\rho}+\ldots\right) l
$$

with sufficient accuracy. Hence, for the whole length of the base,

$$
\Sigma l_{0}=\Sigma l-\frac{\mathrm{x}}{\rho} \Sigma l h
$$

If $L$ denote the total measured length, $L_{0}$ the corresponding total sea level length, and $H$ the mean value of the heights $h$, the above equation gives

$$
L_{0}=L-L \frac{H}{\rho}
$$

## c. The three-point problem.

In this problem the positions of three points $A, B, C$, and hence the elements of the triangle they form, are given together with the two angles $A P C$ and $B P C$ at a point $P$ whose position is required. Denote the angles and the sides of the known triangle by $A, B, C$, and $a, b, c$, respectively. Also put

$$
\begin{array}{ll}
A P C=\beta, & B P C=a, \\
P A C=x, & P B C=y .
\end{array}
$$

Then the sum of the angles in the quadrilateral $P A C B$ is
whence

$$
a+\beta+x+y+C=360^{\circ}
$$

$$
\begin{equation*}
\frac{1}{2}(x+y)=180^{\circ}-\frac{1}{2}(\alpha+\beta+C) . \tag{I}
\end{equation*}
$$

Compute an auxiliary angle $z$ from the equation

$$
\begin{equation*}
\tan z=\frac{a \sin \beta}{b \sin a} ; \tag{2}
\end{equation*}
$$

Then

$$
\begin{equation*}
\tan \frac{1}{2}(x-y)=\tan \left(z-45^{\circ}\right) \tan \frac{1}{2}(x+y) . \tag{3}
\end{equation*}
$$

These three equations give all the data essential to a complete determination of the position of $P$. Any special case should be elucidated by a diagram drawn in accordance with the specifications given above.

When the positions of the points $A, B, C$ are given on a map, the position of $P$ on the same map may be found graphically by drawing lines making angles with each other equal to the given angles $u$ and $\beta$ from a point on a piece of tracing paper, and then placing this tracing on the map so as to meet the required conditions. This ready method of solving the problem is often sufficient.

## 17. Salient Facts of Physical Geodesy.

a. Area of earth's surface, areas of continents, area of oceans.*

Square miles.

b. Average heights of continents and depths of oceans. $\dagger$

c. Volume, surface density, mean density, and mass of earth.

Volume of earth $=259880000000$ cubic miles.
$=1083200000000$ cubic kilometres.
$=260 \times 10^{9}$ cubic miles (about).
$=108 \times{ }_{10}{ }^{10}$ cubic kilometres (about).
Surface density of earth $=2.56 \pm 0.16 \ddagger$
Mean density of earth $=5.576 \pm 0.016$.

* Derived from relative areas given in Helmert's Geodüsie, Band II. p. 3ז3.
$\dagger$ Helmert's Geodäsie, Band II. p. 3 I 3.
$\ddagger$ These densities are given by Professor Wm. Harkness in his memoir on The Solar Parallax and Related Constants. The surface density applies to that portion of the earth's crust which lies above and within a shell ten miles thick, the lower surface of this shell being ten miles beiow sea level.

Assuming the mass of a cubic foot of water to be 62.28 pounds (at $62^{\circ} \mathrm{F}$.),

$$
\begin{aligned}
\text { Mass of earth } & =13284 \times 10^{21} \text { pounds. } \\
& \left.=6642 \times 10^{18} \text { tons (of } 2000 \mathrm{lbs} .\right) . \\
& =60258 \times 10^{20} \text { kilogrammes. }
\end{aligned}
$$

d. Principal moments of inertia and energy of rotation of earth.
$M=$ mass of earth,
$A=$ moment of inertia of earth about an axis in its equator,
$C=$ moment of inertia about axis of rotation,
$a=$ equatorial axis of earth,
$\omega=$ angular velocity of earth,
$=(2 \pi / 86164)$ for mean solar second as unit of time.
Then $\dagger$

$$
\begin{aligned}
A & =0.325 M a^{2}, \\
C & =0.326 M a^{2} .
\end{aligned}
$$

Energy of rotation of earth $=\frac{1}{2} \omega^{2} C$.
$=0.163 \omega^{2} M \alpha^{2}$.
$=504 \times 10^{28}$ foot-poundals.
$=217 \times 10^{26}$ kilogramme-metres.
$=212 \times 10^{85}$ ergs.

## References.

The most exhaustive treatise on the theory of geodesy is found in "Die Mathematischen und Physikalischen Theorieen der Höheren Geodäsie," von Dr. F. R. Helmert. Leipzig: B. G. Teubner ; 8vo, 1880 (vol. i.), 1884 (vol. ii.). An excellent work on the practical as well as theoretical features of the subject is "Die geodätischen Hauptpunkte und ihre Co-ordinaten," von G. Zachariae ; autorisirte deutsche Ausgabe, von E. Lamp. Berlin : Robert Oppenheim, 8vo, 1878. Of works in English the most comprehensive is " Geodesy," by A. R. Clarke. Oxford: The Clarendon Press, 8vo, 188 o .

[^6]
## ASTRONOMY.

## I. The Celestial Sphere. Planes and Circles of Reference.

The celestial sphere is a sphere to which it is convenient to refer stars and other celestial objects. Its centre is assumed to be coincident with the eye of the observer, and the objects referred to it are supposed to lie in its surface. The orientation of this sphere is defined by its equator, which is assumed to be parallel to the earth's equator. The equator is thus the principal plane of reference. Other planes of reference are the plane of the horizon, which is perpendicular to the plumb line at the place; the meridian, which is a plane through the place and the earth's axis of rotation; the prime-vertical, which is a vertical plane at the place at right angles to the meridian ; and the ecliptic, which is a plane parallel to the plane of the earth's orbit. These planes cut the surface of the sphere in great circles called the equator, the horizon, the meridian, etc. The points on the sphere defined by the intersection of the meridians, or the points where the axis of the equator pierces the sphere, are called the poles. Similarly, the prolongation of the plumb line upwards pierces the sphere in the zenith, and its prolongation downwards pierces the sphere in the nadir. Great circles passing through the zenith are called vertical circles.

## 2. Spherical Co-ordinates.

## a. Notation.

The position of a celestial body may be defined by several systems of co-ordinates. The most important of these in practical astronomy are the azimuth and altitude system and the hour angle and declination system. In the first of these the azimuth of a star or other body is the angle between the meridian plane of the place and a vertical plane through the star. It is measured, in general, from the south around by the west through $360^{\circ}$. The altitude of a star is its angular distance above the horizon, and its zenith distance is the complement of the altitude. In the second system the hour angle of a star is the angle between the meridian plane of the place and a meridian plane through the star. It is measured towards the west through $360^{\circ}$. The declination of a star is its angular distance above or below the equator; the complement of the declination is called the polar distance.

The angular distance of the pole above the horizon is equal to the zenith distance of the equator, or to the latitude of the place. Likewise, the altitude of the equator and the zenith distance of the pole are each equal to the complement of the latitude at any place.

These quantities are usually designated by the following notation : -
$A=$ the azimuth of a star or object,
$h=$ its altitude,
$z=$ its zenith distance $=90^{\circ}-h$,
$t=$ its hour angle,
$\delta=$ its declination,
$p=$ its polar distance $=90^{\circ}-\delta$,
$q=$ the parallactic angle, or angle at the star between the pole and the zenith,
$\phi=$ the latitude of the place of observation.
b. Altitude and azimuth in terms of declination and hour angle.

The fundamental relations for this problem are -

$$
\begin{array}{rr}
\sin h & =\sin \phi \sin \delta+\cos \phi \cos \delta \cos t, \\
\cos h \cos A & =-\cos \phi \sin \delta+\sin \phi \cos \delta \cos t,  \tag{I}\\
\cos h \sin A & =r
\end{array}
$$

When it is desired to compute both $A$ and $h$ by means of logarithms, the most convenient formulas are,

$$
\begin{align*}
m \sin M & =\sin \delta, & \tan M & =\frac{\tan \delta}{\cos t}, \\
m \cos M & =\cos \delta \cos t, & & \tan A=\frac{\tan t \cos M}{\sin (\phi-M)}, \\
\sin h & =m \cos (\phi-M), & & \cos ,  \tag{2}\\
\cos h \cos A & =m \sin (\phi-M), & \tan h=\frac{\cos A}{\cos h \sin A}=\cos \delta \sin t, &
\end{align*}
$$

For the computation of $A$ and $z$ separately, the following formulas are useful:

$$
\begin{align*}
\tan A & =-\frac{\sin t}{\cos \phi \tan \delta(\mathrm{I}-\tan \phi \cot \delta \cos t)}  \tag{3}\\
& =-\frac{a \sin t}{1-b \cos t},
\end{align*}
$$

where

$$
a=\sec \phi \cot \delta, \quad b=\tan \phi \cot \delta .
$$

Formulas (3) are especially appropriate for the computation of a series of azimuths of close circumpolar stars, since $a$ and $b$ will be constant for a given place and date.

$$
\begin{gather*}
\cos z=\cos (\phi \sim \delta)-2 \cos \phi \cos \delta \sin ^{2} \frac{1}{2} t \\
\sin ^{2} \frac{1}{2} z=\sin ^{2} \frac{1}{2}(\phi \sim \delta)+\cos \phi \cos \delta \sin ^{2} \frac{1}{2} t  \tag{4}\\
(\phi \sim \delta)=\phi-\delta, \text { for } \phi>\delta \\
=\delta-\phi, \text { for } \phi<\delta .
\end{gather*}
$$

For logarithmic application of (4) we may write

$$
\begin{gather*}
m^{2}=\cos \phi \cos \delta, \quad n^{2}=\sin ^{2} \frac{1}{2}(\phi \sim \delta) \\
\tan N=\frac{m}{n} \sin \frac{1}{2} t  \tag{5}\\
\sin \frac{1}{2} z=\frac{n}{\cos N}=\frac{m}{\sin N} \sin \frac{1}{2} t
\end{gather*}
$$

c. Declination and hour angle in terms of altitude and azimuth.

The fundamental relations for this case are

$$
\begin{align*}
\sin \delta & =\sin \phi \sin h-\cos \phi \cos h \cos A \\
\cos \delta \cos t & =\cos \phi \sin h+\sin \phi \cos h \cos A  \tag{I}\\
\cos \delta \sin t & =\quad \cos h \sin A
\end{align*}
$$

For logarithmic computation by means of an auxiliary angle $M$ one may write

$$
\begin{align*}
& m \sin M=\cos h \cos A, \quad \tan M=\cot h \cos A \\
& m \cos M=\sin h, \\
& \sin \delta=m \sin (\phi-M), \quad \tan t=\frac{\tan A \sin M}{\cos (\phi-M)}  \tag{2}\\
& \cos \delta \cos t=m \cos (\phi-M), \\
& \cos \delta \sin t=\cos h \sin A, \quad \tan \delta=\tan (\phi-M) \cos t
\end{align*}
$$

d. Hour angle and azimuth in terms of zenith distance.

$$
\begin{gathered}
\cos t=\frac{\cos z-\sin \phi \sin \delta}{\cos \phi \cos \delta} \\
\tan ^{2} \frac{1}{2} t=\frac{\sin (\sigma-\phi) \cos (\sigma-\delta)}{\cos \sigma \cos (\sigma-z)}, \quad \sigma=\frac{1}{2}(\phi+\delta+z) . \\
\cos A=\frac{\sin \phi \cos z-\sin \delta}{\cos \phi \sin z} \\
\tan ^{2} \frac{1}{2} A=\frac{\sin (\sigma-\phi) \cos (\sigma-z)}{\cos \sigma \sin (\sigma-\delta)}, \quad \sigma=\frac{1}{2}(\phi+\delta+z)
\end{gathered}
$$

e. Formulas for parallactic angle.
$\cos z=\sin \delta \sin \phi+\cos \delta \cos \phi \cos t$, $\sin z \cos q=\cos \delta \sin \phi-\sin \delta \cos \phi \cos t$, $\sin z \sin q=\quad \cos \phi \sin t$,
$\sin \delta=\cos z \sin \phi+\sin z \cos \phi \cos t$,
$\cos \delta \cos q=\sin z \sin \phi+\cos z \cos \phi \cos A$,
$\cos \delta \sin q=\quad \cos \phi \sin A$.

The first three of these are adapted to logarithmic computation as follows : -

$$
\begin{aligned}
n \sin N & =\cos \phi \cos t, \\
n \cos N & =\sin \phi, \\
\cos z & =n \sin (\delta+N), \\
\sin z \cos q & =n \cos (\delta+N), \\
\sin z \sin q & =\cos \phi \sin t ;
\end{aligned}
$$

whence

$$
\begin{align*}
\tan N & =\cot \phi \cos t, \\
\tan z \sin q & =\frac{\tan t \sin N}{\sin (\delta+N)},  \tag{2}\\
\tan z \cos q & =\cot (\delta+N) .
\end{align*}
$$

A similar adaptation results for the last three of equations (1) by interchanging $\delta$ and $z$. The equations (2) give both $z$ and $q$ in terms of $\phi, \delta$, and $t$, without ambiguity, since $\tan z$ is positive for stars above the horizon.
If $A, z$, and $q$ are all required from $\phi, \delta$, and $t$, they are best given by the Gaussian relations

$$
\begin{align*}
& \sin \frac{1}{2} z \sin \frac{1}{2}(A+q)=\sin \frac{1}{2} t \cos \frac{1}{2}(\phi+\delta), \\
& \sin \frac{1}{2} z \cos \frac{1}{2}(A+q)=\cos \frac{1}{2} t \sin \frac{1}{2}(\phi-\delta),  \tag{3}\\
& \cos \frac{1}{2} z \sin \frac{1}{2}(A-q)=\sin \frac{1}{2} t \sin \frac{1}{2}(\phi+\delta), \\
& \cos \frac{1}{2} z \cos \frac{1}{2}(A-q)=\cos \frac{1}{2} t \cos \frac{1}{2}(\phi-\delta) .
\end{align*}
$$

f. Hour angle, azimuth, and zenith distance of a star at elongation.

In this case the parallactic angle is $90^{\circ}$ and the required quantities are given by the formulas

$$
\begin{align*}
& \cos t=\frac{\tan \phi}{\tan \delta} \\
& \sin A=\frac{\cos \delta}{\cos \phi}  \tag{I}\\
& \cos z=\frac{\sin \phi}{\sin \delta}
\end{align*}
$$

When all of the quantities $t, A$, and $z$ are to be computed the following formulas are more advantageous: -

$$
\begin{align*}
& K^{2}=\sin (\delta+\phi) \sin (\delta-\phi), \\
& \sin t=\frac{K}{\cos \phi \sin \delta}, \quad \cos A=\frac{K}{\cos \phi}, \quad \sin z=\frac{K}{\sin \delta},  \tag{2}\\
& \tan t=\frac{K}{\sin \phi \cos \delta}, \quad \tan A=\frac{\cos \delta}{K}, \quad \tan z=\frac{K}{\sin \phi} .
\end{align*}
$$

g. Hour angle, zenith distance, and parallactic angle for transit of a star across prime vertical.
In this case the azimuth angle is $90^{\circ}$ and the required quantities are given by the formulas

$$
\begin{align*}
& \cos t=\frac{\tan \delta}{\tan \phi} \\
& \cos z=\frac{\sin \delta}{\sin \phi}  \tag{I}\\
& \sin q=\frac{\cos \phi}{\cos \delta}
\end{align*}
$$

or, if all of them are to be computed, by the formulas

$$
\begin{gather*}
K^{2}=\sin (\phi+\delta) \sin (\phi-\delta) \\
\sin t=\frac{K}{\sin \phi \cos \delta}, \quad \sin z=\frac{K}{\sin \phi}, \quad \cos q=\frac{K}{\cos \delta}  \tag{2}\\
\tan t=\frac{K}{\cos \phi \sin \delta}, \quad \tan z=\frac{K}{\sin \widehat{\delta}} \quad \tan q=\frac{\cos \phi}{K}
\end{gather*}
$$

For special accuracy the following group will be preferred : -

$$
\begin{gather*}
\tan ^{2} \frac{1}{2} t=\frac{\sin (\phi-\delta)}{\sin (\phi+\delta)} \\
\tan ^{2} \frac{1}{2} z=\frac{\tan \frac{1}{2}(\phi-\delta)}{\tan \frac{1}{2}(\phi+\delta)}  \tag{3}\\
\tan ^{2}\left(45^{\circ}-\frac{1}{2} q\right)=\tan \frac{1}{2}(\phi+\delta) \tan \frac{1}{2}(\phi-\delta)
\end{gather*}
$$

h. Hour angle and azimuth of a star when in the horizon, or at the time of rising or setting.

In this case the zenith distance of the star is $90^{\circ}$, and the required quantities are given by

$$
\begin{aligned}
\cos t & =-\tan \phi \tan \delta \\
\cos A & =-\frac{\sin \delta}{\cos \phi}
\end{aligned}
$$

or by

$$
\begin{aligned}
\tan ^{2} \frac{1}{2} t & =\frac{\cos (\phi-\delta)}{\cos (\phi+\delta)} \\
\tan ^{2} \frac{1}{2} A & =\frac{\tan \frac{1}{2}\left(90^{\circ}-\phi+\delta\right)}{\tan \frac{1}{2}\left(90^{\circ}-\phi-\delta\right)}
\end{aligned}
$$

On account of refraction, the values of $t$ and $A$ given by these formulas are subject to the following corrections, to wit : -

$$
\Delta t=\frac{R}{\cos \phi \cos \delta \sin t}, \quad \Delta A=\frac{\tan \phi}{\sin A} R
$$

where $R$ is the refraction in the horizon. Thus the actual values of the hour angle and azimuth at the time of rising or setting of a star are

$$
t+\Delta t \text { and } A+\Delta A
$$

## i. Differential formulas.

The general differential relations for the altitude and azimuth and the declination and hour angle systems of coördinates are : -

$$
\begin{align*}
d z & =-\cos q d \delta+\sin q \cos \delta d t+\cos A d \phi  \tag{I}\\
\sin z d A & =\sin q d \delta+\cos q \cos \delta d t-\cos z \sin A d \phi \\
d \delta & =-\cos q d z+\sin q \sin z d A+\cos t d \phi  \tag{2}\\
\cos \delta d t & =\sin q d z+\cos q \sin z d A+\sin \delta \sin t d \phi
\end{align*}
$$

The following values derived from (r) are of interest as showing the dependence of $z$ and $A$ on $t$ in special cases : -

For a star in the meridian $=0,=\frac{\cos \delta}{\sin z}$,
For a star in the prime vertical $=\cos \phi,=\sin \phi$,
For a star at elongation $=\cos \delta,=0$.

## 3. Relations of Different Kinds of Time used in Astronomy.

a. The sidereal and solar days.

The sidereal day is the interval between two successive transits of the vernal equinox over the same meridian. The sidereal time at any instant is the hour angle of the vernal equinox reckoned from the meridian towards the west from o to 24 hours. The sidereal time at any place is o when the vernal equinox is in the meridian of that place.

The solar day is the interval between two successive transits of the sun across any meridian ; and the solar time at any instant is the hour angle of the sun at that instant. The solar day begins at any place when the sun is in the meridian of that place.

The mean solar day is the interval between two successive transits over the same meridian of a fictitious sun, called the mean sun, which is assumed to move uniformly in the equator at such a rate that it returns to the vernal equinox at the same instant with the actual sun.

Time reckoned with respect to the actual sun is called apparent time, while that reckoned with respect to the mean sun is called mean time. The difference between apparent and mean time, which amounts at most to about $16^{m}$, is called the equation of time. This quantity is given for every day in the year in ephemerides.

The sidereal time when a star or other object crosses the meridian is called the right ascension of the object. The right ascension of the mean sun is also called the sidereal time of mean noon. This time is given for every day in the year in ephemerides for particular meridians, and can be found for any meridian by allowing for the difference in longitude.

The time to which ephemerides and most astronomical calculations are referred
is the solar day, beginning at noon, and divided to hours numbered continuously from $0^{h}$ to $24^{h}$. This is called astronomical time; and such a day is called the astronomical day. It begins, therefore, 12 hours later than the civil day.

## b. Relation of apparent and mean time.

$A=$ apparent time $=$ hour angle of real sun,
$M=$ mean time $=$ hour angle of mean sun,
$E=$ equation of time.

$$
M=A+E
$$

In the use of this relation, $\boldsymbol{E}$ may be most conveniently derived (by interpolation for the place of observation) from an ephemeris.

## c. Relation of sidereal and mean solar intervals of time.

$I=$ interval of mean solar time,
$I^{\prime}=$ corresponding interval in sidereal time,
$r=$ the ratio of the tropical year expressed in sidereal days to the tropical year expressed in mean solar days

$$
\begin{aligned}
& =\frac{366.2422}{365.2422}=1.002738 \\
I^{\prime} & =r I=I+(r-\mathrm{s}) I=I+0.00273^{8} I \\
I & =r^{-1} I^{\prime}=I^{\prime}-\left(\mathrm{⿺}-r^{-1}\right) I^{\prime}=I^{\prime}-0.00273^{\circ} I^{\prime} .
\end{aligned}
$$

Tables for making such calculations are usually given in ephemerides (see, for example, the American Ephemeris). Short tables for this purpose are Tables 34 and 35 of this volume.

Frequent reference is made to the relations $24^{h}$ sidereal time $=23^{h} 56^{m} 04 .^{\circ}$ ogr solar time, $24^{h}$ mean time $=24^{h} \circ 3^{m} 56 .^{s} 555$ sidereal time.
d. Interconversion of sidereal and mean solar time.

$$
\begin{aligned}
T_{m} & =\text { mean time at any place }, \\
T_{s} & =\text { corresponding sidereal time } \\
& =\text { right ascension of meridian of the place, } \\
A & =\text { right ascension of mean sun for place and date, } \\
& =\text { sidereal time of mean noon for place and date. }
\end{aligned}
$$

$T_{s}=A+T_{m}$ expressed in sidereal time. $T_{m}=\left(T_{s}-A\right)$ expressed in mean time.

The quantity $A$ is given in the ephemerides for particular meridians, and can be found by interpolation for any meridian whose longitude with respect to the meridian of the ephemeris is known. The formulas assume that $A$ is taken out of the ephemeris for the next preceding mean noon.
e. Relation of sidereal time to the right ascension and hour angle of a star.

```
\(T_{s}=\) sidereal time at any place,
\(=\) right ascension of the meridian of the place,
\(a=\) right ascension of a star,
\(t=\) the hour angle of the star at the time \(T_{0}{ }^{\circ}\)
```

$$
T_{s}=a+t, \quad t=T_{0}-a
$$

## 4. Determination of Time.

## a. By meridian transits.

A determination of time consists in finding the correction to the clock, chronometer, or watch used to record time. If $T_{0}$ denote the true time at any place of an event, $T$ the corresponding observed clock time, and $\Delta T$ the clock correction,

$$
T_{0}=T+\Delta T
$$

The simplest way to determine the clock correction is to observe the transit of a star, whose right ascension is known, across the meridian. In this case the true time $T_{0}=a$, the right ascension of the star ; and if $T$ is the observed clock time of the transit,

$$
\Delta T=a-T
$$

Meridian transits of stars may be observed by means of a theodolite or transit instrument mounted so that its telescope describes the meridian when rotated about its horizontal axis. The meridian transit instrument is specially designed for this purpose, and affords the most precise method of determining time.*

Since it is impossible to place the telescope of such an instrument exactly in the meridian, it is essential in precise work to determine certain constants, which define this defect of adjustment, along with the clock correction. These constants are the azimuth of the telescope when in the horizon, the inclination of the horizontal axis of the telescope, and the error of collimation of the telescope. $\dagger$

Let
$a=$ azimuth constant,
$b=$ inclination or level constant,
$c=$ collimation constant.
$a$ is considered plus when the instrument points east of south; $b$ is plus when the west end of the rotation axis is the higher; and $c$ is intrinsically plus when the star observed crosses the thread (or threads) too soon from lack of collimation. (The latter constant is generally referred to the clamp or circle on the horizontal axis of the instrument.)

[^7]Also let

$$
\begin{aligned}
\phi & =\text { latitude of the place } \\
\delta & =\text { declination of star observed, } \\
\alpha & =\text { right ascension of star observed, } \\
T & =\text { observed clock time of star's transit, } \\
\Delta T & =\text { the clock correction at an assumed epoch } T_{0} \\
r & =\text { the rate of the clock, or other timepiece, } \\
A & =\frac{\sin (\phi-\delta)}{\cos \delta}=\text { the " azimuth factor," } \\
B & =\frac{\cos (\phi-\delta)}{\cos \delta}=\text { the " level factor," } \\
C & =\frac{\mathbf{1}}{\cos \delta}=\text { the "collimation factor." }
\end{aligned}
$$

Then, when $a, b, c$ are small (conveniently less than $10^{8}$ each, and in ordinary practice less than $I^{s}$ each),

$$
T+\Delta T+A a+B b+C c+r\left(T-T_{0}\right)=a
$$

This is known as Mayer's formula for the computation of time from star transits.
The quantity $B b$ is generally observed directly with a striding level. Assuming it to be known and combined with $T$, the above equation gives

$$
\begin{equation*}
\Delta T+A a+C c+r\left(T-T_{0}\right)=\alpha-T \tag{I}
\end{equation*}
$$

This equation involves four unknown quantities, $\Delta T, a, c$, and $r$; so that in general it will be essential to observe at least four different stars in order to get the objective quantity $\Delta T$. Where great precision is not needed, the effect of the rate, for short intervals of time, may be ignored, and the collimation $c$ may be rendered insignificant by adjustment. Then the equation ( I ) is simplified in

$$
\begin{equation*}
\Delta T+A a=a-T \tag{2}
\end{equation*}
$$

This shows that observations of two stars of different declinations will suffice to give $\Delta T$. Since the factor $A$ is plus for stars south of the zenith (in north latitude) and minus for stars north of the zenith, if stars be so chosen as to make the two values of $A$ equal numerically but of opposite signs, $\Delta T$ will result from the mean of two equations of the form (2). With good instrumental adjustments ( $b$ and $c$ small), this simple sort of observation with a theodolite will give $\Delta T$ to the nearest second.

A still better plan for approximate determination of time is to observe a pair of north and south stars as above, and then reverse the telescope and observe another pair similarly situated, since the remaining error of collimation will be partly if not wholly eliminated. Indeed, a well selected and well observed set of four stars will give the error of the timepiece used within a half second or less. This method is especially available to geographers who may desire such an approximate value of the timepiece correction for use in determining azimuth. It will suffice in the application of the method to set up the instrument (theodolite or transit) in the vertical plane of Polaris, which is always close enough to the meridian. The determination will then proceed according to the following programme: -
I. Observe time of transit of a star south of zenith,
2. Observe time of transit of a star north of zenith.

Reverse telescope,
3. Observe time of transit of another star south of zenith,
4. Observe time of transit of another star north of zenith.

Each star observation will give an equation of the form ( $\mathbf{r}$ ), and the mean of the four resulting equations is

$$
\Delta T+a \frac{\Sigma A}{4}+c \frac{\Sigma C}{4}+r \frac{\Sigma\left(T-T_{0}\right)}{4}=\frac{\Sigma(a-T)}{4}
$$

Now the coefficient of $r$ in this equation may be always made zero by taking for the epoch $T_{0}$ the mean of the observed times $T$. Likewise, $\Sigma A$ and $\Sigma C$ may be made small by suitably selected stars, since two of the $A$ 's and $C$ 's are positive and two negative. The value $\frac{1}{4} \Sigma(\alpha-T)$ is thus always a close approximation to $\Delta T$ for the epoch $T_{0}=\frac{1}{4} \Sigma T$, when $\Sigma A$ and $\Sigma C$ approximate to zero. But if these sums are not small, approximate values of $a$ and $c$ may be found from the four equations of the form ( $\mathbf{r}$ ), neglecting the rate, and these substituted in the above formula will give all needful precision.

For refined work, as in determining differences of longitude, several groups of stars are observed, half of them with the telescope in one position and half in the reverse position, and the quantities $\Delta T, a, c$, and $r$ are computed by the method of least squares. In such work it is always advantageous to select the stars with a view to making the sums of the azimuth and collimation coefficients approximate to zero, since this gives the highest precision and entails the simplest computations.*

## b. By a single observed altitude of a star.

An approximate determination of time, often sufficient for the purposes of the geographer, may be had by observing the altitude or zenith distance of a known star. The method requires also a knowledge of the latitude of the place.

Let

$$
\begin{aligned}
z_{1} & =\text { the observed zenith distance of the star, } \\
R & =\text { the refraction, } \\
z & =\text { the true zenith distance of the star, } \\
& =z_{1}+R, \\
a, \delta, & =\text { the right ascension and declination of the star, } \\
t & =\text { hour angle of star at time of observation } \\
T & =\text { observed time when } z_{1} \text { is measured, } \\
\Delta T & =\text { correction to timepiece, } \\
\phi & =\text { latitude of place. }
\end{aligned}
$$

Then the hour angle $t$ may be computed by

$$
\tan ^{2} \frac{1}{2} t=\frac{\sin (\sigma-\phi) \sin (\sigma-\delta)}{\cos \sigma \cos (\sigma-z)}, \quad \sigma=\frac{1}{2}(\phi+\delta+z)
$$

[^8]Having the hour angle the clock correction $\Delta T$ is given by

$$
\Delta T=a+t-T
$$

in which all terms must be expressed in the same unit; i.e., in sidereal or in mean time.

The refraction $R$ may be taken from Table 3 r.
The most advantageous position of the star observed, so far as the effect of an error in the measured quantity $z_{1}$ is concerned, is in the prime vertical, but stars near the horizon should be avoided on account of uncertainties in refraction. The least favorable position of the star is in the meridian.

Compared with the preceding method the present method is inferior in precision, but it is often available when the other cannot be applied.

## c. By equal altitudes of a star.

This method is an obvious extension of the preceding method, and has the advantage of eliminating the effect of constant instrumental errors in the measured altitudes or zenith distances. Thus it is plain that the mean of the times when a (fixed) star has the same altitude east and west of the meridian, whether one can measure that altitude correctly or not, is the time of meridian transit.

This method may, therefore, give a good approximation to the timepiece correction when nothing better than an engineer's transit, whose telescope can be clamped, is available. When the instrument has a vertical circle (or when a sextant is used) a series of altitudes may be observed before meridian passage of the star, and a similar series in the reverse order with equal altitudes respectively after meridian passage. The half sums of the times of equal altitudes on the two sides of the meridian will give a series of values for the time of meridian transit from which the precision attained may be inferred.

This method is frequently applied to the sun, observations being made before ind after noon. For the theory of the corrections essential in this case on account of the changing position of the sun, on account of inequalities in the observed altitudes, etc., the reader must be referred to special treatises on practical astronomy.*

## 5. Determination of Latitude.

## a. By meridian altitudes.

The readiest method of determining the latitude of a place is to measure the meridian zenith distance or altitude of a known star. When precision is not required this process is a very simple one, since it is only essential to follow a (fixed) star near the meridian until its altitude is greatest, or zenith distance least. Thus, if the observed zenith distance is $z_{1}$, the true zenith distance $z$, and the refrac tion $R$,

$$
z=z_{1}+R ;
$$

[^9]and if the declination of the star is $\delta$ and the latitude of the place $\phi$,
$$
\phi=\delta \pm z
$$
according as the star is south or north of the zenith.
A more accurate application of the same principle is to observe the altitudes of a circumpolar star at upper and lower culmination (above and below the pole). The mean of these altitudes, corrected for refraction, is the latitude of the place. This process, it will be observed, does not require a knowledge of the star's declination.

## b. By the measured altitude of a star at a known time.

$h=$ measured altitude corrected for refraction, $T_{s}=$ observed sidereal time, $a, \delta=$ right ascension and declination of star,
$t=$ hour angle of star,
$\phi=$ latitude of place.
Then $\phi$ may be computed by means of the following formulas:-

$$
\begin{gathered}
t=T-a \\
\tan \beta=\frac{\tan \delta}{\cos t} \quad \cos \gamma=\frac{\sin h \sin \beta}{\sin \delta} \\
\phi=\beta \pm \gamma
\end{gathered}
$$

In the application of these $\beta$ may be taken numerically less than $90^{\circ}$, and since $t$ may also be taken less than $90^{\circ}, \beta$ may be taken with the same sign as $\delta . \quad \gamma$ is indeterminate as to sign analytically, but whether it should be taken as positive or negative can be decided in general by an approximate knowledge of the latitude, which is always had except in localities near the equator.

The most advantageous position of a star in determining latitude by this method is in the meridian, and the least advantageous in the prime vertical. When a series of observations on the same star is made, they should be equally distributed about the meridian ; and when more than one star is observed it is advantageous to observe equal numbers of them on the north and south of the zenith.

The application of this method to the pole star is especially well adapted to the means available to the geographer and engineer, namely, a good theodolite and a good timepiece. In this case the following simple formula for the latitude may be used : -

$$
\phi=h-p \cos t+\frac{1}{2} p^{2} \sin \mathrm{r}^{\prime \prime} \sin ^{2} t \tan h
$$

where $p$ is the polar distance of Polaris in seconds (about $5400^{\prime \prime}$ ), and the other symbols have the same meaning as defined above. Tables giving the logarithms of $p$ and $\frac{1}{2} p^{2} \sin \mathrm{t}^{\prime \prime}$ are published in the American Ephemeris.

## c. By the zenith telescope.

The zenith telescope furnishes the most precise means known for the determination of the latitude of a place. For the theory of the instrument and method when applied to refined work the reader must be referred to special treatises.* It will suffice here to state the principle of the method, which may sometimes be advantageously applied by the geographer. Let $z_{\mathrm{s}}$ be the meridian zenith distance of a star south of the zenith, and $z_{n}$ the meridian zenith distance of another star north of the zenith. Let $\delta_{s}$ and $\delta_{n}$ denote the declinations of these stars respectively. Then

$$
\begin{aligned}
& z_{3}=\phi-\delta_{n} \\
& z_{n}=\delta_{n}-\phi,
\end{aligned}
$$

whence

$$
\phi=\frac{1}{2}\left(\delta_{s}+\delta_{n}\right)+\frac{1}{2}\left(z_{s}-z_{n}\right) .
$$

It appears, therefore, that this method requires only that the difference $\left(z_{s}-z_{n}\right)$ be measured. Herein lies the advantage of the method, since that difference may be made small by a suitable selection of pairs of stars. With the zenith telescope the stars are so chosen that the difference $\left(z_{s}-z_{n}\right)$ may be measured by means of a micrometer in the telescope.

The essential principles and advantages of this method may be realized also with a theodolite, or other telescope, to which a vertical circle is attached, the difference $\left(z_{s}-z_{n}\right)$ being measured on the circle ; and a determination of latitude within $5^{\prime \prime}$ or less is thus easy with small theodolites of the best class (i.e., with those whose circles read to $1 \mathbf{o}^{\prime \prime}$ or less by opposite verniers or microscopes).

## 6. Determination of Azimuth.

## a. By observation of a star at a known time.

$T_{s}=$ sidereal time of observation,
a, $\delta=$ right ascension and declination of star observed,
$t=$ hour angle of star,
$=T_{s}-a$,
$\phi=$ latitude of place,
$A=$ azimuth of the star at the time $T_{s}$ counted from the south around by the west through $360^{\circ}$.

The azimuth $A$ may be computed by the formulas

$$
\begin{gather*}
a=\sec \phi \cot \delta, \quad b=\tan \phi \cot \delta, \\
\tan A=-\frac{a \sin t}{\mathrm{r}-b \cos t} \tag{I}
\end{gather*}
$$

The angle $A$ will fall in the same semicircle as $t$, and $A$ is thus determined by its tangent without ambiguity. The quantities $a$ and $b$ will be sensibly constant for

[^10]a given star and date; and hence they need be computed but once for a series of observations on the same star on one date.

The effects of small errors $\Delta t, \Delta \phi$, and $\Delta \delta$ in the assumed time, latitude, and declination are expressed by

$$
\frac{\cos \delta \cos q}{\sin z} \Delta t, \quad-\sin A \cot z \Delta \phi, \quad \frac{\sin q}{\sin z} \Delta \delta
$$

respectively, where $z$ and $q$ are the zenith distance and parallactic angle of the star. Hence the effect of $\Delta t$ will vanish for a star at elongation; the effect of $\Delta \phi$ vanishes for a star in the meridian, and is always small (in middle latitudes) for a close circumpolar star; the effect of $\Delta \delta$ vanishes for a star in the meridian. It appears advantageous, therefore, to observe for azimuth (in middle latitudes) close circumpolar stars at elongations, since the effect of the time error is then least, and the effects of errors in the latitude and declination are small and may be eliminated entirely by observing the same star at both elongations.

The hour angle $t_{e}$, the azimuth $A_{e}$, and the altitude $h_{e}$ of a star at elongation are given by the formulas (2) of section $2, f$. Those best suited to the purpose are

$$
K^{2}=\sin (\delta+\phi) \sin (\delta-\phi)
$$

$$
\begin{equation*}
\tan t_{e}=\frac{K}{\sin \phi \cos \delta}, \quad \tan A_{e}=\frac{\cos \delta}{K}, \quad \tan h_{e}=\frac{\sin \phi}{K} \tag{2}
\end{equation*}
$$

Knowing the time of elongation of a close circumpolar star, it suffices for many purposes to observe the angle between the star and some reference terrestrial mark at or in the vicinity of that time.

For precise determinations of azimuth it is customary to observe a star near its elongation repeatedly, thus obtaining a series of results whose mean will be sensibly free from errors of observation and errors due to instrumental defects.

The computation of the azimuth $A$ may be made accurately in all cases by the formulas (1); but when a close circumpolar star is observed near elongation, it may be more convenient to use the following formulas:-
$\Delta t=\left(t-t_{e}\right)$, or the interval before or after elongation at the time of observation,
$\Delta A=\left(A-A_{c}\right)$, or the difference in azimuths of the star at the time of elongation and at the time of observation,
$\Delta A^{\prime \prime}=\frac{(15)^{2}}{2 \rho^{\prime \prime}} \frac{\sin \delta \cos \delta}{\sin t_{e} \cos \phi}\left(\Delta t^{s}\right)^{2} \pm \frac{(15)^{8}}{2\left(\rho^{\prime \prime}\right)^{2}} \frac{\sin \delta \cos \delta}{\sin t_{e} \tan t_{e} \cos \phi}\left(\Delta t^{\circ}\right)^{8 .} \cdot *$

* To the same order of approximation one may write in the first term of this expression

$$
\frac{(\mathrm{I} 5)^{2}}{2 \rho^{\prime \prime}}\left(\Delta t^{t}\right)^{2}=\rho^{\prime \prime} 2 \sin ^{2} \frac{1}{2} \Delta t=\frac{2 \sin ^{2} \frac{1}{2} \Delta t}{\sin \mathrm{I}^{\prime \prime}},
$$

which latter is the most convenient form when tables giving $\log \frac{\left(2 \sin ^{2} \frac{1}{2} \Delta t\right)}{\sin x^{\prime \prime}}$ for the argument $\Delta t$ in time are at hand. Such tables are given in Chauvenet's Manual of Spherical and Practical Astronomy (for full title see p. lxxxii), and in Formeln und Hiilfstafeln fïr Geographische Orts bestimmungen, von Dr. Th. Albrecht. Leipzig: Wilhelm Engelmann, 4to, 2d ed., 1879.

This last formula gives $\Delta A$ in seconds of arc when $\Delta t$ is expressed in seconds of time; $\Delta t$ is considered positive in all cases (in the use of the formula), and with this convention the positive sign is used when the star is between either elongation and upper culmination, and the negative sign when the star is between either elongation and lower culmination. For a given star, place, and date the coefficients of $\left(\Delta t^{s}\right)^{2}$ and $\left(\Delta t^{s}\right)^{8}$ will be sensibly constant and their logarithms will thus be constant for a series of observations of a star on any date. By reason of the large factors $\left(\rho^{\prime \prime}=206264 .{ }^{\prime \prime} 8\right)^{2}$ and $\tan t_{e}$ in the denominator of the second term, it will be very small unless $\Delta t^{s}$ is large. Hence this term may often be neglected. Using both terms, the formula will give $\Delta A$ for Polaris to the nearest o."or when $\Delta t<40^{m}$ and when observations are made in middle latitudes.

By reference to formulas ( 2 ) of section $2, f$, it is seen that

$$
\begin{gathered}
\frac{\sin \delta \cos \delta}{\sin t_{e} \cos \phi}=\frac{\sin ^{2} \delta \cos \delta}{K} \\
\frac{\sin \delta \cos \delta}{\sin t_{e} \tan t_{e} \cos \phi}=\frac{\sin ^{2} \delta \cos ^{2} \delta \sin \phi}{K^{2}}, \\
K^{2}=\sin (\delta+\phi) \sin (\delta-\phi) .^{*}
\end{gathered}
$$

## b. By an observed altitude of a star.

$h=$ true altitude of star observed ; i.e., the observed altitude less the refraction,
$\phi=$ latitude of place,
$p=$ polar distance of star,
$A=$ azimuth of star.

$$
\begin{aligned}
\tan ^{2} \frac{1}{2} A & =\frac{\sin (\sigma-\phi) \sin (\sigma-h)}{\cos \sigma \cos (\sigma-p)} \\
\sigma & =\frac{1}{2}(\phi+h+p)
\end{aligned}
$$

The most advantageous position of the star, on account of possible error in the observed value of $h$, is that in which $\sin A$ is a maximum. This position is then at elongation for stars which elongate, in the prime vertical for stars which cross this great circle, and in the horizon for a star which neither elongates nor crosses the prime vertical. A star will elongate when $p<90^{\circ}-\phi$; it will cross the prime vertical when $p$ lies between $90^{\circ}-\phi$ and $90^{\circ}$; and it will neither elongate nor cross the prime vertical when $p>90^{\circ}$, or when the declination ( $\delta$ ) of the star is negative.

## c. By equal altitudes of a star.

By this method, when a fixed star is observed first east of the meridian and then west of the meridian at the same altitude, the direction of the meridian will

[^11]obviously be given by the mean of the azimuth circle readings for the two observed directions. This process will thus give the direction of the meridian free from the effect of any instrumental errors common to the equal altitudes observed. Neither does it require any knowledge of the star's position (right ascension and declination). It is therefore available to one provided with nothing but an instrument for measuring altitudes and azimuths, and is susceptible of considerable precision when a series of such equal altitudes is carefully referred to a terrestrial mark.

When the sun is observed, it is essential to take account of its change in declination between the first and the second observation. Let $A_{1}$ and $A_{2}$ be the true azimuths counted from the meridian toward the east and west respectively at the times $t_{1}$ and $t_{2}$ of the two observations. Also, let $\Delta \delta$ be the increase in declination of the sun in the interval $\left(t_{2}-t_{1}\right)$. Then

$$
A_{2}-A_{1}=\frac{\Delta \delta}{\cos \phi \sin \frac{1}{2}\left(t_{2}-t_{1}\right)} .
$$

Calling the azimuth circle readings for the east and west observations $R_{1}$ and $R_{2}$, respectively, the resulting azimuths are

$$
\begin{aligned}
& A_{1}=\frac{1}{2}\left(R_{2}-R_{1}\right)-\frac{1}{2}\left(A_{2}-A_{1}\right), \\
& A_{2}=\frac{1}{2}\left(R_{2}-R_{1}\right)+\frac{1}{2}\left(A_{2}-A_{1}\right) .
\end{aligned}
$$

## References.

Many excellent treatises on spherical and practical astronomy are available. Among these the most complete are the following: -
"A Manual of Spherical and Practical Astronomy," by William Chauvenet. Philadelphia: J. B. Lippincott \& Co., 2 vols., 8vo, 5th ed., 1887. "A Treatise on Practical Astronomy, as applied to Astronomy and Geodesy," by C. L. Doolittle. New York: John Wiley \& Sons, 8vo, 2d ed., 1888. "Lehrbuch der Sphärischen Astronomie," von F. Brünnow. Berlin: Fred. Dümler, 8vo, 185 r. "Spherical Astronomy," by F. Brünnow. Translated by the author from the second German edition. London : Asher \& Co., 8vo, 8865.

## THEORY OF ERRORS.

## I. Laws of Error.

The theory of errors is that branch of mathematical science which considers the nature and extent of errors in derived quantities due to errors in the data on which such quantities depend. A law of error is a relation between the magnitude of an error and the probability of its occurrence. The simplest case of a law of error is that in which all possible errors (in the system of errors) are equally likely to occur. An example of such a case is had in the errors of tabular logarithms, natural trigonometric functions, etc.; all errors from zero to a half unit in the last tabular place being equally likely to occur.

When quantities subject to errors following simple laws are combined in any manner, the law of error of the quantity resulting from the combination is in general more complex than that of either component.
Let $\varepsilon$ denote the magnitude of any error in a system of errors whose law of error is defined by $\phi(\xi)$. Then if $\epsilon$ vary continuously the probability of its occurrence will be expressed by $\phi(\epsilon) d \epsilon$. If $\epsilon$ vary continuously between equal positive and negative limits whose magnitude is $a$, the sum of all the probabilities $\phi(\epsilon) d \epsilon$ must be unity, or

$$
\int_{-a}^{+a} \phi(\epsilon) d \epsilon=\mathrm{r}
$$

For the case of tabular logarithms, etc., alluded to above, $\phi(\boldsymbol{\epsilon})=c$, a constant whose value is $\mathrm{I} /(2 a)=\mathrm{I}$, since $a=0.5$.
For the case of a logarithm interpolated between two consecutive tabular values, by the formula $v=v_{1}+\left(v_{2}-v_{1}\right) t=v_{1}(\mathrm{r}-t)+v_{2} t$, where $v_{1}$ and $v_{2}$ are the tabular values, and $t$ the interval between $v_{1}$ and the derived value $v, \phi(\epsilon)$ has the following remarkable forms when the extra decimals (practically the first of them) in $\left(v_{2}-v_{1}\right) t$ are retained $:$ -

$$
\begin{align*}
\phi(\epsilon) & =\frac{\frac{1}{2}+\epsilon}{(\mathrm{I}-t) t} \text { for values of } \epsilon \text { between }-\frac{1}{2} \text { and }-\left(\frac{1}{2}-t\right), \\
& =\frac{\mathrm{r}}{\mathrm{I}-t} \text { for values of } \epsilon \text { between }-\left(\frac{1}{2}-t\right) \text { and }+\left(\frac{1}{2}-t\right),  \tag{I}\\
& =\frac{\frac{1}{2}-\epsilon}{(\mathrm{I}-t) t} \text { for values of } \epsilon \text { between }+\left(\frac{1}{2}-t\right) \text { and }+\frac{1}{2} .
\end{align*}
$$

It thus appears that $\phi(\epsilon)$ in this case is represented by the upper base and the two sides of a trapezoid.

When, as is usually the practice, the quantity $\left(v_{2}-v_{1}\right) t$ is rounded to the nearest unit of the last tabular place, $\phi(\epsilon)$ becomes more complex, but is still represented by a series of straight lines. It is worthy of remark that the latter species of interpolated value is considerably less precise than the former, wherein an additional figure beyond the last tabular place is retained.

When an infinite number of infinitesimal errors, each subject to the law of constant probability and each as likely to be positive as negative, are combined by addition, the law of the resultant error is of remarkable simplicity and generality. It is expressed by

$$
\begin{equation*}
\phi(\epsilon)=\frac{h}{\sqrt{\pi}} e^{-k^{2} e^{2}} \tag{2}
\end{equation*}
$$

where $e$ is the Napierian base, $\pi=3.14 \mathrm{I} 59+$, and $h$ is a constant dependent on the relative magnitude of the errors in the system. This is the law of error of least squares. It is the law followed more or less closely by most species of observational errors. Its general use is justified by experience rather than by mathematical deduction.

## a. Probable, mean, and average errors.

For the purposes of comparison of different systems of errors following the same law, three different terms are in use. These are the probable error,* or that error in the system which is as likely to be exceeded as not; the mean error, or that error which is the square root of the mean of the squares of all errors in the system; and the average error, which is the average, regardless of sign, of all errors in the system. Denote these errors by $\epsilon_{p}, \epsilon_{m}, \epsilon_{a}$, respectively. Then in all systems in which positive and negative errors of equal magnitude are equally likely to occur, and in which the limits of error are denoted by $-a$ and $+a$, the analytical definitions of the probable, mean, and average errors are : -

$$
\begin{align*}
& \int_{-a}^{-\epsilon_{p}} \phi(\epsilon) d \epsilon=\int_{-\epsilon_{p}}^{o} \phi(\epsilon) d \epsilon=\int_{0}^{+} \phi(\epsilon) d \epsilon=\int_{-\epsilon_{p}}^{+a} \phi(\epsilon) d \epsilon=\frac{1}{4},  \tag{3}\\
& \epsilon_{m}^{2}=\int_{-a}^{+a} \phi(\epsilon) \epsilon^{2} d \epsilon, \quad \epsilon_{a}=\int_{-a}^{+a} \phi(\epsilon) \epsilon d \epsilon .
\end{align*}
$$

[^12]b. Probable, mean, average, and maximum actual errors of interpolated logarithms, trigonometric functions, etc.

When values of logarithms, etc., are interpolated from numerical tables by means of first differences, as explained above, the probable and other errors depend on the magnitude of the interpolating factor. Thus, the interpolated value is

$$
v=v_{1}+\left(v_{2}-v_{1}\right) t
$$

where $v_{1}$ and $v_{2}$ are consecutive tabular values and $t$ is the interpolating factor.
For the species of interpolated value wherein the quantity $\left(v_{2}-v_{1}\right) t$ is not rounded to the nearest unit of the last tabular place (or wherein the next figure beyond that place is retained) the maximum possible actual error is 0.5 of a unit of the last tabular place, and formulas ( 1 ) and (3) show that the probable, mean, and average errors are given by the following expressions :-

$$
\begin{array}{rlr}
\epsilon_{p} & =\frac{1}{4}(\mathrm{I}-t) & \text { for } t \text { between } \circ \text { and } \frac{1}{3} \\
& =\frac{1}{2}-\frac{1}{2} \sqrt{2 t(\mathrm{I}-t)} & \text { for } t \text { between } \frac{1}{3} \text { and } \frac{2}{3}, \\
& =\frac{1}{4} t & \text { for } t \text { between } \frac{3}{3} \text { and } \mathrm{I} . \\
\epsilon_{m} & =\left\{\frac{\mathrm{I}-(\mathrm{I}-2 t)^{4}}{96(\mathrm{I}-t) t}\right\}^{\frac{1}{2}} . \\
\epsilon_{a} & =\frac{\mathrm{r}-(\mathrm{I}-2 t)^{8}}{24(\mathrm{I}-t) t} & \text { for } t \text { between } o \text { and } \frac{1}{2} \\
& =\frac{1-(2 t-\mathrm{r})^{8}}{24(\mathrm{I}-t) t} & \text { for } t \text { between } \frac{1}{2} \text { and } \mathrm{I} .
\end{array}
$$

It thus appears that the probable error of an interpolated value of the species under consideration decreases from 0.25 to 0.15 of a unit of the last tabular place as $t$ increases from o to 0.5 . Hence such interpolated values are more precise than tabular values.
For the species of interpolated values ordinarily used, wherein $\left(v_{2}-v_{1}\right) t$ is rounded to the nearest unit of the last tabular place, the probable, mean, and average errors are greater than the corresponding errors for tabular values. The laws of error for thls ordinary species of interpolated value are similar to but in general more complex than those defined by equations ( I ). It must suffice here to give the practical results which flow from these laws for special values of the interpolating factor $t .^{*}$ The following table gives the probable, mean, average,
 will be observed that $t=\mathrm{r}$ corresponds to a tabular value.

[^13]Characteristic Errors of Interpolated Logarithms, etc.

| Interpolating factor $t$ | Probable error $\epsilon_{p}$ | Mean error $\boldsymbol{\epsilon}_{\boldsymbol{m}}$ | Average error $\boldsymbol{\epsilon}_{\boldsymbol{a}}$ | $\underset{\text { error }}{\text { Maximual }}$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.250 | 0.289 | 0.250 | $\frac{1}{2}$ |
| 2 | . 292 | . 408 | . 333 | 1 |
| $\frac{1}{3}$ | . 256 | . 347 | . 287 | $\stackrel{8}{8}$ |
| 1 | .276 | .382 | . 313 | 1 |
| $\frac{1}{6}$ | . 268 | . 370 | . 303 | 20 |
| $t$ | . 277 | .385 | .315 | 1 |
| $\frac{1}{4}$ | . 274 | . 380 | .311 | 13 |
| $\frac{1}{1}$ | .279 | . $3^{89}$ | . 318 | 1 |
| $t$ | . 278 | . 386 | . 316 | 17 |
| $\frac{18}{10}$ | .281 | . 392 | . 320 | 1 |

## 2. The Method of Least Squares.

a. General statement of method.

When the errors to which observed quantities are subject follow the law expressed by

$$
\phi(\epsilon)=\frac{h}{\sqrt{\pi}} e^{-h^{2} \varepsilon^{2}},
$$

a unique method results for the computation of the most probable values of the observed quantities and of quantities dependent on the observed quantities. The method requires that the sum of the weighted squares of the corrections to the observed quantities shall be a minimum,* subject to whatever theoretical conditions the corrections must satisfy. These conditions are of two kinds, namely, those expressing relations between the corrections only, and those expressing relations between the corrections and other unknown quantities whose values are disposable in determining the minimum. A familiar illustration of the first class of conditions is presented by the case of a triangle each of whose angles is measured, the condition being that the sum of the corrections is a constant. An equally familiar illustration of the second class of conditions is found in the case where the sum and difference of two unknown quantities are separately observed; in this case the two unknowns are to be found along with the corrections.

Mathematically, the general problem of least squares may be stated in two

[^14]equations. Thus, let $x, y, z, \ldots$ be the observed quantities with weights $p, q$, $r, \ldots$ Let the corrections to the observed quantities be denoted by $\Delta x, \Delta y$, $\Delta z, \ldots$; so that the corrected quantities are $x+\Delta x, y+\Delta y, z+\Delta z, \ldots$ Let the disposable quantities whose values are to be determined along with the corrections be denoted by $\xi, \eta, \zeta, \ldots$ Then, the theoretical conditions which must be satisfied by $x+\Delta x, y+\Delta y, z+\Delta z, \ldots$ and by $\xi, \eta, \zeta, \ldots$ may be symbolized by
\[

$$
\begin{equation*}
F_{n}(\xi, \eta, \zeta, \ldots x+\Delta x, y+\Delta y, z+\Delta z, \ldots)=0 \tag{4}
\end{equation*}
$$

\]

Subject to the conditions specified by the $n$ equations (4), we must also have

$$
\begin{align*}
p(\Delta x)^{2}+q(\Delta y)^{2}+r(\Delta z)^{2}+\ldots & =\text { a minimum }  \tag{5}\\
& =u, \text { say }
\end{align*}
$$

Equations (4) and (5) contain the solution of every problem of adjustment by the method of least squares. Two examples may suffice to illustrate their use.

First, take the case of the observed angles of a triangle alluded to above. Calling the observed angles $x, y, z$, we have
or

$$
\begin{gathered}
x+\Delta x+y+\Delta y+z+\Delta z=180^{\circ}+\text { spherical excess } \\
\begin{aligned}
\Delta x+\Delta y+\Delta z & =180^{\circ}+\text { spherical excess }-(x+y+z) \\
& =c, \text { say. }
\end{aligned}
\end{gathered}
$$

This is the only condition of the form (4). The problem is completely stated, then, in the two equations

$$
\begin{aligned}
\Delta x+\Delta y+\Delta z & =c \\
p(\Delta x)^{2}+q(\Delta y)^{2}+r(\Delta z)^{2} & =\text { a min. }=u
\end{aligned}
$$

To solve this problem the simplest mode of procedure is to eliminate one of the corrections by means of the first equation and then make $u$ a minimum. Thus, eliminating $\Delta z$, there results

$$
u=p(\Delta x)^{2}+q(\Delta y)^{2}+r(c-\Delta x-\Delta y)^{2}
$$

The conditions for a minimum of $u$ are : -

$$
\begin{aligned}
& \frac{\partial u}{\partial \Delta x}=(p+r) \Delta x+r \Delta y-r c=0 \\
& \frac{\partial u}{\partial \Delta y}=r \Delta x+(q+r) \Delta y-r c=0
\end{aligned}
$$

and these give, in connection with the value $\Delta z=c-\Delta x-\Delta y$,

$$
\Delta x=\frac{Q}{p}, \quad \Delta y=\frac{Q}{q}, \quad \Delta z=\frac{Q}{r}
$$

where

$$
Q=\frac{c}{\frac{1}{p}+\frac{1}{q}+\frac{\mathbf{x}}{r}}
$$

When the weights are equal, or when $p=q=r$, the corrections are -

$$
\Delta x=\Delta y=\Delta z=\frac{1}{3} c
$$

Secondly, take the case, also alluded to above, of the observed sum and the observed difference of two numbers. Denote the numbers by $\xi$ and $\eta$, the latter being the smaller. Let the observed values of the sum $(\xi+\eta)$ be denoted by $x_{1}, x_{2}, \ldots x_{m}$ and their weights $p_{1}, p_{2}, \ldots p_{m}$ respectively. Likewise, call the observed values of the difference $(\xi-\eta), y_{1}, y_{2}, \ldots y_{n}$, and their weights $q_{1}, q_{2} \ldots q_{n}$ respectively. Then there will be $m+n$ equations of the type (4), namely: 一

$$
\begin{align*}
& \xi+\eta-\left(x_{1}+\Delta x_{1}\right)=0 \\
& \xi+\eta-\left(x_{2}+\Delta x_{2}\right)=0 \\
& \dot{\xi}+\eta-\left(x_{m}+\Delta x_{m}\right)=0  \tag{a}\\
& \xi-\eta-\left(y_{1}+\Delta y_{1}\right)=0 \\
& \xi-\eta-\left(y_{2}+\Delta y_{2}\right)=0 \\
& \cdot \cdot \cdot \cdot \cdot \\
& \xi-\eta-\left(y_{n}+\Delta y_{n}\right)=0
\end{align*}
$$

and the minimum equation is

$$
u=p_{1}\left(\Delta x_{1}\right)^{2}+p_{2}\left(\Delta x_{2}\right)^{2}+\ldots+q_{1}\left(\Delta y_{1}\right)^{2}+q_{2}\left(\Delta y_{2}\right)^{2}+\ldots=\text { a min. }(\mathrm{b})
$$

The equations of group (a) give

$$
\begin{align*}
& \Delta x_{1}=\xi+\eta-x_{1} \\
& \Delta x_{2}=\xi+\eta-x_{2} \\
& \cdots  \tag{c}\\
& \Delta y_{1}=\xi-\eta-y_{1} \\
& \Delta y_{2}=\xi-\eta-y_{2},
\end{align*}
$$

and these values in (b) give

$$
\begin{equation*}
u=p_{1}\left(\xi+\eta-x_{1}\right)^{2}+\ldots+q_{1}\left(\xi-\eta-y_{1}\right)^{2}+\ldots \tag{d}
\end{equation*}
$$

Thus it appears that all conditions will be satisfied if $\xi$ and $\eta$ are so determined as to make $u$ in (d) a minimum. Hence, using square brackets to denote summation of like quantities, the values of $\xi$ and $\eta$ must be found from

$$
\begin{align*}
& \frac{\partial u}{\partial \xi}=[p+q] \xi+[p-q] \eta-[p x+q y]=0  \tag{e}\\
& \frac{\partial u}{\partial \eta}=[p-q] \xi+[p+q] \eta-[p x-q y]=0
\end{align*}
$$

Equations (e) give $\xi$ and $\eta$, and these substituted in (c) will give the corrections to the observed quantities.

## b. Relation of probable, mean, and average errors.

The introduction of the law of error (2) in equations (3) furnishes the following relations, when it is assumed that the limits of possible error are $-\infty$ and $+\infty$ :

$$
\begin{equation*}
\epsilon_{p}=0.6745 \epsilon_{m}=0.8453 \epsilon_{a} . \tag{6}
\end{equation*}
$$

## c. Case of a single unknown quantity.

The case of a single unknown quantity whose observed values are of equal or unequal weight is comprised in the following formulas: -

$$
\begin{aligned}
x_{1}, x_{2}, \ldots x_{m} & =\text { observed values of unknown quantity, } \\
p_{1}, p_{2}, \ldots p_{m} & =\text { the weights of } x_{1}, x_{2}, \ldots \\
v_{1}, v_{2}, \ldots v_{m} & =\text { most probable corrections to } x_{1}, x_{2}, \ldots \\
x & =\text { most probable value of the unknown quantity }, \\
m & =\text { the number of independent observations. }
\end{aligned}
$$

Then the conditional equations (4) are

$$
\begin{aligned}
& x-x_{1}=v_{1} \\
& x-x_{2}=v_{2} \\
& \cdot \cdot \cdot \cdot \\
& x-x_{m}=v_{m}
\end{aligned}
$$

the minimum equation (5) is

$$
p_{1} v_{1}^{2}+p_{2} v_{2}^{2}+\ldots=\left[p v^{2}\right]=\left[p\left(x-x_{i}\right)^{2}\right]=\text { a min. }
$$

where $i=\mathrm{x}, 2, \ldots m$, and

$$
x=\frac{p_{1} x_{1}+p_{2} x_{2}+\ldots p_{m} x_{m}}{p_{1}+p_{2}+\cdots p_{m}}=\frac{[p x]}{[p]} .
$$

When the weights are equal, $p_{1}=p_{2}=\ldots=p_{m}$, and

$$
x=\frac{[x]}{m}
$$

or the arithmetic mean of the observed values.

$$
\begin{aligned}
\text { Weight of } x & =[p] \text { when the } p \text { 's are unequal, } \\
& =m \text { when the } p \text { 's are equal. }
\end{aligned}
$$

Mean error of an observed value of weight unity $=\sqrt{\frac{[p v v]}{m-\mathbf{I}}}$ for unequal weights,

$$
=\sqrt{\frac{[v v]}{m-1}} \text { for equal weights. }
$$

Mean error of an observed value of weight $p=\sqrt{\frac{[p v v]}{(m-r) p}}$ for unequal weights.

$$
\text { Mean error of } \begin{aligned}
x & =\sqrt{\frac{[p v v]}{(m-1)[p]}} \text { for unequal weights, } \\
& =\sqrt{\frac{[v v]}{m(m-1)}} \text { for equal weights. }
\end{aligned}
$$

The corresponding probable errors are found by multiplying these values by 0.6745 . See equation (6).

A formula for the average error sometimes useful is

$$
\begin{aligned}
\text { Average error } & =\frac{[p v]}{\sqrt{(m-1)[p]}} \text { for unequal weights. } \\
& =\frac{[v]}{\sqrt{m(m-1)}} \text { for equal weights. }
\end{aligned}
$$

In these the residuals $v$ are all taken with the same sign. A sufficient approximation in many cases of equal weights is $\frac{[v]}{m}$; but the above formulas dependent on the squares of the residuals are in general more precise.

An important check on the computation of $x$ is $[p \tau]=o ; i . e$., the sum of the residuals $v$, each multiplied by its weight, is zero if the computation is correct.

## d. Case of observed function of several unknown quantities $\xi, \eta, \zeta \ldots$

A case of frequent occurrence, and one which includes the preceding case, is that in which a function of several unknown quantities is observed. Thus, for example, the observed time of passage of a star across the middle thread of a transit instrument is a function of the azimuth and collimation of the transit instrument and the error of the timepiece used. In cases of this kind the conditional equations of the type (4) assume the form

$$
F(\xi, \eta, \zeta \ldots x+\Delta x)=0
$$

that is, each of them contains but one observed quantity $x$ along with several disposable (disposable in satisfying the minimum equation) quantities $\xi, \eta, \zeta \ldots$

The process of solution in this case consists in eliminating the corrections $\Delta x_{1}, \Delta x_{2}, \ldots$ from the above conditional equations, substituting their values in the minimum equation (5), and then placing the differential coefficients of $u$ with respect to $\xi, \eta, \zeta \ldots$ separately equal to zero. There will thus result as many independent equations as there are unknown quantities of the class in which $\xi, \eta$, $\zeta \ldots$ fall, the remaining unknown quantities $\Delta x_{1}, \Delta x_{2}, \ldots$, or the corrections to the observed values, are then found from the conditional equations.

In many applications it happens that the conditional equations

$$
F(\xi, \eta, \zeta, \ldots x+\Delta x)=0
$$

are not of the linear form. But they may be rendered linear in the following manner. First, eliminate the quantities $x+\Delta x$ from the conditional equations. The result of this elimination may be written

$$
f(\xi, \eta, \zeta \ldots)-x-\Delta x=0
$$

Secondly, put

$$
\begin{aligned}
& \xi=\xi_{0}+\Delta \xi, \\
& \eta=\eta_{0}+\Delta \eta,
\end{aligned}
$$

where $\xi_{0}, \eta_{0}, \ldots$ are approximate values of $\xi, \eta, \ldots$, found in any manner, and $\Delta \xi, \Delta \eta, \ldots$ are corrections thereto. Then supposing the approximate values
$\xi_{0}, \eta_{0}, \ldots$ so close that we may neglect the squares, products, and higher powers of $\Delta \xi, \Delta \eta, \ldots$, Taylor's series gives

$$
f\left(\xi_{0}, \eta_{0}, \xi_{0}, \ldots\right)+\frac{\partial f}{\partial \xi} \Delta \xi+\frac{\partial f}{\partial \eta} \Delta \eta+\frac{\partial f}{\partial \zeta} \Delta \zeta+. \quad-x-\Delta x=0
$$

which is linear with respect to the corrections $\Delta \xi, \Delta \eta, \ldots$ For brevity, and for the sake of conformity with notation generally used, put

$$
\begin{gathered}
n=x-f\left(\xi_{0}, \eta_{0}, \zeta_{0} \ldots\right) \\
v=\Delta x, \\
a=\frac{\partial f}{\partial \xi}, \quad b=\frac{\partial f}{\partial \eta}, \quad c=\frac{\partial f}{\partial \xi}, \ldots \\
x=\Delta \xi, \quad y=\Delta \eta, \quad z=\Delta \zeta, \ldots
\end{gathered}
$$

Then the conditional equations will assume the form

$$
a x+b y+c z+\ldots-n=v ;
$$

and if they are $m$ in number they may be written individually thus : -

$$
\begin{align*}
& a_{1} x+b_{1} y+c_{1} z+\ldots-n_{1}=v_{1}, \\
& a_{2}+b_{2}+c_{2}+\ldots-n_{2}=v_{2},  \tag{a}\\
& \cdots \\
& a_{m}+b_{m}+c_{m}+\ldots-n_{m}=v_{m} .
\end{align*}
$$

The minimum equation (5) becomes

$$
u=\left[p v^{2}\right]=\left[p(a x+b y+c z+\ldots-n)^{2}\right] ;
$$

so that placing $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}, \ldots$ separately equal to zero will give as many independent equations as there are values of $x, y, z, \ldots$ The resulting equations are in the usual (Gaussian) notation of least squares:-

$$
\begin{align*}
& {[p a a] x+[p a b] y+[p a c] z+\ldots-[p a n]=0,} \\
& {[p a b]+[p b b]+[p b c]+\ldots-[p b n]=0,}  \tag{b}\\
& {[p a c]+[p b c]+[p c c]+\ldots-[p c n]=0,}
\end{align*}
$$

The equations (a) are sometimes called observation-equations. The absolute term $n$ is called the observed quantity. It is always equal to the observed quantity minus the computed quantity $f\left(\xi_{0}, \eta_{0}, \zeta \ldots\right)$, which latter is assumed to be free from errors of observation. The term $v$ is called the residual. It is sometimes, though quite erroneously, replaced by zero in the equations (a).
The equations (b) are called normal equations. They are usually formed directly from equations (a) by the following process: Multiply each equation by the coefficient of $x$ and by the weight $p$ of the $y$ in the same equation, and add the products. The result is the first equation of (b), or the normal equation in $x_{0}$ The normal equations in $y, z, \ldots$ are found in a similar manner.

A noteworthy peculiarity of the normal equations is their symmetry. Hence in forming equations (b) from (a) it is not essential to compute all the coefficients of $x, y, z, \ldots$ except in the first equation.

Checks on the computed values of the numerical terins in the normal equations are found thus : Add the coefficients $a, b, c, \ldots$ of $x, y, z, \ldots$ in (a) and put

$$
\begin{aligned}
& a_{1}+b_{1}+c_{1}+\ldots=s_{1}, \\
& a_{2}+b_{2}+c_{2}+\ldots=s_{2},
\end{aligned}
$$

Multiply each of these, first, by its $p a$; secondly, by its $p b$, etc., and then add the products. The results are

$$
\begin{aligned}
& {[p a a]+[p a b]+[p a c]+\ldots=[p a s]} \\
& {[p a b]+[p b b]+[p b c]+\ldots=[p b s]}
\end{aligned}
$$

These will check the coefficients of $x, y, z, \ldots$ in (b). To check the absolute terms, multiply each of the above sums by its $n p$, and add the products. The result is

$$
[p a n]+[p b n]+[p c n]+\ldots=[p s n]
$$

which must be satisfied if the absolute terms are correct.
Checks on the computation of $x, y, z, \ldots$ from (b) and of $v_{1}, v_{2}, \ldots$ from (a) are furnished by

$$
[p a v]=0, \quad[p b v]=0, \quad[p c v]=0, \quad \cdots
$$

To get the unknowns $x, y, z$, and their weights simultaneously, the best method of procedure is, in general, the following : For brevity replace the absolute terms in (b) by $A, B, C, \ldots$ respectively. Then the solution of (b) will be expressed by

$$
\begin{align*}
& x=a_{1} A+\beta_{1} B+\gamma_{1} C+\ldots \\
& y=a_{2}+\beta_{2}+\gamma_{2}+\ldots  \tag{c}\\
& z=a_{3}+\beta_{3}+\gamma_{3}+\ldots,
\end{align*}
$$

in which $\alpha_{1}, \beta_{1}, \gamma_{1}, \ldots$ are numerical quantities; and

$$
\begin{align*}
& \text { weight of } x=\frac{\mathrm{r}}{a_{1}}, \\
& \text { weight of } y=\frac{\mathrm{r}}{\beta_{2}}  \tag{d}\\
& \text { weight of } z=\frac{\mathrm{r}}{\gamma_{3}}
\end{align*}
$$

To compute mean (and hence probable) errors the following formulas apply:$m=$ the number of observed quantities $n$
$=$ number of equations of condition,
$\mu=$ number of the quantities $x, y, z, \ldots$
$\epsilon_{m}=$ mean error of an observed quantity ( $n$ ) of weight unity,
$\epsilon_{p}=$ corresponding probable error $=0.6745 \epsilon_{m}$.

$$
\begin{aligned}
\overline{\epsilon_{m}} & =\sqrt{\frac{[p v v]}{m-\mu}} \text { for unequal weights }, \\
& =\sqrt{\frac{[v v]}{m-\mu}} \text { for equal weights, }
\end{aligned}
$$

Mean error of any observed quantity ( $n$ ) of weight $p=\frac{\epsilon_{m}}{\sqrt{p}}$,
Mean error of $x=\epsilon_{m} \sqrt{\alpha_{1}}$,
Mean error of $y=\epsilon_{m} \sqrt{\bar{\beta}_{2}}$,
Mean error of $z=\epsilon_{m} \sqrt{\gamma_{3}}$,
where $a_{1}, \beta_{2}, \gamma_{3}, \ldots$ are defined by equations (c) and (d) above.

## e. Case of functions of several observed quantities $x, y, z, \ldots$

This case is that in which the conditional equations (4) contain no disposable quantities $\xi, \eta, \zeta, \ldots$ It is the opposite extreme to that represented by the case of the preceding section.* It finds its most important and extensive application in the adjustment of triangulation, wherein the observed quantities are the angles and bases of the triangulation, and the conditions (4) arise from the geometrical relations which the observed quantities plus their respective corrections mus ${ }^{+}$ satisfy.

An outline of the general method of procedure in this case is the following :-
The first step consists in stating the conditional equations and in reducing them to the linear form if they are not originally so. The form in which they present themselves is (4) with $\underline{\xi}, \eta, \zeta, \ldots$ suppressed, or

$$
F\left(x_{1}+\Delta x_{1}, x_{2}+\Delta x_{2}, x_{3}+\Delta x_{3}, \ldots\right)=\mathrm{o}
$$

wherein $x, y, z, \ldots$ of (4) are replaced by $x_{1}, x_{2}, x_{3} \ldots$ for the purpose of simplicity in the sequel. If this equation is not linear, Taylor's series gives

$$
F\left(x_{1}, x_{2}, x_{3} \ldots\right)+\frac{\partial F}{\partial x_{1}} \Delta x_{1}+\frac{\partial F}{\partial x_{2}} \Delta x_{2}=\ldots=0
$$

since the method supposes that the squares, products, etc., of $\Delta x_{1}, \Delta x_{2} \ldots$ may be neglected. The last equation is then linear with respect to the corrections $\Delta x_{1}, \Delta x_{2} \ldots$ which it is desired to find.

For brevity put

$$
F\left(x_{1}, x_{2}, x_{3} \ldots\right)=q_{1}, \text { a known quantity, }
$$

$$
\frac{\partial F}{\partial x_{1}}=a_{1}, \quad \frac{\partial F}{\partial x_{2}}=a_{2}, \quad \frac{\partial F}{\partial x_{8}}=a_{3}, \ldots
$$

Then the conditional equations will be of the type

$$
a_{1} \Delta x_{1}+a_{2} \Delta x_{2}+a_{3} \Delta x_{3}+\ldots+q_{1}=0
$$

[^15]There will be as many equations of this type as there are independent relations which the quantities $x_{1}+\Delta x_{1}, x_{2}+\Delta x_{2}, \ldots$ must satisfy. Suppose there are $k$ such relations, and let the differential coefficients $\partial F / \partial x_{1}, \partial F / \partial x_{2}, \ldots$ for the second relation be denoted by $b_{1}, b_{2}, b_{3}, \ldots$; for the third relation by $c_{1}, c_{2}, c_{8}, \ldots$, etc. Then all of the conditional equations may be written thus:

the number of these equations being $k$.
Call the weights of the observed quantities $x_{1}, x_{2}, \ldots p_{1}, p_{2}, \ldots$ Then, subject to the conditions (a) we must have (in accordance with (5))
a minimum.

$$
\begin{equation*}
u=p_{1}\left(\Delta x_{1}\right)^{2}+p_{2}\left(\Delta x_{2}\right)^{2}+\ldots=\left[p(\Delta x)^{2}\right] \tag{b}
\end{equation*}
$$

Equations (a) and (b) contain the solution of all problems falling under the present case. Obviously, the number of conditions (a) must be less than the number of observed quantities $x$, or less than the number of $\Delta x$ 's in (b); in other words, if $m$ denote the number of observed quantities, $m>k$, for if $m \overline{<} k$ the minimum equation (b) has no meaning.

The question presented by $(a)$ and $(b)$ is one of elimination only. Two methods, the one direct and the other indirect, are available. Thus, by the direct method one finds from (a) as many $\Delta x$ 's as there are equations (a), or $k$ such values, and substitutes them in (b). The remaining ( $m-k$ ) values of $\Delta x$ in (b) may then be treated as independent and the differential coefficients of $u$ with respect to each of them placed equal to zero. Thus all of the corrections $\Delta x$ become known.

By the indirect process, one multiplies the first of equations (a) by a factor $Q_{1}$, the second by $Q_{2}$, the third by $Q_{3}, \ldots$ and subtracts the differential (with respect to the $\Delta x$ 's) of the sum of these products from half the differential of (b). The result of these operations is

$$
\begin{aligned}
\frac{1}{2} d u & =\left\{p_{1} \Delta x_{1}-\left(a_{1} Q_{1}+b_{1} Q_{2}+c_{1} Q_{3}+\ldots\right)\right\} d \Delta x_{1} \\
& +\left\{p_{2} \Delta x_{2}-\left(a_{2} Q_{1}+b_{2} Q_{2}+c_{2} Q_{3}+\ldots\right)\right\} d \Delta x_{2} \\
& +\cdots \\
& +\left\{p_{m} \Delta x_{m}-\left(a_{m} Q_{1}+b_{m} Q_{2}+c_{m} Q_{3}+\ldots\right)\right\} d \Delta x_{m}
\end{aligned}
$$

Now we may choose the factors $Q_{1}, Q_{2}, \ldots Q_{k}$ in such a way as to make $k$ of the coefficients of the differentials in this equation disappear; and after thus eliminating $k$ of these differentials we are at liberty to place the coefficients of the remaining ( $m-k$ ) differentials equal to zero. Thus all conditions are satisfied by making

$$
\begin{align*}
& a_{1} Q_{1}+b_{1} Q_{2}+c_{1} Q_{3}+\ldots-p_{1} \Delta x_{1}=0 \\
& a_{2}+b_{2}+c_{2}+\ldots-p_{2} \Delta x_{2}=0  \tag{c}\\
& a_{m}+b_{m}+c_{m}+\ldots-p_{m} \Delta x_{m}=0
\end{align*}
$$

and the values of the corrections will be given by these equations when the factors $Q_{1}, Q_{2}, \ldots$ are known. To find the latter it suffices to substitute the values
of $\Delta x, \Delta x_{2}, \ldots$ from (c) in (a), whereby there will result $k$ equations containing the $Q_{1}, Q_{2} \ldots Q_{k}$ alone as unknowns. The result of this substitution is

$$
\begin{align*}
& {\left[\frac{a a}{p}\right] Q_{1}+\left[\frac{a b}{p}\right] Q_{2}+\left[\frac{a c}{p}\right] Q_{3}+\ldots+q_{1}=0} \\
& {\left[\frac{a b}{p}\right]+\left[\frac{b b}{p}\right]+\left[\frac{b c}{p}\right]+\ldots+q_{2}=0}  \tag{d}\\
& {\left[\frac{a c}{p}\right]+\left[\frac{b c}{p}\right]+\left[\frac{c c}{p}\right]+\ldots+q_{3}=0}
\end{align*}
$$

These equations (d) are derived directly from (c) in the following manner: multiply the first of $(c)$ by $\frac{a_{1}}{p_{1}}$, the second by $\frac{a_{2}}{p_{2}}$, etc., sum the products, and compare the sum with the first of $(a)$. The first of $(d)$ is then evident ; the others are obtained in a similar way.

The mean error of an observed quantity of weight unity is in this case given by the formula

$$
\epsilon_{m}=\sqrt{\frac{\left[p(\Delta x)^{2}\right]}{k}}
$$

where $k$ is the number of conditions (a); and the mean error of any observed value of weight $p$ is

$$
\frac{\epsilon_{m}}{\sqrt{\ddot{p}}}
$$

## f. Computation of mean and probable errors of functions of observed quantities.

Let $V$ denote any function of one or more independently observed quantities $x, y, z, \ldots$; that is, let

$$
V=f(x, y, z \ldots)
$$

A question of frequent occurrence with respect to such functions is, What is the mean* error of $V$ in terms of the mean errors of $x, y, z, \ldots$ ? The answer to this question given by the method of least squares assumes that the actual errors (whatever they may be) of $x, y, z, \ldots$ are so small that the actual error of $V$ is a linear function of the errors of $x, y, z$. In other words, if $e_{x}, e_{y}, e_{z}, \ldots$ denote the actual errors of $x, y, z, \ldots$, and $\Delta V$ denote the corresponding actual error of $V$, the method assumes that

$$
\begin{equation*}
\Delta V=\frac{\partial V}{\partial x} e_{x}+\frac{\partial V}{\partial y} e_{y}+\frac{\partial V}{\partial z} e_{x}+\ldots \tag{a}
\end{equation*}
$$

wherein the squares, products, etc., of $e_{x}, e_{y}, e_{z}, \ldots$ are omitted.
This condition being fulfilled, let $\epsilon$ denote the mean error of $V$, and $\epsilon_{x}, \epsilon_{y}, \epsilon_{z} \ldots$ denote those of $x, y, z, \ldots$ respectively. Then the law of error of least squares requires that

$$
\begin{equation*}
\epsilon^{2}=\left(\frac{\partial V}{\partial x}\right)^{2} \epsilon_{x}{ }^{2}+\left(\frac{\partial V}{\partial y}\right)^{2} \epsilon_{y}{ }^{2}+\left(\frac{\partial V}{\partial z}\right)^{2} \epsilon_{z}{ }^{2}+\ldots \tag{b}
\end{equation*}
$$

[^16]This equation includes all cases. Its analogy with (a) should be noted, since the step from $(a)$ to $(b)$ is clear when the correct form of $(a)$ is known. Mistakes in the application of (b) are most likely to arise from a lack of knowledge of the independently observed quantities $x, y, z, \ldots$ or from a lack of knowledge of the true form of (a). Hence,* in deriving probable errors of functions of observed quantities attention should be given first to the construction of the expression for the actual error (a).

A few examples may serve to illustrate the use of (a) and (b).
(I.) Suppose

Then

$$
V=f(x, y, z, \ldots)=a(x-y)+b(y+z)+c(z-\mathrm{I})
$$

$$
\begin{aligned}
\frac{\partial V}{\partial x} & =a, \quad \frac{\partial V}{\partial y}=b-a, \quad \frac{\partial V}{\partial z}=b+c \\
\Delta V & =a e_{x}+(b-a) e_{y}+(b+c) e_{y,} \\
\epsilon^{2} & =a^{2} \epsilon_{x}^{2}+(b-a)^{2} \epsilon_{y}{ }^{2}+(b+c)^{2} \varepsilon_{z}^{2} .
\end{aligned}
$$

(2.) Suppose

$$
V=f(x, y, z \ldots)=\frac{a}{x}+b \frac{y}{z^{2}}
$$

Then

$$
\begin{gathered}
\frac{\partial V}{\partial x}=-\frac{a}{x^{2}} \quad \frac{\partial V}{\partial y}=\frac{b}{z^{2}} \quad \frac{\partial V}{\partial z}=-\frac{2 b y}{z^{8}}, \\
\Delta V=-\frac{a}{x^{2}} e_{x}+\frac{b}{z^{2}} e_{y}-\frac{2 b y}{z^{3}} e_{z,} \\
\epsilon^{2}=\frac{a^{2}}{x^{4} \epsilon_{x}^{2}+\frac{b^{2}}{z^{2}} \epsilon_{y}^{2}+\frac{4 b^{2} y^{2}}{z^{6}} \epsilon_{z}^{2} .}
\end{gathered}
$$

(3.) Suppose

Then

$$
\frac{\partial V}{\partial x}=\frac{a \mu}{x}, \quad \frac{\partial V}{\partial y}=b \cos y, \quad \frac{\partial V}{\partial z}=\frac{c \mu}{\sin z \cos z},
$$

and

$$
\epsilon^{2}=\left(\frac{a \mu}{x}\right)^{2} \epsilon_{x}^{2}+(b \cos y)^{2} \epsilon_{y}^{2}+\left(\frac{2 c \mu}{\sin 2 z}\right)^{2} \epsilon_{x}^{2} .
$$

(4.) Suppose the case of a single triangle all of whose angles are observed. What is the mean error, rst, of an observed angle ; 2 d , of the correction to an observed angle ; and 3d, of the corrected or adjusted angle ?
Let $x, y, z$ denote the observed angles, $p, q, r$ their weights, and $\Delta x, \Delta y, \Delta z$ the corresponding corrections.

Then, as shown on p . lxxxvii,

$$
\begin{gathered}
\Delta x+\Delta y+\Delta z=c=180^{\circ}+\text { sph. excess }-(x+y+z) \\
=\text { error of closure of triangle, } \\
Q=\frac{c}{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}}, \\
\Delta x=\frac{Q}{p}, \quad \Delta y=\frac{Q}{q}, \quad \Delta z=\frac{Q}{r}
\end{gathered}
$$

* As remarked by Sir George Airy in his Theory of Errors.
$\dagger \mu=$ modulus of common logarithms.

For brevity, put

Then

$$
g=180^{\circ}+\text { spherical excess, } \quad h=\frac{1}{\frac{1}{p}+\frac{1}{q}+\frac{1}{r}} .
$$

$$
\begin{aligned}
Q & =\frac{h}{(g-x-y-z)}=h c, \\
\Delta x & =\frac{h}{p}(g-x-y-z), \\
x+\Delta x & =\frac{h}{p}(g-x-y-z)+x,
\end{aligned}
$$

with similar expressions for the other two angles.
Now by the formula on p. xcv the square of the mean error of an observed angle of weight unity is (since there is but one condition to which $\Delta x, \Delta y, \Delta z$ are subject),

$$
p(\Delta x)^{2}+q(\Delta y)^{2}+r(\Delta z)^{2}=\frac{Q^{2}}{h}=h c^{2} .
$$

Hence, the squares of the mean errors of the observed angles $x, y, z$, their weights being $p, q, r$ respectively, are

$$
\frac{h c^{2}}{p}, \quad \frac{h c^{2}}{q}, \quad \frac{h c^{2}}{r}
$$

respectively.
To get the mean error of a correction, $\Delta x$ for example, formula (a) gives

$$
\Delta V=\Delta(\Delta x)=-\frac{h}{\bar{p}}\left(e_{x}+e_{y}+e_{z}\right),
$$

and the corresponding expressions for the actual errors of $\Delta y$ and $\Delta z$ are found from this by replacing $p$ by $q$ and $r$ respectively. Thus by (b), observing that the mean errors of $x, y, z$ are given above, there result

Square of mean error of $\Delta x=(h c / p)^{2}$,

$$
\begin{array}{llll}
\text { " } & \text { " } & \text { " } & \Delta y=(h c / q)^{2} \\
" & " & " & \Delta z=(h c / r)^{2}
\end{array}
$$

Likewise, the formula for the actual error of $x+\Delta x$ is

$$
\Delta V=\Delta(x+\Delta x)=\left(\mathrm{I}-\frac{h}{p}\right) e_{x}-\frac{h}{\bar{p}} e_{y}-\frac{h}{\bar{p}} e_{x}
$$

and the corresponding expressions for the actual errors of $y+\Delta y$ and $z+\Delta z$ are found by interchange of $q$ and $r$ with $p$. Thus the squares of the mean errors of the adjusted angles are:-

$$
\begin{array}{ll}
\text { for }(x+\Delta x), & \frac{h c^{2}}{p}\left(\mathrm{x}-\frac{h}{p}\right), \\
\text { for }(y+\Delta y), & \frac{h c^{2}}{q}\left(\mathrm{x}-\frac{h}{q}\right), \\
\text { for }(z+\Delta z), & \frac{h c^{2}}{r}\left(\mathrm{r}-\frac{h}{r}\right) .
\end{array}
$$

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In case the weights are equal, or in case $p=q=r, h=\frac{1}{3}$, and there result, -

Square of mean error of observed angle $\quad=\frac{1}{3} c^{2}$,

" " " " adjusted angle = 号 $c^{a}$,
where $c$ is the error of closure of the triangle; so that in this case of equal weights the three mean errors are to one another as $\frac{1}{3} \sqrt{3}, \frac{1}{3}$, and $\frac{1}{3} \sqrt{2}$.

## References.

The literature of the theory of errors, especially as exemplified by the method of least squares, is very extensive. Amongst the best treatises the following are worthy of special mention : Method of Least Squares, Appendix to vol. ii. of Chauvenet's "Spherical and Practical Astronomy." Philadelphia: J. B. Lippincott \& Co., 8vo, 5th ed., 1887. "A Treatise on the Adjustment of Observations, with Applications to Geodetic Work and Other Measures of Precision," by T. W. Wright. New York: D. Van Nostrand, 8vo, 1884. "On the Algebraical and Numerical Theory of Errors of Observation and on the Combination of Observations," by Sir George Biddle Airy. London: Macmillan \& Co., I2mo, 2d ed., 1875. "Die Ausgleichungsrechnung nach der Methode der Kleinsten Quadrate, mit Anwendungen auf die Geodäsie und die Theorie der Messinstrumente," von F. R. Helmert. Leipzig: B. G. Teubner, 8vo, 1872.

## EXPLANATION OF SOURCE AND USE OF THE TABLES.

TABLES I and 2 are copies of tables issued by the Office of Standard Weights and Measures of the United States, edition of November, $\mathbf{1} 89 \mathbf{1}$.

Table 3 is derived from standard tables giving such data. The arrangement is that given in "Des Ingenieurs Taschenbuch, herausgegeben von dem Verein 'Hütte'"* (1rth edition, 1877). The numbers have been compared with those given in the latter work, and also with those in Barlow's "Tables." The logarithms have been checked by comparison with Vega's 7-place tables.

Table 4 is abridged from a similar table in the Taschenbuch just referred to.
Tables 5 and 6 are copies of standard forms for such table. They have been checked by comparison with standard higher-place tables. The mode of using these tables will be evident from the following examples :-
(土.) To find the logarithm of any number, as 0.06944 , we look in Table 5 in the column headed $\mathbf{N}$ for the first two significant figures of the number, which are in this case 69. In the same horizontal line with 69 we now look for the number in the column headed with the next figure of the given number, which is in the present case 4 . We thus find $.84^{14}$ for the mantissa of the logarithm of the number 694. To get the increase due to the additional figure 4 , we look in the same horizontal line under Prop. Parts in the colunnn headed 4 and find the number 2, which is the amount in units of the fourth place to be added to the part of the mantissa previously found. Thus the mantissa of $\log$ ( 0.06944 ) is 8416. The characteristic for the logarithm in question is $-2=8-10$. Hence $\log (0.06944)=8.8416-10$.
(2.) To find the number corresponding to any logarithm, as $8.8416-\mathrm{Io}$, we look in Table 6 in the column headed L for the first two figures of the mantissa, which are in this case 84 . In the same horizontal line with 84 we now look for the number in the column headed by the next figure of the mantissa, which is in this case 1 . We thus find 6394 for the number corresponding to the mantissa 84 ro. To get the increase due to the additional figure 6 , we look in the same horizontal line under Prop. Parts in the column headed 6 and find 10 , which is the amount in units of the fourth place to be added to the number previously found. Thus the significant figures of the number are 6944 , and since the characteristic of the logarithm is $8-10=-2$, the required number is 0.06944 .

[^17]Tables 7 and 8 are taken from "Smithsonian Meteorological Tables" (the first volume of this series). Their mode of use will be apparent from the following example: Required the sine and tangent for $28^{\circ} \mathrm{I} 7^{\prime}$.


Table 9 is a copy of a similar table published in "Professional Papers, Corps Engineers," U. S. A., No. 12. It has been checked by comparison with other tables in general use. This table is useful in computing latitudes and departures in traverse surveys wherein the bearings of the lines are observed to the nearest quarter of a degree, and in other work where multiples of sines and cosines are required. Thus, if $L$ denote the length and $B$ the bearing from the meridian of any line, the latitude and departure of the line are given by

$$
L \cos B \quad \text { and } L \sin B
$$

respectively; the " latitude" being the distance approximately between the parallels of latitude at the ends of the line, and the "departure" being the distance approximately between the meridians at the ends of the line. As an example, let it be required to compute the latitude and departure for $L=4837$, in any unit, and $B=36^{\circ} 15^{\prime}$. The computation runs thus : -


Tables Io and II give the logarithms of the principal radii of curvature of the earth's spheroid. They were computed by Mr. B. C. Washington, Jr., and carefully checked by differences. They depend on the elements of Clarke's spheroid of 1866 . The use of these tables is sufficiently explained on p. xlv-xlix.

Table 12 gives logarithms of radii of curvature of the earth's spheroid in sections inclined to the meridian sections. It is abridged to 5 places from a 6-place table published in the " Report of the U. S. Coast and Geodetic Survey for $1876 . "$ Its use is explained on pp. lxi-lxiv.

Tables 13 and 14 give logarithms of factors needed to compute the spheroidal excess of triangles on the earth's spheroid. No. 13 is constructed for the Eng. lish foot as unit, and No. 14 for the metre. These tables were computed by Mr.

Charles H. Kummell. Their use is explained on p. lviii. The following example will illustrate their use :-


Tables 15 and 16 give logarithms of factors for computing differences of latitude, longitude, and azimuth in secondary triangulation whose lines are 12 miles ( 20 kilometres) or less in length. These tables were computed by Mr. Charles H. Kummell. Table 15 gives factors for the English foot as unit, and Table 16 for the metre as unit. The use of these tables is illustrated by a numerical example given on pp. lx and lxi. For lines not exceeding the length mentioned, the tables will give differences of latitude and longitude to the nearest hundredth of a second of arc, using 5 -place logarithms of the lengths of the lines.

Table I7 gives lengths of terrestrial arcs of meridians corresponding to latitude intervals of $10^{\prime \prime}, 20^{\prime \prime}, \ldots 60^{\prime \prime}$, and $10^{\prime}, 20^{\prime}, \ldots 60^{\prime}$, or lengths corresponding to arcs less than $\mathrm{I}^{\circ}$. The unit of length is the English foot. The table was computed by Mr. B. C. Washington, Jr.

The length corresponding to any latitude interval is the distance along the meridian between parallels whose latitudes are less and greater respectively than the given latitude by half the interval. Thus, for example, the length corresponding to the interval $30^{\prime}$ and latitude $37^{\circ}$ ( $\mathbf{r} 82047.3$ feet) is the distance along the meridian from latitude $36^{\circ} 45^{\prime}$ to latitude $37^{\circ} 15^{\prime}$.
By interpolation, we may get from this table the meridional distance corresponding to any interval. The following example illustrates this use : Required the distance between latitude $41^{\circ} \quad 28^{\prime} 17 . .^{\prime \prime} 8$ and latitude $41^{\circ} 39^{\prime} \quad 53 . .^{\prime \prime} 4$. The difference of these latitudes is $1 \mathrm{I}^{\prime} 35 . .^{\prime \prime} 6$, and their mean is $41^{\circ} 34^{\prime} 05 .^{\prime \prime} 6$. The computation runs thus :-

| $10^{\prime}$ | Latitude $41^{\circ}$. |  | Tabular difference. |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 60724.60 feet |  | 10.70 feet |  |
| $\mathrm{I}^{\prime}$ | 6072.46 | " | 1.07 | " |
| $30^{\prime \prime}$ | 3036.23 | ، | . 54 | " |
| $5{ }^{\prime \prime}$ | 506.04 | " | . 09 | " |
| 0."6 | 60.72 | " | . 01 | " |
| ${ }_{\frac{34}{}{ }_{60} 09} \times 12.41$ | 7.05 | " | sum, 12.41 | " |
| Dista | 70407.10 |  |  |  |

When the degree of precision required is as great as that of the example just
given, it will be more convenient to use formulas (2) on p. xlvi. Thus, in this example, -

| $\Delta \phi=695 .{ }^{\prime \prime} 6$ | $\begin{gathered} \log . \\ 2.8423596 \end{gathered}$ |
| :---: | :---: |
| $\phi=4 \mathrm{I}^{\circ} 34^{\prime} 05 . \mathrm{C}$, $\rho_{m}$ (Table 10) | 7.3196820 |
| cons't | 4.6855749 |
| Length $=70407.10$ feet | 4.8476165 |

Table 18 gives lengths of terrestrial arcs of parallels corresponding to longitude intervals of $10^{\prime \prime}, 20^{\prime \prime}, \ldots 60^{\prime \prime}$, and $10^{\prime}, 20^{\prime}, \ldots 60^{\prime}$, or lengths corresponding to arcs less than $1^{\circ}$. The unit is the English foot. This table was computed by Mr. B. C. Washington, Jr.

The method of using this table is similar to that applicable to Table 17 explained above. For the computation of long arcs it will in general be less laborious to use the formulas ( r ) on p . xlix than to resort to interpolation from Table 18.

Tables 19-24 give the rectangular co-ordinates for the projection of maps, in accordance with the polyconic system explained on pp . liii-lvi, for the following scales respectively : -


These tables were computed by Mr. B. C. Washington, Jr.
The use of these tables and their application in the construction of maps may be best explained by an example. Suppose it is required to draw meridians and parallels for a map of an area of $I^{0}$ extent in longitude, lying between the parallels of $34^{\circ}$ and $35^{\circ}$. Let the scale of the map be one mile to the inch, or $\mathrm{I} / 63360$, and let the meridians and parallels be $10^{\prime}$ apart respectively. Draw on the projection paper an indefinite straight line $A B$, Fig. 4, to represent the middle meridian of the map. Take any convenient point, as $C$, on this line for the latitude $34^{\circ}$, and lay off from this point the meridional distances $C D, C E, C F, \ldots C I$, given in the second column of Table 22, p. iI4.* Through the points $D, E, F$, . . . I, thus found, draw indefinite straight lines perpendicular to $A B$. By means of these lines and the tabular co-ordinates, points on the developed parallels and meridians are readily found. Thus, for example, the abscissas for points ten minutes apart on the parallel $34^{\circ} 20^{\prime}$ are $9.53,19.06$, and 28.59 inches. These distances are to be laid off on $N N^{\prime}$ in both directions from $A B$. At the points $L, M, N, L^{\prime}, M^{\prime}, N^{\prime}$, so determined, erect perpendiculars to $N N^{\prime}$ equal in length, respectively, to the ordinates corresponding to the longitude intervals

[^18]$10^{\prime}, 20^{\prime}, 30^{\prime}$. The curved line joining the extremities of these perpendiculars is the parallel required. It may be drawn by means of a flexible ruler. The other parallels are constructed in the same manner. They are all concave towards the north or south according as the map shows a portion of the northern or southern hemisphere. The meridians are drawn in a similar manner through the points (e.g., $P, Q, M, R, S, T, U$ in Fig. 4) having the same longitude relative to the middle meridian. All meridians are concave towards the middle meridian.

A test of the graphical work which should always be applied is the approximation to equality of corresponding diagonals in the various quadrilaterals formed. Thus in Fig. 4, $V X$ should be equal to $W Y, C N$ to $C N^{\prime}, E V$ to $E W$, etc.*


Fig. 4.
Tables 25-29 give areas of quadrilaterals, bounded by meridians and parallels, of the earth's surface. They are taken from "Bulletin 50, U. S. Geological Survey." The unit of length used is the English mile, and the areas are thus given in square miles. The method of using these tables is obvious.

Table 30 gives data for the computation of heights, from barometric measures, in accordance with the formula of Babinet. $\dagger$ This table is taken from the "Smithsonian Meteorological Tables" (the first volume of this series). The manner of using it is explained in connection with the table.

[^19]Table 31 gives the mean astronomical refraction in terms of the apparent altitude of a star or other object outside the earth's atmosphere. It is taken from Vega's 7 -place table of logarithms. Its use will be evident from the following example:-


Tables 32 and 33 facilitate the interconversion of arc and time. They are taken from the "Smithsonian Meteorological Tables" (the first volume of this series). The following examples illustrate their use : -
(r.) To convert $68^{\circ} 29^{\prime} 48 .^{\prime \prime} 8$ into time we have from Table 32 -

$$
\begin{array}{rlrl}
68^{\circ} & =4^{\mathrm{h}} & 32^{\mathrm{m}} 00^{\mathrm{s}} \\
29^{\prime} & = & 1 & 56 \\
4^{4 \prime} & = & & 3.20 \\
0 . .^{\prime \prime} & = & & .05 \\
\text { Equivalent in time } & =4 & 33 & 59.25
\end{array}
$$

(2.) To con ert $5^{\mathrm{h}} 43^{\mathrm{m}} 28.8$ into arc we have from Table $33-$


Tables 34 and 35 facilitate the interconversion of mean solar and sidereal time intervals. They are taken from Vega's 7-place table of logarithms. The mode of using them is explained in the tables themselves.

Tables 36 and 37 give the lengths of degrees of terrestrial arcs of meridians and parallels expressed in metres,* statute miles (English), and geographic miles (distance corresponding to $I^{\prime}$ on the earth's equator). These tables are taken from the "Smithsonian Meteorological Tables" (the first volume of this series).

Table 38 facilitates the interconversion of statute (English) miles and nautical miles. The nautical mile used is that defined by the U. S. Coast and Geodetic Survey, namely : the length of a minute of arc of a great circle of the sphere whose surface equals that of the earth (Clarke's spheroid of 1866). For formula for radius of such sphere see p. lii. This table is taken from the "Smithsonian Meteorological Tables" (the first volume of this series).

Table 39 gives the English and metric equivalents of other standards of length still in use or obsolescent. It is taken from the "Smithsonian Meteorological Tables" (the first volume of this series).

Table 40 gives values of the acceleration ( $g$ ) of gravity, $\log g, \log (1 / 2 g), \log \sqrt{2 g}$,

[^20]and $\left(g / \pi^{2}\right)$ or the length of a seconds pendulum, for intervals of $5^{\circ}$ of geographical latitude. It was computed by the editor, and is based on the formula for $g$ given by Professor William Harkness in his memoir "On the Solar Parallax and its Related Constants." *

Table 41 gives the linear expansions of the principal metals. It was compiled by the editor from various sources. The values given for the expansion per degree Centigrade have been rounded (with one exception) to the nearest unit in the millionths place, or to the nearest micron, since different specimens of the same metal vary more or less in the ten-millionths place.

Table 42 gives the fractional changes in numbers corresponding to changes in the $4^{\text {th }}, 5$ th,. .7 th place of their logarithms. These fractions are often convenient in showing the approximate error in a number due to a given error in its logarithm, or the converse. Thus, for example, referring to the remark in a foot-note under explanation of Tables 36 and 37 above, the error in the logarithm of Clarke's ratio of the yard to the metre is about 4 units in the sixth place of decimals; the Table 42 shows, then, that the metric equivalents in Tables 36 and 37 are erroneous by about $1 / 100$ oooth part.

[^21]
## GEOGRAPHICAL TABLES

FOR CONVERTING U. S. WEICHTS AND MEASURES.* CUSTOMARY TO METRIC.


The only authorized material standard of customary length is the Troughton scale belonging to this office, whose length at $59^{\circ} .62$ Fahr. conforms to the British standard. The yard in use in the United States is therefore equal to the British yard.

The only anthorized material standard of customary weight is the Troy pound of the Mint. It is of brass of unknown density, and therefore not suitable for a standard of mass. It was derived from the British standard Troy pound of 1758 by direct comparison. The British Avoirdupois pound was also derived from the latter, and contains 7,000 grains Troy.

The grain Troy is therefore the same as the grain Avoirdupois, and the pound Avoirdupois in use in the Uuited States is equal to the British pound Avoirdupois.

The British galion $=4.54346$ litres. The British bushel $=36.3477$ litres.
The length of the nautical mile given above and adopted by the U.S. Coast and Geodetic Survey many years ago is defined as that of a minute of arc of a great circle of a sphere whose surface equals that of the earth (Clarke's Spheroid of 1866).

* Issued by U. S. Office of Standard Weights and Measures, and republished here by permission of Superintendent of Coast and Geodetic Survey.


By the concurrent action of the principal governments of the world an International Bureau of Weights and Measures has been established near Paris. Under the direction of the International Committee, two ingots were cast of pure platinum-iridium in the proportion of 9 parts of the former to y of the latter metal. From one of these a certain number of kilogrammes were prepared, from the other a definite number of metre bars. These standards of weight and length were intercompared, without preference, and certain ones were selected as International prototype standards. The others were distributed by lot, in September, 1889 , to the different governments and are called National prototype standards. Those apportioned to the United States were received in r8go and are in the keeping of this office.

The metric system was legalized in the United States in 1866.
The International Standard Metre is derived from the Metre des Archives, and its length is defined by the distance between two Iines at $0^{\circ}$ Centigrade, on a platinum-iridium bar deposited at the International Bureau of Weights and Measures.

The International Standard Kilogramme is a mass of platinum-iridium deposited at the same place, and its weight in vacuo is the same as that of the Kilogramme des Archives.

The litre is equal to a cubic decimetre, and it is measured by the quantity of distilled water which, at its maximum density, will counterpoise the standard kilogramme in a vacuum, the volume of such a quantity of water being, as nearly as has been ascertained, equal to a cubic decimetre.
Smithsonian Tables.

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{n}$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1000.000 | 1 | 1 | 1.0000 | 1.0000 | 0.00000 |
| 2 | 500.000 | 4 | 8 | 1.4142 | 1.2599 | 0.30103 |
| 3 | 333.333 | 9 | 27 | 1.7321 | 1. 4422 | 0.47712 |
| 4 | 250.000 | 16 | 64 | 2.0000 | 1.5874 | 0.60206 |
| 5 | 200.000 | 25 | 125 | 2.2361 | 1.7100 | 0.69897 |
| 6 | 166.667 | 36 | 216 | 2.4495 | 1.8171 | 0.77815 |
| 7 | 142.857 | 49 | 343 | 2.6458 | 1.9129 | 0.84510 |
| 8 | 125.000 | 64 | 512 | 2.8284 | 2.0000 | 0.90309 |
| 9 | III.III | 8 I | 729 | 3.0000 | 2.0801 | 0.95424 |
| 10 | 100.000 | 100 | 1000 | 3.1623 | 2.1544 | 1.00000 |
| 11 | 90.9091 | 121 | 1331 | 3.3166 | 2.2240 | 1.04139 |
| 12 | 83.3333 | 144 | 1728 | 3.4641 | 2.2894 | 1.07918 |
| 13 | 76.9231 | 169 | 2197 | 3.6056 | 2.3513 | I.11394 |
| 14 | 71.4286 | 196 | 2744 | 3.7417 | 2.4101 | 1.14613 |
| 15 | 66.6667 | 225 | 3375 | 3.8730 | 2.4662 | 1.17609 |
| 16 | 62.5000 | 256 | 4096 | 4.0000 | 2.5198 | 1.20412 |
| 17 | 58.8235 | 289 | 4913 | 4.1231 | 2.5713 | 1.23045 |
| 18 | 55.5556 | 324 | 5832 | 4.2426 | 2.6207 | 1.25527 |
| 19 | 52.6316 | 361 | 6859 | 4.3589 | 2.6684 | 1.27875 |
| 20 | 50.0000 | 400 | 8000 | 4-4721 | 2.7144 | 1.30103 |
| 21 | 47.6190 | 441 | 9261 | 4.5826 | 2.7589 | 1.32222 |
| 22 | 45.4545 | 484 | 10648 | 4.6904 | 2.8020 | 1.34242 |
| 23 | 43.4783 | 529 | 12167 | 4.7958 | 2.8439 | 1.36173 |
| 24 | 41.6667 | 576 | $13^{824}$ | 4.8990 | 2.8845 | I.3802 |
| 25 | 40.0000 | 625 | 15625 | 5.0000 | 2.9240 | 1. 39794 |
| 26 | 38.46 I 5 | 676 | 17576 | 5.0990 | 2.9625 | 1.41497 |
| 27 | 37.0370 | 729 | 19683 | 5.1962 | 3.0000 | I. 43136 |
| 28 | 35.7143 | 784 | 21952 | 5.2915 | 3.0366 | r. 44716 |
| 29 | 34.4828 | 841 | 24389 | 5.3852 | 3.0723 | 1.46240 |
| 30 | 33.3333 | 900 | 27000 | $5 \cdot 4772$ | 3.1072 |  |
| 31 | 32.2581 | 961 | 29791 | $5 \cdot 5678$ | 3.1414 | 1.49136 |
| 32 | 31.2500 | 1024 | 32768 | 5.6569 | 3.1748 | 1.50515 |
| 33 | 30.3030 | 1089 | 35937 | 5.7446 | 3.2075 | 1.5185I |
| 34 | 29.4118 | II 56 | 39304 | 5.8310 | 3.2396 | I.53148 |
|  | 28.5714 |  |  | 5.9161 | 3.2711 |  |
| 36 | 27.7778 | 1296 | 46656 | 6.0000 | 3.3019 | 1.545630 |
| 37 | 27.0270 | 1369 | 50653 | 6.0828 | 3.3322 | 1.56820 |
| 38 | 26.3158 | 1444 | 54872 | 6.1644 | 3.3620 | 1.57978 |
| 39 | 25.6410 | I 521 | 59319 | 6.2450 | $3 \cdot 3912$ | 1.59106 |
| 40 | 25.0000 | 1600 | 64000 | 6.3246 | 3.4200 | 1.60206 |
| 41 | 24.3902 | 1681 | 68921 | 6.4031 | $3 \cdot 4482$ | 1. 61278 |
| 42 | 23.8095 | 1764 | 74088 | 6.4807 | 3.4760 | 1.62325 |
| 43 | 23.2558 | 1849 | 79507 | 6.5574 | 3.5034 | 1.63347 |
| 44 | 22.7273 | 1936 | 85184 | 6.6332 | 3.5303 | 1. 64345 |
| 45 | 22.2222 | 2025 | 91125 | 6.7082 | 3.5569 |  |
| 46 | 21.7391 | 2116 | 97336 | 6.7823 | 3.5830 | 1.66276 |
| 47 | 21.2766 | 2209 | 103823 | 6.8557 | 3.6088 | 1.67210 |
| 48 | 20.8333 | 2304 | 110592 | 6.9282 | 3.6342 | 1.68124 |
| 49 | 20.4082 | 2401 | 117649 | 7.0000 | 3.6593 | 1.69020 |
| 50 | 20.0000 |  |  | 7.0711 | 3.6840 |  |
| 51 | 19.6078 | 2601 | 132651 | 7.1414 | 3.7084 | $1.70757$ |
| 52 53 | 19.2308 | 2704 | 140608 | 7.2111 | 3.7325 | 1.71600 |
| 53 | 18.8679 | 2809 | 148877 | 7.2801 | 3.7563 | 1.72428 |
| 54 | 18.5185 | 2916 | 157464 | 7.3485 | 3.7798 | 1.73239 |

Smithsonian Tables.

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SOUARE ROOTS, CUBE ROOTS, AND COMMON LOCARITHMS OF NATURAL NUMBERS.

| $n$ | $1000 \cdot \frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{n}$ | $\log$. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 55 | 18.1818 | 3025 | 166375 | 7.4162 | 3.8030 | 1.74036 |
| 56 | 17.857 | 3136 | 175616 | 7.4833 | 3.8259 | 1.74819 |
| 57 | 17.5439 | 3249 | 185193 | $7 \cdot 5498$ | 3.8485 | 1.75587 |
| 58 | 17.2414 | 3364 | 195112 | 7.6158 | 3.8709 | 1.76343 |
| 59 | 16.9492 | 348 I | 205379 | 7.6811 | 3.8930 | 1.77085 |
| 60 | 16.6667 | 3600 | 216000 | 7.7460 | 3.9149 | 1.77815 |
| 61 | 16.3934 | 3721 | 22698 I | 7.8102 | 3.9365 | 1.78533 |
| 62 | 16.1290 | 3844 | 238328 | 7.8740 | 3.9579 | 1.79239 |
| 63 | 15.8730 | 3969 | 250047 | 7.9373 | 3.9791 | 1.79934 |
| 64 | $15.625^{\circ}$ | 4096 | 262144 | 8.0000 | 4.0000 | 1.80618 |
| 65 | 15.3846 | 4225 | 274625 | 8.0623 | 4.0207 | 1.81291 |
| 66 | 15.1515 | 4356 | 287496 | 8.1240 | 4.0412 | 1.81954 |
| 67 | 14.9254 | 4489 | 300763 | 8.1854 | 4.0615 | 1.82607 |
| 68 | 14.7059 | 4624 | 314432 | 8.2462 | 4.0817 |  |
| 69 | 14.4928 | 4761 | 328509 | 8.3066 | 4.1016 | 1.83885 |
| 70 | 14.2857 | 4900 | 343000 | 8.3666 | 4.1213 | 1.84510 |
| 71 | 14.0845 | 5041 | 35791 1- | 8.4261 | 4.1408 | 1.85126 |
| 72 | 13.8889 | 5184 | 373248 | 8.4853 | 4.1602 | 1.85733 |
| 73 | 13.6986 | 5329 | 389017 | 8.5440 | 4.1793 | 1.86332 |
| 74 | 13.5135 | 5476 | 405224 | 8.6023 | 4.1983 | 1.86923 |
| 75 | 13.3333 | 5625 | 421875 | 8.6603 | 4.2172 | 1:87506 |
| 76 | 13.1579 | 5776 | 438976 | 8.7178 | 4.2358 | 1.88081 |
| 77 | 12.9870 | 5929 | 456533 | 8.7750 | 4.2543 | 1.88649 |
| 78 | 12.8205 | 6084 | 474552 | 8.8318 | 4.2727 | 1.89209 |
| 79 | 12.6582 | 6241 | 493039 | 8.8882 | 4.2908 | 1.89763 |
| 80 | 12.5000 | 6400 | 512000 | 8.9443 | 4.3089 | 1.90309 |
| 81 | 12.3457 | 656I | 531441 | 9.0000 | 4.3267 | 1.90849 |
| 82 | 12.1951 | 6724 | 551368 | 9.0554 | 4.3445 | 1.91381 |
| 83 | 12.0482 | 6889 | 571787 | 9.1104 | 4.362 I | 1.91908 |
| 84 | 11.9048 | 7056 | 592704 | 9.1652 | 4.3795 | 1.92428 |
| 85 | 11.7647 | 7225 | 614125 | 9.2195 | 4.3968 | 1. 92942 |
| 86 | 11.6279 | 7396 | 636056 | 9.2736 | 4.4140 | 1.93450 |
| 87 | 11.4943 | 7569 | 658503 | 9.3274 | 4.4310 | 1.93952 |
| 88 | 11.3636 | 7744 | 681472 | 9.3808 | $4 \cdot 4480$ | 1.94448 |
| 89 | 11.2360 | 7921 | 704969 | 9.4340 | $4 \cdot 4647$ | 1.94939 |
| 90 | 11.1111 | 8100 | 729000 | 9.4868 | 4.4814 | 1.95424 |
| 91 | 10.9890 | 8281 | 753571 | 9.5394 | $4 \cdot 4979$ | 1.95904 |
| 92 | 10.8696 | 8464 | 778688 | 9.5917 | 4.5144 | 1.96379 |
| 93 | 10.7527 | 8649 | 804357 | 9.6437 | 4.5307 | 1.96848 |
| 94 | 10.6383 | 8836 | 830584 | 9.6954 | $4 \cdot 5468$ | 1.97313 |
| 95 | 10.5263 | 9025 |  |  |  |  |
| 96 | 10.4167 | 9216 | 884736 | 9.7980 | 4.5789 | $1.98227$ |
|  | 10.3093 |  | 912673 | 9.8489 | 4.5947 | 1.98677 |
| 98 | 10.2041 | 9604 | 941192 | 9.8995 | 4.6104 | 1.99123 |
| 99 | 10.1010 | 9801 | 970299 | 9.9499 | 4.6261 | 1.99564 |
| 100 | 10.0000 | 10000 | 1000000 | 10.0000 | 4.6416 | 2.00000 |
| 101 | 9.90099 | 10201 | 1030301 | 10.0499 | 4.6570 | 2.00432 |
| 102 | 9.80392 | 10404 | 1061208 | 10.0995 | 4.6723 | 2.00860 |
| 103 | 9.70874 | 10609 | 1092727 | 10.1489 | 4.6875 | 2.01284 |
| 104 | 9.61538 | 10816 | 1124864 | 10.1980 | 4.7027 | 2.01703 |
| 105 | 9.52381 | 11025 | 1157625 | 10.2470 | 4.7177 | 2.02119 |
| 106 | 9.43396 | 11236 | 1191016 | 10.2956 | 4.7326 | 2.02531 |
|  | 9.34579 | 11449 | 1225043 | 10.3441 | 4.7475 | 2.02938 |
| 108 | 9.25926 | 11664 | 1259712 | 10.3923 | 4.7622 | 2.03342 |
| 109 | 9.17431 | 11881 | 1295029 | 10.4403 | 4.7769 | 2.03743 |

VALUES OF RECIPROCALS, SOUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOCARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{n}$ | $\sqrt[3]{n}$ | log. 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 110 | 9.09091 | 12100 | 1331000 | 10.4881 | 4.7914 | 2.04139 |
| 111 | 9.00901 | 12321 | 1367631 | 10.5357 | 4.8059 | 2.04532 |
| 112 | 8.92857 | 12544 | 1404928 | 10.5830 | 4.8203 | 2.04922 |
| 113 | 8.84956 | 12769 | 1442897 | 10.6301 | 4.8346 | 2.05308 |
| 114 | 8.77 I93 | 12996 | 1481544 | 10.6771 | 4.8488 | 2.05690 |
| 115 | 8.69565 | 13225 | 1520875 | $10.723^{8}$ | 4.8629 | 2.06070 |
| 116 | 8.62069 | 13456 | 1560896 | 10.7703 | 4.8770 | 2.06446 |
| 117 | 8.54701 | 13689 | 1601613 | 10.8167 | 4.8910 | 2.06819 |
| 118 | 8.47458 | 13924 | 1643032 | 10.8628 | 4.9049 | 2.07188 |
| 119 | 8.40336 | 14161 | 1685159 | 10.9087 | 4.9187 | 2.07555 |
| 120 | 8.33333 | 14400 | 1728000 | 10.9545 | 4.9324 | 2.07918 |
| 121 | 8.26446 | 1464 I | 1771561 | 11.0000 | 4.9461 | 2.08279 |
| 122 | 8.19672 | 14884 | 1815848 | 11.0454 | 4.9597 | 2.08636 |
| 123 | 8.13008 | 15129 | 1860867 | 11.0905 | 4.9732 | 2.08991 |
| 124 | 8.06452 | 15376 | 1906624 | 11.1355 | 4.9866 | 2.09342 |
| 125 | 8.00000 | I 5625 | 1953125 | 11.1803 | 5.0000 | 2.09691 |
| 126 | 7.93651 | I5876 | 2000376 | $11.225^{\circ}$ | 5.0133 | 2.10037 |
| 127 | 7.87402 | 16129 | 2048383 | 11.2694 | 5.0265 | $2.103^{80}$ |
| 128 | 7.81250 | 16384 | 2097152 | 11.3137 | 5.0397 | 2.10721 |
| 129 | 7.75194 | 1664 I | 2146689 | 11.3578 | 5.0528 | 2.11059 |
| 130. | 7.69231 | 16900 | 2197000 | 11.4018 | 5.0658 | 2.11394 |
| 131 | 7.63359 | 17161 | 2248091 | II. 4455 | 5.0788 | 2.11727 |
| 132 | 7.57576 | 17424 | 2299968 | 11.4891 | 5.0916 | 2.12057 |
| 133 | 7.51880 | 17689 | 2352637 | 11.5326 | 5.1045 | 2.12385 |
| 134 | 7.46269 | 17956 | 2406104 | $11.575^{8}$ | 5.1172 | 2.12710 |
| 135 | 7.40741 | 18225 | 2460375 | 11.6190 | 5.1299 | 2.13033 |
| 136 | 7.35294 | 18496 | 2515456 | 11.6619 | 5.1426 | 2.13354 |
| 137 | 7.29927 | 18769 | 2571353 | 11.7047 | 5.1551 | 2.13672 |
| 138 | 7.24638 | 19044 | 2628072 | 11.7473 | 5.1676 | 2.I 3988 |
| 139 | 7.19424 | 19321 | 2685619 | 11.7898 | 5.1801 | 2.14301 |
| 140 | 7.14286 | 19600 | 2744000 | 11.8322 | 5.1925 | 2.14613 |
| 141 | 7.09220 | 19881 | 2803221 | 11.8743 | 5.2048 | 2.14922 |
| 142 | 7.04225 | 20164 | 2863288 | II.9164 | 5.2171 | 2.15229 |
| 143 | 6.99301 | 20449 | 2924207 | 11.9583 | 5.2293 | 2.15534 |
| 144 | 6.94444 | 20736 | 2985984 | 12.0000 | 5.2415 | 2.15836 |
| 145 | 6.89655 | 21025 | 3048625 | 12.0416 |  |  |
| 146 | 6.84932 | 21316 | 3112136 | 12.0830 | 5.2656 | $2.16435$ |
| 147 | 6.80272 6.75676 | 21609 21904 | 3176523 3241792 | 12.1244 | 5.2776 | 2.16732 |
| 148 | 6.75676 | 21904 | 3241792 | 12.1655 | 5.2896 | 2.17026 |
| 149 | 6.71141 | 22201 | 3307949 | 12.2066 | $5 \cdot 3015$ | 2.17319 |
| 150 | 6.66667 | 22500 | 3375000 | 12.2474 |  | 2.17609 |
| ${ }^{1} 51$ | 6.62252 | 22801 | 3442951 | 12.2882 | $5 \cdot 3251$ | 2.17898 |
| 152 | 6.57893 | 23104 | 3511808 | 12.3288 | 5.3368 | 2.18184 |
| ${ }^{1} 53$ | 6.53595 | 23409 | 3581577 | 12.3693 | 5.3485 | 2.18469 |
| 154 | 6.49351 | 23716 | 3652264 | 12.4097 | $5 \cdot 3601$ | $2.1875^{2}$ |
| 155 | 6.45161 |  |  | 12.4499 |  |  |
| ${ }_{1}^{156}$ | 6.41026 | 24336 | 3796416 | 12.4900 | 5.3832 | 2.19312 |
| 157 | 6.36943 | 24649 | 3869893 | 12.5300 | $5 \cdot 3947$ | 2.19590 |
| I 58 | 6.32911 | 24964 | 3944312 | 12.5698 | 5.406 I | 2.19866 |
| I 59 | 6.28931 | 25281 | 4019679 | 12.6095 | 5.4175 | 2.20140 |
| 160 |  | 25600 | 4096000 | 12.6491 | 5.4288 |  |
| 161 | 6.21118 | 25921 | 4173281 | 12.6886 | $5 \cdot 4401$ | 2.20683 |
| 162 | 6.17284 | 26244 | 4251528 | 12.7279 | 5.4514 | 2.20952 |
| 163 | 6.13497 | 26569 | 4330747 | 12.7671 | 5.4626 | 2.21219 |
| 164 | 6.09756 | 26896 | 4410944 | 12.8062 | $5 \cdot 4737$ | 2.21484 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | $1000 \cdot \frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[3]{2}$ | log. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 165 | 6.06061 | 27225 | 4492125 | 12.8452 | 5.4848 | 2.21748 |
| 166 | 6.02410 | 27556 | 4574296 | 12.884 I | 5.4959 | 2.22011 |
| 167 | 5.98802 | 27889 | 4657463 | 12.9228 | 5.5069 | 2.22272 |
| 168 | 5.95238 | 28224 | 4741632 | 12.9665 | 5.5178 | 2.22531 |
| 169 | 5.91716 | 28561 | 4826809 | 13.0000 | $5 \cdot 5288$ | 2.22789 |
| 170 | 5.88235 | 28900 | 4913000 | 13.0384 | $5 \cdot 5397$ | 2.23045 |
| 171 | 5.84795 | 29241 | 50002 II | 13.0767 | 5.5505 | 2.23300 |
| 172 | 5.81395 | 29584 | 5088448 | 13.1149 | 5.5613 | 2.23553 |
| 173 | 5.78035 | 29929 | 5177717 | 13.1529 | $5 \cdot 572 \mathrm{I}$ | 2.23805 |
| 174 | $5 \cdot 74713$ | 30276 | 5268024 | I 3.1909 | 5.5828 | 2.24055 |
| 175 | 5.71429 | 30625 | 5359375 | 13.2288 | 5.5934 | 2.24304 |
| 176 | 5.68182 | 30976 | 5451776 | 13.2665 | 5.6041 | 2.2455 I |
| 177 | 5.64972 | 31329 | 5545233 | 13.3041 | 5.6147 | 2.24797 |
| 178 | 5.61798 | 31684 | 5639752 | 13.3417 | 5.6252 | 2.25042 |
| 179 | $5 \cdot 58659$ | 32041 | 5735339 | ${ }^{1} 3.3791$ | 5.6357 | 2.25285 |
| 180 | 5.55556 | 32400 | 5832000 | 13.4164 | 5.6462 | 2.25527 |
| 181 | 5.52486 | 32761 | 5929741 | I 3.4536 | 5.6567 | 2.25768 |
| 182 | 5.49451 | 33124 | 6028568 | 13.4907 | 5.667 I | 2.26007 |
| 183 | $5 \cdot 46448$ | 33489 | 6 I 28487 | 13.5277 | 5.6774 | 2.26245 |
| 184 | 5.43478 | 33856 | 6229504 | I3.5647 | 5.6877 | 2.26482 |
| 185 | 5.40541 | 34225 | 6331625 | 13.6015 | 5.6980 | 2.26717 |
| 186 | 5.37634 | 34596 | 6434856 | 13.6382 | 5.7083 | 2.2695 I |
| 187 | $5 \cdot 34759$ | 34969 | 6539203 | 13.6748 | 5.7185 | 2.27184 |
| 188 | $5 \cdot 31915$ | 35344 | 6644672 | 13.7113 | 5.7287 | 2.27416 |
| 189 | $5 \cdot 29101$ | 35721 | 6751269 | 13.7477 | 5.7388 | 2.27646 |
| 190 | 5.26316 | 36100 | 6859000 | 13.7840 | 5.7489 | 2.27875 |
| 191 | 5.23560 | 3648 I | 6967871 | 13.8203 | 5.7590 | 2.28103 |
| 192 | 5.20833 | 36864 | 7077888 | 13.8564 | 5.7690 | 2.28330 |
| 193 | 5.18135 | 37249 | 7189057 | 13.8924 | 5.7790 | 2.28556 |
| 194 | 5.15464 | 37636 | 7301384 | 13.9284 | 5.7890 | 2.28780 |
| 195 | 5.12821 | 38025 |  | 13.9642 | 5.7989 | 2.29003 |
| 196 | 5.10204 | 38416 | 7529536 | 14.0000 | 5.8088 | 2.29226 |
| 197 | 5.07614 | 38809 | 7645373 | r4.0357 | 5.8186 | 2.29447 |
| 198 | 5.05051 | 39204 | 7762392 | 14.0712 | 5.8285 | 2.29667 |
| 199 | 5.02513 | 39601 | 7880599 | 14.1067 | 5.8383 | 2.29885 |
| 200 | 5.00000 | 40000 | 8000000 | 14.1421 | 5.8480 | 2.30103 |
| 201 | 4.97512 | 40401 | 8120601 | 14.1774 | 5.8578 | 2.30320 |
| 202 | 4.95050 | 40804 | 8242408 | 14.2127 | 5.8675 | 2.30535 |
| 203 | 4.92611 | 41209 | 8365427 | 14.2478 | 5.877 I |  |
| 204 | 4.90196 | 41616 | 8489664 | 14.2829 | 5.8868 | 2.30963 |
|  | 4.87805 |  | 8615125 | 14.3178 | 5.8964 |  |
| 206 | 4.85437 | 42436 | 8741816 | 14.3527 | 5.9059 | $2.31387$ |
| 207 | 4.83092 | 42849 | 8869743 | 14.3875 | 5.9155 | $2.31597$ |
| 208 | 4.80769 | 43264 | 8998912 | 14.4222 | 5.9250 | $2.31806$ |
| 209 | 4.78469 | 43681 | 9129.329 | 14.4568 | 5.9345 | 2.32015 |
| 210 | 4.76190 | 44100 | 9261000 | 14.4914 | 5.9439 | 2.32222 |
| 211 | 4.73934 | 44521 | 9393931 | 14.5258 | 5.9533 | 2.32428 |
| 212 | 4.71698 | 44944 | 9528128 | 14.5602 | 5.9627 | 2.32634 |
| 213 | 4.69484 | 45369 | 9663597 | 14.5945 | 5.9721 | 2.32838 |
| 214 | 4.67290 | 45796 | 9800344 | 14.6287 | 5.9814 | 2.33041 |
| 215 | 4.65116 |  |  | $14.6629$ |  |  |
| 216 | 4.62963 | 46656 | 10077696 | 14.6969 | 6.0000 | 2.33445 |
| 217 | 4.60829 | 47089 | 10218313 | 14.7309 | 6.0092 | 2.33646 |
| 218 | 4.58716 | 47524 | 10360232 | 14.7648 | 6.0185 | 2.33846 |
| 219 | 4.56621 | 4796 r | 10503459 | 14.7986 | 6.0277 | 2.34044 |

Gmithsonian Tables.

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| 22 | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{n}$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 220 | 4.54545 | 48400 | 10648000 | 14.8324 | 6.0368 | 2.34242 |
| 221 | 4.52489 | 4884 I | 10793861 | 14.8661 | 6.0459 | 2.34439 |
| 222 | 4.50450 | 49284 | 10941048 | 14.8997 | 6.0550 | 2.34635 |
| 223 | 4.48431 | 49729 | 11089567 | 14.9332 | 6.0641 | 2.34830 |
| 224 | 4.46429 | 50176 | 11239424 | 14.9666 | 6.0732 | 2.35025 |
| 225 | 4.44444 | 50625 | 11390625 | 15.0000 | 6.0822 | 2.35218 |
| 226 | 4.42478 | 51076 | 11543176 | 15.0333 | 6.0912 | 2.35411 |
| 227 | 4.40529 | 51529 | 11697083 | 15.0665 | 6.1002 | 2.35603 |
| 228 | 4.38596 | 51984 | 11852352 | 15.0997 | 6.1091 | 2.35793 |
| 229 | 4-3668ı | 52441 | 12008989 | 15.1327 | 6.i̇I80 | 2.35984 |
| 230 | 4.34783 | 52900 | 12167000 | 15.1658 | 6.1269 | 2.36173 |
| 231 | 4.32900 | 53361 | 12326391 | 15.1987 | 6.1358 | 2.36361 |
| 232 | 4.31034 | 53824 | 12487168 | 15.2315 | 6.1446 | 2.36549 |
| 233 | 4.29185 | 54289 | 12649337 | 15.2643 | 6.1534 | 2.36736 |
| 234 | $4.2735^{\circ}$ | 54756 | 12812904 | 15.2971 | 6.1622 | 2.36922 |
| 235 | 4.25532 | 55225 | 12977875 | 15.3297 | 6.1710 | 2.37107 |
| 236 | 4.23729 | 55696 | 13144256 | 15.3623 | 6.1797 | 2.37291 |
| 237 | 4.21941 | 56169 | 13312053 | I 5-3948 | 6.1885 | 2.37475 |
| 238 | 4.20168 | 56644 | 13481272 | 15.4272 | 6.1972 | 2.37658 |
| 239 | 418410 | 57121 | 13651919 | 15.4596 | 6.2058 | 2.37840 |
| 240 | 4.16667 | 57600 | 13824000 | 15.4919 | 6.2145 | 2.38021 |
| 241 | 4.14938 | 58081 | 13997521 | 15.5242 | 6.2231 | 2.38802 |
| 242 | 4.13223 | 58564 | 14172488 | 15.5563 | 6.2317 | $2.383^{82}$ |
| 243 | 4.11523 | 59049 | 14348907 | 15.5885 | 6.2403 | 2.38561 |
| 244 | 4.09836 | 59536 | 14526784 | 15.6205 | 6.2488 | 2.38739 |
| 245 | 4.08163 | 60025 | 14706125 | 15.6525 | 6.2573 | 2.38917 |
| 246 | 4.06504 | 60516 | 14886936 | 15.6844 | . 6.2658 | 2.39094 |
| 247 | 4.04858 | 61009 | 15069223 | 15.7162 | 6.2743 | 2.39270 |
| 248 | 4.03226 | 61504 | 15252992 | 15.7480 | 6.2828 | 2.39445 |
| 249 | 4.01606 | 62001 | 15438249 | 15.7797 | 6.2912 | 2.39620 |
| 250 | 4.00000 | 62500 | 15625000 | 15.8114 | 6.2996 | 2.39794 |
| 251 | 3.98406 | 63001 | 15813251 | 15.8430 | 6.3080 | 2.39967 |
| 252 | 3.96825 | 63504 | 16003008 | 15.8745 | 6.3164 | 2.40140 |
| 253 | 3.95257 | 64009 | 16194277 | 15.9060 | 6.3247 | 2.40312 |
| 254 | 3.93701 | 64516 | 16387064 | 15.9374 | 6.3330 | 2.40483 |
| 255 | 3.92I 57 | 65025 | 16581375 | 15.9687 | 6.3413 |  |
| 256 | 3.90625 | 65536 | 16777216 | 16.0000 | 6.3496 | 2.40824 |
| 257 | 3.89105 | 66049 | 16974593 | 16.0312 | 6.3579 | 2.40993 |
| 258 | 3.87597 | 66564 | 17173512 | 16.0624 | 6.3661 | 2.41162 |
| 259 | 3.86100 | 67081 | 17373979 | 16.0935 | 6.3743 | 2.41330 |
| 260 | 3.84615 | 67600 | 17576000 | 16.1245 | 6.3825 | 2.41497 |
| 261 | 3.83142 | 68121 | 17779581 | 16.1555 | 6.3907 | 2.41664 |
| 262 | 3.81679 | 68644 | 17984728 | 16.1864 | 6.3988 | 2.41830 |
| 263 264 | 3.80228 3.78788 | 69169 69696 | 18191447 18399744 | 16.2173 16.2481 | 6.4070 6.4151 | 2.41996 2.42160 |
|  |  |  |  |  |  |  |
| 265 | 3.77358 | 70225 | 18609625 | 16.2788 | 6.4232 | 2.42325 |
| 266 | 3.75940 | 70756 | 18821096 | 16.3095 | 6.4312 | 2.42488 |
| 267 | 3.74532 | 71289 | 19034163 | 16.3401 | 6.4393 | 2.42651 |
| 268 269 | 3.73134 3.71747 | 71824 | 19248832 19465109 | 16.3707 16.4012 | 6.4473 | 2.42813 |
| 269 | 3.71747 | 72361 | 19465109 | 16.4012 | 6.4553 | 2.42975 |
| 270 | 3:70370 | 72900 | 19683000 | 16.4317 | 6.4633 | $2.43136$ |
| 271 | 3.69004 | 73441 | 19902511 | 16.4621 | 6.4713 | 2.43297 |
| 272 | 3.67647 3.66300 | 73984 | 20123648 | 16.4924 | 6.4792 | 2.43457 2.43616 |
| 273 | 3.66300 | 74529 | 20346417 | 16.5227 | 6.4872 | 2.43616 |
| 274 | 3.64964 | 75076 | 20570824 | 16.5529 | 6.4951 | 2.43775 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON'LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[3]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 275 | 3.63636 | 75625 | 20796875 | 16.583 I | 6.5030 | 2.43933 |
| 276 | 3.62319 | 76176 | 21024576 | 16.6132 | 6.5108 | 2.4409 I |
| 277 | 3.6ioII | 76729 | 21253933 | 16.6433 | 6.5187 | 2.44248 |
| 278 | $3 \cdot 59712$ | 77284 | 21484952 | 16.6733 | 6.5265 | 2.44404 |
| 279 | $3 \cdot 58423$ | 77841 | 21717639 | 16.7033 | 6.5343 | 2.44560 |
| 280 | 3.57143 | 78400 | 21952000 | 16.7332 | 6.542 I | 2.44716 |
| 281 | 3.55872 | 78961 | 22188041 | 16.763 I | 6.5499 | 2.44871 |
| 282 | 3.54610 | 79524 | 22425768 | 16.7929 | 6.5577 | 2.45025 |
| 283 | 3.53357 | 80089 | 22665187 | 16.8226 | 6.5654 | 2.45179 |
| 284 | 3.52II3 | 80656 | 22906304 | 16.8523 | 6.5731 | $2.4533{ }^{2}$ |
| 285 | $3 \cdot 50877$ | 81225 | 23149125 | 16.8819 | 6.5808 | 2.45484 |
| 286 | 3.49650 | 81796 | 23393656 | 16.9115 | 6.5885 | 2.45637 |
| 287 | 3.48432 | 82369 | 23639903 | 16.9411 | 6.5962 | 2.45788 |
| 288 | 3.47222 | 82944 | 23887872 | 16.9706 | 6.6039 | 2.45939 |
| 289 | 3.46021 | 8352 I | 24137569 | 17.0000 | 6.6115 | 2.46090 |
| 290 | 3.44828 | 84100 | 24389000 | 17.0294 | $6.619{ }^{-}$ | 2.46240 |
| 291 | 3.43643 | 84681 | 24642171 | 17.0587 | 6.6267 | 2.46389 |
| 292 | 3.42466 | 85264 | 24897088 | 17.0880 | 6.6343 | 2.46538 |
| 293 | 3.41297 | 85849 | 25153757 | 17.1172 | 6.6419 | 2.46687 |
| 294 | $3 \cdot 40136$ | 86436 | 25412184 | 17.1464 | 6.6494 | 2.46835 |
| 295 | 3.38983 | 87025 | 25672375 | 17.1756 | 6.6569 | 2.46982 |
| 296 | $3 \cdot 37838$ | 87616 | 25934336 | 17.2047 | 6.6644 | 2.47129 |
| 297 | 3.36700 | 88209 | 26198073 | 17.2337 | 6.67 I9 | 2.47276 |
| 298 | $3 \cdot 35570$ | 88804 | 26463592 | 17.2627 | 6.6794 | 2.47422 |
| 299 | 3.34448 | 89401 | 26730899 | 17.2916 | 6.6869 | 2.47567 |
| 300 | 3.33333 | 90000 | 27000000 | 17.3205 | 6.6943 | 2.47712 |
| 301 | $3 \cdot 32226$ | 90601 | 27270901 | 17.3494 | 6.7018 | 2.47857 |
| 302 | $3 \cdot 31126$ | 91204 | 27543608 | 17.3781 | 6.7092 | 2.48001 |
| 303 | 3.30033 | 91809 | 27818127 | 17.4069 | 6.7166 | 2.48144 |
| 304 | 3.28947 | 92416 | 28094464 | 17.4356 | 6.7240 | 2.48287 |
| 305 | 3.27869 | 93025 | 28372625 | 17.4642 | 6.7313 | 2.48430 |
| 306 | 3.26797 | 93636 | 28652616 | 17.4929 | 6.7387 | 2.48572 |
| 307 | 3.25733 | 94249 | 28934443 | 17.5214 | 6.7460 | 2.48714 |
| 308 | 3.24675 | 94864 | 29218112 | 17.5499 | 6.7533 | $2.48855$ |
| 309 | 3.23625 | 9548 I | 29503629 | 17.5784 | 6.7606 | 2.48996 |
| 310 | 3.22581 | 96100 | 29791000 | 17.6068 | 6.7679 | 2.49136 |
| 311 | 3.21543 | 96721 | 30080231 | 17.6352 | 6.7752 | 2.49276 |
| 312 | 3.20513 | 97344 | 30371328 | 17.6635 | 6.7824 | 2.494 I 5 |
| 313 | 3.19489 3.18471 | 97969 98596 | 30664297 | 17.6918 | 6.7897 |  |
| 314 | 3.18471 | 98596 | 30959144 | 17.7200 | 6.7969 | 2.49693 |
| 315 | 3.17460 | 99225 | 31255875 | 17.7482 | 6.8041 |  |
| 316 | 3.16456 | 99856 | 31554496 | 17.7764 | 6.8113 | 2.49969 |
| 317 | 3.15457 | 100489 | 31855013 | 17.8045 | 6.8185 | 2.50106 |
| 318 | 3.14465 3.13480 | 101124 | 32157432 32461759 | 17.8326 17.8606 | 6.8256 6.8328 | 2.50243 |
| 319 | 3.13480 | 101761 | 32461759 | 17.8606 | 6.8328 | 2.50379 |
| 320 | 3.12500 | 102400 | 32768000 | 17.8885 | 6.8399 | 2.50515 |
| 321 | 3.11527 | 103041 | 33076161 | 17.9165 | 6.8470 | 2.5065 I |
| 322 | 3.10559 | 103684 | 33386248 | 17.9444 | 6.8541 | 2.50786 |
| 323 | 3.09598 | 104329 | 33698267 | 17.9722 | 6.8612 | 2.50920 |
| 324 | 3.08642 | 104976 | 34012224 | 18.0000 | 6.8683 | 2.51055 |
| 325 | 3.07692 |  |  | 18.0278 | 6.8753 | 2.51188 |
| 326 | 3.06748 | 106276 | 34645976 | 18.0555 | $\cdot 6.8824$ | 2.51322 |
| 327 | 3.05810 | 106929 | 34965783 | 18.0831 | 6.8894 | 2.51455 |
| 328 329 | 3.04878 | 107584 | 35287552 3561289 | 18.1108 | 6.8964 | $2.51587$ |
| 329 | 3.03951 | 108241 | 35611289 | $18.1{ }^{884}$ | 6.9034 | 2.51720 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{n}$ | $\sqrt[3]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 330 | 3.03030 | 108900 | 35937000 | 18.1659 | 6.9104 | 2.51851 |
| 331 | 3.02115 | 109561 | 36264691 | 18.1934 | 6.9174 | 2.51983 |
| 332 | 3.01205 | 110224 | 36594368 | 18.2209 | 6.9244 | $2.52 \times 14$ |
| 333 | 3.00300 | 110889 | 36926037 | 18.2483 | 6.9313 | 2.52244 |
| 334 | 2.99401 | $11155^{\circ}$ | 37259704 | 18.2757 | 6.9382 | 2.52375 |
| 335 | 2.98507 | 112225 | 37595375 | 18.3030 | 6.9451 | 2.52504 |
| 336 | 2.97619 | 112896 | 37933056 | 18.3303 | 6.952 I | 2.52634 |
| 337 | 2.96736 | 113569 | 38272753 | 18.3576 | 6.9589 | 2.52763 |
| 338 | 2.95858 | 114244 | 38614472 | 18.3848 | 6.9658 | 2.52892 |
| 339 | 2.94985 | 114921 | 38958219 | 18.4120 | 6.9727 | 2.53020 |
| 340 | 2.94118 | 115600 | 39304000 | 18.4391 | 6.9795 | 2.53148 |
| 341 | 2.93255 | 116281 | 39651821 | 18.4662 | 6.9864 | 2.53275 |
| 342 | 2.92398 | 116964 | 40001688 | 18.4932 | 6.9932 | 2.53403 |
| 343 | 2.91545 | 117649 | 40353607 | 18.5203 | 7.0000 | 2.53529 |
| 344 | 2.90698 | 118336 | 40707584 | 18.5472 | 7.0068 | 2.53656 |
| 345 | 2.89855 | 119025 | 41063625 | 18.5742 | 7.0136 | 2.53782 |
| 346 | 2.89017 | 119716 | 41421736 | 18.6011 | 7.0203 | 2.53908 |
| 347 | 2.88184 | 120409 | 41781923 | 18.6279 | 7.0271 | 2.54033 |
| 348 | 2.87356 | 121104 | 42144192 | 18.6548 | 7.0338 | 2.54158 |
| 349 | 2.86533 | 121801 | 42508549 | 18.6815 | 7.0406 | 2.54283 |
| 350 | 2.85714 | 122500 | 42875000 | 18.7083 | 7.0473 | 2.54407 |
| 351 | 2.84900 | 123201 | 43243551 | 18.7350 | 7.0540 | 2.54531 |
| 352 | 2.84091 | 123904 | 43614208 | 18.7617 | 7.0607 | 2.54654 |
| 353 | 2.83286 | 124609 | 43986977 | 18.7883 | 7.0674 | 2.54777 |
| 354 | 2.82486 | 125316 | 44361864 | 18.8149 | 7.0740 | 2.54900 |
| 355 | 2.81690 | 126025 | 44738875 | 18.8414 | 7.0807 | 2.55023 |
| 356 | 2.80899 | 126736 | 45118016 | 18.8680 | 7.0873 | 2.55145 |
| 357 | 2.80112 | 127449 | 45499293 | 18.8944 | 7.0940 | 2.55267 |
| 358 | 2.79330 | 128164 | 45882712 | 18.9209 | 7.1006 | 2.55388 |
| 359 | 2.78552 | 128881 | 46268279 | 18.9473 | 7.1072 | 2.55509 |
| 360 | 2.77778 | 129600 | 46656000 | 18.9737 | 7.11138 | 2.55630 |
| 36 I | 2.77008 | 130321 | 47045881 | 19.0000 | 7.1204 | 2.55751 |
| 362 | 2.76243 | 131044 | 47437928 | 19.0263 | 7.1269 | 2.55871 |
| 363 | 2.75482 | 131769 | 47832147 | 19.0526 | 7.1335 | 2.55991 |
| 364 | 2.74725 | 132496 | 48228544 | 19.0788 | 7.1400 | 2.56110 |
| 365 | 2.73973 | 133225 | 48627125 | 19.1050 | 7.1466 | 2. 56229 |
| 366 | 2.73224 | 133956 | 49027896 | 19.13II | 7.1531 | 2.56348 |
| 367 | 2.72480 | 134689 | 49430863 | 19.1572 | 7.1596 | 2.56467 |
| 368 | 2.71739 | 135424 | 49836032 | 19.1833 | 7.1661 | 2.56585 |
| 369 | 2.71003 | 136161 | 50243409 | 19.2094 | 7.1726 | 2. 56703 |
| 370 | 2.70270 | 136900 | 50653000 |  | 7.1791 | 2.56820 |
| 371 | 2.69542 | 137641 | 510648 II | 19.26 I 4 | 7.1855 | 2.56937 |
| 372 | 2.68817 | 138384 | 51478848 | 19.2873 | 7.1920 | 2.57054 |
| 373 | 2.68097 | 139129 | 51895117 | 19.3132 | 7.1984 | 2.57171 |
| 374 | 2.67380 | 139876 | 52313624 | 19.3391 | $7 \cdot 2048$ | 2.57287 |
| 375 | 2.66667 | 140625 | 52734375 | 19.3649 | 7.2112 |  |
| 376 377 | 2.65957 | 141376 | 53157376 | 19.3907 | 7.2177 | 2.57519 |
| 377 378 | 2.65252 2.64550 | 142129 142884 | 53582633 | 19.4165 | 7.2240 | 2.57634 |
| 378 | 2.64550 | 142884 | 54010152 | 19.4422 | 7.2304 | 2.57749 |
| 379 | 2.63852 | 143641 | 54439939 | 19.4679 | 7.2368 | 2.57864 |
|  | $2.63158$ | 144400 |  | 19.4936 | 7.2432 |  |
| 381 382 | 2.62467 2.61780 | 145161 | 55306341 | 19.5192 | 7.2495 | 2.58092 |
| 382 | 2.61780 | 145924 | 55742968 | 19.5448 | $7.255^{8}$ | 2.58206 |
| 383 | 2.61097 | 146689 | 56181887 | 19.5704 | 7.2622 | 2.58320 |
| 384 | 2.60417 | 147456 | 56623104 | 19.5959 | 7.2685 | 2.58433 |

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| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt{1}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 385 | 2.59740 | 148225 | 57066625 | 19.6214 | 7.2748 | 2.58546 |
| $3^{86}$ | 2.59067 | 148996 | 57512456 | 19.6469 | 7.2811 | 2.58659 |
| 387 | 2.58398 | 149769 | 57960603 | 19.6723 | 7.2874 | 2.58771 |
| 388 | 2.57732 | 150544 | 58411072 | 19.6977 | 7.2936 | 2.58883 |
| 389 | 2.57069 | 151321 | 58863869 | 19.7231 | 7.2999 | 2.58995 |
| 390 | 2.56410 | 152100 | 59319000 | 19.7484 | 7.3061 | 2.59106 |
| 391 | 2.55754 | 152881 | 59776471 | 19.7737 | $7 \cdot 3124$ | 2.59218 |
| 392 | 2.55102 | 153664 | 60236288 | 19.7990 | 7.3186 | 2.59329 |
| 393 | 2.54453 | 154449 | 60698457 | 19.8242 | 7.3248 | 2.59439 |
| 394 | 2.53807 | 155236 | 6II62984 | 19.8494 | 7.3310 | $2.5955^{\circ}$ |
| 395 | 2.53165 | 156025 | 61629875 | 19.8746 | $7 \cdot 3372$ | 2.59660 |
| 396 | 2.52525 | 156816 | 62099136 | 19.8997 | 7.3434 | 2.59770 |
| 397 | 2.51889 | 157609 | 62570773 | 19.9249 | 7.3496 | 2.59879 |
| 398 | 2.51256 | 158404 | 63044792 | 19.9499 | $7 \cdot 355$ | 2.59988 |
| 399 | 2.50627 | 159201 | 63521199 | $19.975^{\circ}$ | 7.3619 | 2.60097 |
| 400 | 2.50000 | 160000 | 64000000 | 20.0000 | $7 \cdot 3681$ | 2.60206 |
| 401 | 2.49377 | 160801 | 64481201 | 20.0250 | 7.3742 | 2.60314 |
| 402 | 2.48756 | 161604 | 64964808 | 20.0499 | 7.3803 | 2.60423 |
| 403 | 2.48139 | 162409 | 65450827 | 20.0749 | $7 \cdot 3864$ | 2.60531 |
| 404 | 2.47525 | 163216 | 65939264 | 20.0998 | $7 \cdot 3925$ | 2.60638 |
| 405 | 2.46914 | 164025 | 66430125 | 20.1246 | 7.3986 | 2.60746 |
| 406 | 2.46305 | 164836 | 66923416 | 20.1494 | 7.4047 | 2.60853 |
| 407 | 2.45700 | 165649 | 67419143 | 20.1742 | 7.4108 | 2.60959 |
| 408 | 2.45098 | 166464 | 67917312 | 20.1990 | 7.4169 | 2.61066 |
| 409 | 2.44499 | 167281 | 68417929 | 20.2237 | $7 \cdot 4229$ | 2.61172 |
| 410 | 2.43902 | 168100 | 68921000 | 20.2485 | 7.4290 | 2.61278 |
| 411 | 2.43309 | 168921 | 69426531 | 20.2731 | 7.4350 | 2.61384 |
| 412 | 2.42718 | 169744 | 69934528 | 20.2978 | 7.4410 | 2.61490 |
| 413 | 2.42131 | 170569 | 70444997 | 20.3224 | 7.4470 | 2.61595 |
| 414 | 2.41546 | 171396 | 70957944 | 20.3470 | 7.4530 | 2.61700 |
| 415 | 2.40964 2.40385 | 172225 173056 |  | 20.3715 20.3961 | 7.4590 7.4650 | 2.61805 2.61909 |
| 416 417 | 2.40385 2.39808 | 173056 173889 | 71991296 72511713 | 20.3961 20.4206. | 7.4650 7.4710 | 2.61909 2.62014 |
| 418 | 2.39234 | 174724 | $7303463^{2}$ | 20.4450 | 7.4770 | 2.62118 |
| 419 | 2.38663 | 175561 | 73560059 | 20.4695 | $7 \cdot 4829$ | 2.62221 |
| 420 | 2.38095 | 176400 | 74088000 | 20.4939 20.5183 | 7.4889 |  |
| 421 | 2.37530 | 177241 | 74618461 | 20.5183 | 7.4948 | $2.62428$ |
| 422 | 2.36967 | 178084 | 75151448 | 20.5426 | 7.5007 | 2.62531 |
| 423 | 2.36407 | 178929 | 75686967 | 20.5670 | $7 \cdot 5067$ | 2.62634 2.62737 |
| 424 | 2.35849 | 179776 | 76225024 | 20.5913 | $7 \cdot 5126$ | 2.62737 |
| 425 | 2.35294 | 180625 | 76765625 | 20.6155 | 7.5185 | 2.62839 |
| 426 | 2.34742 | 181476 | 77308776 | 20.6398 | 7.5244 | 2.62941 |
| 427 | 2.34192 | 182329 | 77854483 | 20.6640 | 7.5302 | 2.63043 |
| 428 | 2.33645 | 183184 | 78402752 | 20.6882 | $7 \cdot 5361$ | 2.63144 |
| 429 | 2.33100 | 184041 | 78953589 | 20.7123 | $7 \cdot 5420$. | 2.63246 |
| 430 | 2.32558 | 184900 | 79507000 | 20.7364 | 7.5478 | 2.63347 |
| 43 I | 2.32019 | 185761 | 80062991 80621568 | 20.7605 20.7846 | 7.5537 7.5595 | 2.63448 2.63548 |
| 432 433 | 2.31481 2.30947 | 186624 18748 | 80621568 81182737 | 20.7846 20.8087 | 7.5595 7.5654 | 2.63548 2.63649 |
| 433 434 | 2.30947 2.30415 | 188356 | 81746504 | 20.8327 | 7-5712 | 2.63749 |
| 435 | 2.29885 | $189225$ |  | $20.8567$ |  |  |
| 436 437 | 2.29358 2.28833 | 190096 | 82881856 | 20.8806 | 7.5828 7.5886 | 2.63949 2.64048 |
| 437 | 2.28833 | 190969 | 83453453 84027672 | 20.9045 20.9284 | 7.5886 | 2.64048 |
| 438 439 | 2.28311 2.27790 | 191844 192721 | 84027672 84604519 | 20.9284 20.9523 | 7.5944 7.6001 | 2.64147 2.64246 |

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| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 440 | 2.27273 | 193600 | 85184000 | 20.9762 | 7.6059 | 2.64345 |
| 441 | 2.26757 | 194481 | 85766121 | 21.0000 | 7.6117 | 2.64444 |
| 442 | 2.26244 | 195364 | 86350888 | 21.0238 | 7.6174 | 2.64542 |
| 443 | 2.25734 | 196249 | 86938307 | 21.0476 | 7.6232 | 2.64640 |
| 444 | 2.25225 | 1971 $3^{6}$ | 87528384 | 21.0713 | 7.6289 | 2.64738 |
| 445 | 2.24719 | 198025 | 88121125 | 21.0950 | 7.6346 | 2.64836 |
| 446 | 2.24215 | 198916 | 88716536 | 21.1187 | 7.6403 | 2.64933 |
| 447 | 2.23714 | 199809 | 89314623 | 21.1424 | 7.6460 | 2.65031 |
| 448 | 2.23214 | 200704 | 89915392 | 21.1660 | 7.6517 | 2.65128 |
| 449 | 2.22717 | 201601 | 90518849 | 21.1896 | 7.6574 | 2.65225 |
| 450 | 2.22222 | 202500 | 91125000 | 21.2132 | 7.6631 | 2.65321 |
| 451 | 2.21730 | 203401 | 9173385 | 21.2368 | 7.6688 | 2.65418 |
| 452 | 2.21239 | 204304 | 92345408 | 21.2603 | 7.6744 | 2.65514 |
| 453 | 2.20751 | 205209 | 92959677 | 21.2838 | 7.6801 | 2.65610 |
| 454 | 2.20264 | 206116 | 93576664 | 21.3073 | 7.6857 | 2.65706 |
| 455 | 2.19780 | 207025 | 94196375 | 21.3307 | 7.6914 | 2.65801 |
| 456 | 2.19298 | 207936 | 94818816 | 21.3542 | 7.6970 | 2.65896 |
| 457 | 2.18818 | 208849 | 95443993 | 21.3776 | 7.7026 | 2.65992 |
| 458 | 2.18341 | 209764 | 96071912 | 21.4009 | 7.7082 | 2.66087 |
| 459 | 2.17865 | 210681 | 96702579 | 21.4243 | $7 \cdot 7138$ | 2.66181 |
| 460 | 2.17391 | 211600 | 97336000 | 21.4476 | 7.7194 | 2.66276 |
| 461 | 2.16920 | 212521 | 97972181 | 21.4709 | 7.7250 | 2.66370 |
| 462 | 2.16450 | 213444 | 98611128 | 21.4942 | 7.7306 | 2.66464 |
| 463 | 2.15983 | 214369 | 99252847 | 21.5174 | 7.7362 | 2.66558 |
| 464 | 2.15517 | 215296 | 99897344 | 21.5407 | .7.7418 | 2.66652 |
| 465 | 2.15054 | 216225 | 100544625 | 21.5639 | 7.7473 |  |
| 466 | 2.14592 | 217156 | 101194696 | 21.5870 | 7.7529 | $2.66839$ |
| 467 | 2.14133 | 218089 | 101847563 | 21.6102 | 7.7584 | 2.66932 |
| 468 | 2.13675 | 219024 | 102503232 | 21.6333 | 7.7639 | 2.67025 |
| 469 | 2.13220 | 219961 | 103161709 | 21.6564 | 7.7695 | 2.67117 |
| 470 | 2.12766 | 220900 | 103823000 | 21.6795 | 7.7750 | 2.67210 |
| 471 | 2.12314 | 221841 | 104487111 | 21.7025 | 7.7805 | 2.67302 |
| 472 | 2.11864 | 222784 | 105154048 | 21.7256 | 7.7860 | 2.67394 |
| 473 | 2.11416 | 223729 | 105823817 | 21.7486 | 7.7915 | 2.67486 |
| 474 | 2.10970 | 224677 | 106496424 | 21.7715 | 7.7970 | 2.67578 |
|  | 2.10526 | 225625 | 107171875 | 21.7945 | 7.8025 | 2.67669 |
| 476 | 2.10084 | 226576 | 107850176 | 21.8174 | 7.8079 | 2.67761 |
| 477 478 | 2.09644 | 227529 | 108531333 | 21.8403 | 7.8134 | 2.67852 |
| 478 | 2.09205 | 228484 | 109215352 | 21.8632 | 7.8188 | 2.67943 |
| 479 | 2.08768 | 229441 | 109902239 | 21.8861 | 7.8243 | 2.68034 |
| 480 | 2.08333 | 230400 | 110592000 | 21.9089 | 7.8297 | 2.68124 |
| 481 | 2.07900 | 231361 | 111284641 | 21.9317 | 7.8352 | 2.68215 |
| 482 | 2.07469 | 232324 | 111980168 | 21.9545 | 7.8406 | 2.68305 |
| 483 484 | 2.07039 2.06612 | 233289 234256 | 112678587 | 21.9773 | 7.8460 | 2.68395 |
| 484 | 2.06612 | 234256 | 113379904 | 22.0000 | 7.8514 | 2.68485 |
| 485 | 2.06186 |  | 114084125 | 22.0227 | 7.8568 |  |
| 486 | 2.05761 | 236196 | 114791256 | 22.0454 | 7.8622 | 2.68664 |
| 487 488 | 2.05339 | 237169 | 115501303 | 22.0681 | 7.8676 | 2.68753 |
| 488 489 | 2.04918 | 238144 | 116214272 | 22.0907 | 7.8730 | 2.68842 |
| 489 | 2.04499 | 239121 | 116930169 | 22.1133 | 7.8784 | 2.68931 |
| 490 | 2.04082 | 240100 |  |  | 7.8837 | 2.69020 |
| 491 492 | 2.03666 | 241081 | 118370771 | 22.1585 | 7.8891 | $2.69108$ |
| 492 493 | 2.03252 2.02840 | 242064 | 119095488 | 22.1811 | 7.8944 | 2.69197 |
| 493 494 | 2.02840 | 243049 244036 | 119823157 120553784 | 22.2036 22.2261 | 7.8998 | 2.69285 |
| 494 | 2.02429 | 244036 | 120553784 | 22.2261 | 7.9051 | 2.69373 | ROOTS, AND COMMON'LOCARITHMS OF NATURAL NUMBERS.


| $n$ | $1000 \cdot \frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \boldsymbol{n}$ | $\sqrt{8}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 495 | 2.02020 | 245025 | $121287375$ | 22.2486 | 7.9105 | $2.6946 \mathrm{I}$ |
| 496 | 2.01613 | 246016 | $122023936$ | 22.2711 | 7.9158 | $2.69548$ |
| 497 | 2.01207 | 247009 | 122763473 | 22.2935 | 7.9211 | 2.69636 |
| 498 | 2.00803 | 248004 | 123505992 | 22.3159 | 7.9264 | 2.69723 |
| 499 | 2.00401 | 249001 | 124251499 | 22.3383 | 7.9317 | 2.69810 |
| 500 | 2.00000 | 250000 | 125000000 | 22.3607 | 7.9370 | 2.69897 |
| 501 | 1.99601 | 251001 | 125751501 | 22.3830 | 7.9420 | 2.69984 |
| 502 | 1.99203 | 252004 | 126506008 | 22.4054 | 7.9476 | 2.70070 |
| 503 | 1. 98807 | 253009 | 127263527 | 22.4277 | 7.9528 | 2.70157 |
| 504 | 1. 98413 | 254016 | 128024064 | 22.4499 | 7.9581 | $2.70243$ |
| 505 | 1.98020 | 255025 | 128787625 | 22.4722 | 7.9634 | 2.70329 |
| 506 | 1.97628 | 256036 | 129554216 | 22.4944 | 7.9686 | 2.70415 |
| 507 | 1.97239 | 257049 | 130323843 | 22.5167 | 7.9739 | 2.70501 |
| 508 | I. 96850 | 258064 | 131096512 | 22.5389 | 7.9791 | 2.70586 |
| 509 | 1.96464 | 259081 | 131872229 | 22.5610 | 7.9843 | 2.70672 |
| 510 | 1.96078 | 260100 | 132651000 | 22.5832 | 7.9896 | 2.70757 |
| 511 | 1.95695 | 261121 | 13343283 I | 22.6053 | 7.9948 | 2.70842 |
| 512 | 1.95312 | 262144 | 134217728 | 22.6274 | 8.0000 | 2.70927 |
| 513 | 1.94932 | 263169 | 125005697 | 22.6495 | 8.0052 | 2.71012 |
| 514 | 1.94553 | 264196 | 155796744 | 22.6716 | 8.0104 | 2.71096 |
| 515 | 1.94175 | 265225 | 136590875 | 22.6936 | 8.0156 | 2.71181 |
| 516 | 1.93798 | 266256 | 137388096 | 22.7156 | 8.0208 | 2.71265 |
| 517 | 1.93424 | 267289 | ${ }^{1} 38188413$ | 22.7376 | 8.0260 | 2.71349 |
| 518 | 1.93050 | 268324 | 138991832 | 22.7596 | 8.0311 | $2.71433$ |
| 519 | 1.92678 | 269361 | 139798359 | 22.7816 | 8.0363 | 2.71517 |
| 520 | 1.92308 | 270400 | 140608000 | 22.8035 | 8.0415 |  |
| 521 | 1.91939 | 271441 | 141420761 | 22.8254 | 8.0466 | $2.71684$ |
| 522 | 1.91571 | 272484 | 142236648 | 22.8473 | 8.0517 | 2.71767 |
| 523 | 1.91205 | 273529 | 143055667 | 22.8692 | 8.0569 | 2.71850 |
| 524 | 1.90840 | 274576 | 143877824 | 22.8910 | 8.0620 | 2.71933 |
| 525 | 1.90476 | 275625 | 144703125 | 22.9129 | 8.0671 | 2.72016 |
| 526 | 1.90114 | 276676 | 145531576 | 22.9347 | 8.0723 | 2.72099 |
| 527 | 1. 89753 | 277729 | 146363183 | 22.9565 | 8.0774 | 2.72181 |
| 528 | 1. 89394 | 278784 | 147197952 | 22.9783 | 8.0825 | $2.72263$ |
| 529 | 1.89036 | 279841 | 148035889 | 23.0000 | 8.0876 | 2.72346 |
| 530 | 1.88679 | 280900 | 148877000 | 23.0217 | 8.0927 | 2.72428 |
| 531 | 1.88324 | 281961 | 149721291 | 23.0434 | 8.0978 | 2.72509 |
| 532 | 1.87970 | 283024 | 150568768 | 23.0651 | 8.1028 | 2.72591 |
| 533 | 1. 87617 | 284089 | 151419437 | 23.0868 | 8.1079 | 2.72673 |
| 534 | 1.87266 | 285156 | 152273304 | 23.1084 | 8.11 30 | 2.72754 |
|  | 1.86916 |  | 153130375 | 23.1301 | 8.1180 |  |
| 536 | 1.86567 | 287296 | 153990656 | 23.1517 | 8.1231 | 2.72916 |
| 537 | 1.86220 | 288369 | 154854153 | 23.1733 | 8.1281 | $2.72997$ |
| 538 539 | 1.85874 1.85529 | 289444 | 155720872 156500819 | 23.1948 | 8.1332 8.1382 | $2.73078$ |
| 539 | 1.85529 | 290521 | 156590819 | 23.2164 | 8.1382 | 2.73159 |
| 540 | 1.85185 | 291600 | 157464000 | 23.2379 |  | 2.73239 |
| 541 | 1.84843 | 292681 | I58340421 | 23.2594 | 8.1483 | 2.73320 |
| 542 | 1.84502 | 293764 | 159220088 | 23.2809 | 8.1533 | 2.73400 |
| 543 | 1.84162 | 294849 | 160103007 | 23.3024 | 8.1583 | $2.73480$ |
| 544 | 1.83824 | 295936 | 160989184 | 23.3238 | 8.1633 | 2.73560 |
| 545 | 1.83486 | 297025 |  |  | 8.1683 | 2.73640 |
| 546 | 1.83150 | 298116 | 162771336 | 23.3666 | 8.1733 | 2.73719 |
| 547 | 1.82815 | 299209 | 163667323 | 23.3880 | 8.1583 | $2.73799$ |
| 548 | 1.82482 | 300304 | 164566592 | 23.4094 | 8.1833 | $2.73878$ |
| 549 | 1.82149 | 301401 | 165469149 | 23.4307 |  | 2.73957 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SOUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 550 | 1.81818 | 302500 | 166375000 | 23.4521 | 8.1932 | 2.74036 |
| 551 | 1.81488 | 303601 | 167284151 | 23.4734 | 8.1982 | 2.74115 |
| 552 | 1.81159 | 304704 | 168196608 | 23.4947 | 8.2031 | 2.74194 |
| 553 | 1.80832 | 305809 | 169112377 | 23.5160 | 8.2081 | 2.74273 |
| 554 | 1.80505 | 306916 | 170031464 | 23.5372 | 8.2130 | 2.74351 |
| 555 | 1.80180 | 308025 | 170953875 | 23.5584 | 8.2180 | 2.74429 |
| 556 | 1.79856 | 309136 | 171879616 | 23.5797 | 8.2229 | 2.74507 |
| 557 | 1.79533 | 310249 | 172808693 | 23.6008 | 8.2278 | 2.74586 |
| 558 | 1.7921 I | 311364 | 173741112 | 23.6220 | 8.2327 | 2.74663 |
| 559 | 1.78891 | 31248 r | 174676879 | 23.6432 | 8.2377 | 2.7474 r |
| 560 | 1.78571 | 313600 | 175616000 | 23.6643 | 8.2426 | 2.74819 |
| 561 | I.78253 | 314721 | 176558481 | 23.6854 | 8.2475 | 2.74896 |
| 562 | 1.77936 | 315844 | 177504328 | 23.7065 | 8.2524 | 2.74974 |
| 563 | 1.77620 | 316969 | 178453547 | 23.7276 | 8.2573 | 2.75051 |
| 564 | 1.77305 | 318096 | 179406I44 | 23.7487 | 8.2621 | 2.75128 |
| 565 | 1.76991 | 319225 | 180362125 | 23.7697 | 8.2670 | 2.75205 |
| 566 | 1.76678 | 320356 | 1813221496 | 23.7908 | 8.2719 | 2.75282 |
| 567 | 1.76367 | 321489 | 182284263 | 23.8118 | 8.2768 | 2.75358 |
| 568 | 1.76056 | 322624 | 183250432 | 23.8328 | 8.2816 | 2.75435 |
| 569 | 1.75747 | 323761 | 184220009 | 23.8537 | 8.2865 | 2.755 II |
| 570 | 1.75439 | 324900 | 185193000 | 23.8747 | 8.2913 | 2.75587 |
| 571 | 1.7513 I | 326041 | 186169411 | 23.8956 | 8.2962 | 2.75664 |
| 572 | 1.74825 1.74520 | 327184 | 187149248 | 23.9165 | 8.3010 | 2.75740 |
| 573 | 1.74520 | 328329 | 188132517 | 23.9374 | 8.3059 | 2.75815 |
| 574 | 1.74216 | 329476 | 189r19224 | 23.9583 | 8.3107 | 2.75891 |
| 575 | 1.73913 | 330625 | 190109375 | 23.9792 | 8.3155 | 2.75967 |
| 576 | 1.736 I | 331776 | 191102976 | 24.0000 | 8.3203 | 2.76042 |
| 577 | 1.73310 | 332929 | 192100033 | 24.0208 | 8.3251 | 2.76118 |
| 578 579 | 1.73010 1.72712 | 334084 | 193100552 | 24.0416 | 8.3300 | 2.76193 |
| 579 | 1.72712 | 335241 | 194104539 | 24.0624 | 8.3348 | 2.76268 |
| 580 | 1.72414 | 336400 | 195112000 | 24.0832 | 8.3396 |  |
| 581 | 1.72117 | 33756 I | 196122941 | 24.1039 | 8.3443 | 2.76418 |
| 582 | 1.71821 | 338724 | 197137368 | 24.1247 | 8.3491 | 2.76492 |
| 58 | 1.71527 | 339889 | 198155287 | 24.1454 | 8.3539 | $2.76567$ |
| 584 | 1.71233 | 341056 | 199176704 | 24.166r | 8.3587 | 2.7664 I |
| 585 |  |  | 200201625 | 24.1868 | 8.3634 |  |
| 586 | 1.70648 | 343396 | 201230056 | 24.2074 | 8.3682 | $2.76790$ |
| 587 588 | 1.70358 | 344569 | 202262003 | 24.2281 | 8.3730 | 2.76864 |
| 588 589 | 1.70068 1.69779 | 345744 346921 | 203297472 204336469 | 24.2487 | 8.3777 8.387 | 2.76938 |
| 589 | 1.69779 | 34692 I | 204336469 | 24.2693 | 8.3825 | 2.77012 |
| 590 | 1. 69492 | 348100 | 205379000 | 24.2899 | 8.3872 | 2.77085 |
| 591 | 1.69205 | 34928 I | 206425071 | 24.3105 | 8.3919 | 2.77159 |
| 592 593 | 1.68919 1.68634 | 350464 351649 | 207474688 20852785 | 24.3311 | 8.3967 | 2.77232 |
| 594 | 1.68350 | 351849 35286 | 208527857 209584584 | $24 \cdot 3516$ 24.3721 | 8.4014 8.4061 | $\begin{aligned} & 2.77305 \\ & 2.77379 \end{aligned}$ |
| 595 | 1.68067 |  | 210644875 |  | 8.4108 |  |
| 596 | 1.67785 | 355216 | 211708736 | 24.3926 24.4531 | 8.4108 8.4155 | 2.77452 2.77525 |
| 597 | 1.67504 | 356409 | 212776173 | 24.4336 | 8.4202 | 2.77597 |
| 598 | 1.67224 | 357604 | 213847192 | 24.4540 | 8.4249 | 2.77670 |
| 599 | 1.66945 | 3588 or | 214921799 | 24.4745 | 8.4296 | 2.77743 |
|  |  | 360000 |  |  |  |  |
| 601 | 1.66389 | 36 I 201 | 217081801 | 24.5153 | 8.4390 | 2.77887 |
| 602 | 1.66113 | 362404 | 218167208 | 24.5357 | 8.4437 | 2.77960 |
| 603 | 1.65837 $\times 6563$ | 363609 | 219256227 | 24.5561 | 8.4484 | 2.78032 |
| 604 | 1.65563 | 364816 | 220348864 | 24.5764 | 8.4530 | 2.\%SIO4 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOCARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{ } n$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 605 | 1.65289 | 366025 | 221445125 | 24.5967 | 8.4577 | 2.78176 |
| 606 | 1.65017 | 367236 | 222545016 | 24.6171 | 8.4623 | 2.78247 |
| 607 | 1. 64745 | 368449 | 223648543 | 24.6374 | 8.4670 | 2.78319 |
| 608 | 1. 64474 | 369664 | 224755712 | 24.6577 | $8.47{ }^{16}$ | 2.78390 |
| 609 | 1.64204 | 37088 I | 225866529 | 24.6779 | 8.4763 | 2.78462 |
| 610 | 1.63934 | 372100 | 226981000 | 24.6982 | 8.4809 | 2.78533 |
| 611 | 1.63666 | 373321 | 228099131 | 24.7184 | 8.4856 | 2.78604 |
| 612 | 1. 63399 | 374544 | 229220928 | 24.7386 | 8.4902 | 2.78675. |
| 613 | 1.63132 | 375769 | 230346397 | 24.7588 | 8.4948 | 2.78746 |
| 614 | 1.62866 | 376996 | 231475544 | 24.7790 | 8.4994 | 2.788 I 7 |
| 615 | 1.62602 | 378225 | 232608375 | 24.7992 | 8. 5040 | 2.78888 |
| 616 | 1.62338 | 379456 | 233744896 | 24.8193 | 8.5086 | 2.78958 |
| 617 | 1.62075 | 380689 | 234885113 | 24.8395 | 8.5132 | 2.79029 |
| 618 | 1.61812 | 381924 | 236029032 | 24.8596 | 8.5178 | 2.79099 |
| 619 | 1.61551 | 383 I 61 | 237176659 | 24.8797 | 8.5224 | 2.79169 |
| 620 | 1.61290 | 384400 | 238328000 | 24.8998 | -8.5270 | 2.79239 |
| 621 | 1.61031 | 385641 | 239483061 | 24.9199 | 8.5316 | 2.79309 |
| 622 | 1.60772 | 386884 | 240641848 | 24.9399 | 8.5362 | 2.79379 |
| 623 | 1.60514 | 388129 | 241804367 | 24.9600 | 8.5408 | 2.79449 |
| 624 | 1.60256 | 389376 | 242970624 | 24.9800 | 8.5453 | 2.79518 |
| 625 | 1.60000 | 390625 | 244140625 | 25.0000 | 8.5499 | 2.79934 |
| 626 | I. 59744 | 391876 | 245314376 | 25.0200 | 8.5544 | 2.79657 |
| 627 | I. 59490 | 3931 29 | 246491883 | 25.0400 | 8.5590 | 2.79727 |
| 628 | 1.59236 | 394384 | 247673152 | 25.0599 | 8.5635 | 2.79796 |
| 629 | I. 58983 | 395641 | 248858189 | 25.0799 | 8.5681 | 2.79865 |
| 630 | 1. 58730 | 396900 | 250047000 | 25.0998 | 8.5726 |  |
| 631 | I. 58479 | 398161 | 251239591 | 25.1197 | 8.5772 | 2.80003 |
| 632 | 1.58228 | 399424 | 252435968 | 25.1396 | 8.5817 | 2.80072 |
| 633 | 1. 57978 | 400689 | 253636137 | 25.1595 | 8.5862 | 2.80140 |
| 634 | 1.57729 | 401956 | 254840104 | 25.1794 | 8.5907 | 2.80209 |
| 635 | 1.57480 | 403225 | 256047875 | 25.1992 | 8.5952 | 2.80277 |
| 636 | 1. 57233 | 404496 | 257259456 | 25.2190 | 8. 5997 | 2.80346 |
| 637 | 1.56986 | 405769 | 258474853 | 25.2389 | 8.6043 | 2.80414 |
| 638 | 1.56740 | 407044 | 259694072 | 25.2587 | 8.6088 | 2.80482 |
| 639 | 1. 56495 | 40832 I | 260917119 | 25.2784 | 8.6132 | $2.8055^{\circ}$ |
| 640 | 1. 56250 | 409600 | 262144000 | 25.2982 | 8.6177 | 2.80618 |
| 641 | I. 56006 | 41088 I | 263374721 | 25.3180 | 8.6222 | 2.80686 |
| 642 | I. 55763 | 412164 | 264609288 | 25.3377 | 8.6267 | 2.80754 |
| 643 | 1.55521 | 413449 | 265847707 | 25.3574 | 8.6312 | 2.80821 |
| 644 | 1. 55280 | 414736 | 267089984 | 25.3772 | 8.6357 | 2.80889 |
| 645 | 1. 55039 | 416025 |  | 25.3969 |  |  |
| 646 | I. 54799 | 417316 418609 | 269586136 270840023 | 25.4165 25.4362 | 8.6446 8.6490 | 2.81023 2.8 rogo |
| 647 648 | 1.54560 1.54321 | 418609 419904 | 270840023 272097792 | 25.4362 25.4558 | 8.6490 8.6535 | 2.81090 2.81158 |
| 649 | 1.54083 | 42 I 201 | 273359449 | 25.4755 | 8.6579 | 2.81224 |
| 650 | 1. 53846 | 422500 | 274625000 | $25 \cdot 4951$ | $8.6624^{\circ}$ | 2.81291 |
| 651 | 1.53610 | 42380 I | 275894451 | 25.5147 | 8.6668 | 2.81358 |
| 652 | 1. 53374 | 425104 | 277167808 | 25.5343 | 8.6713 | 2.81425 |
| 653 | 1.53139 | 426409 | 278445077 | 25.5539 | 8.6757 | $2.81491$ |
| 654 | 1.52905 | 427716 | 279726264 | 25.5734 | 8.6801 | $2.8155^{8}$ |
| 655 | 1. 52672 |  |  | 25.5930 | 8.6845 |  |
| 656 | I. 52439 | 430336 | 282300416 | 25.6125 | 8.6890 | 2.81690 |
| 657 658 | 1. 52207 | 431649 | $283593393$ | $25.6320$ | $8.6934$ | $2.81757$ |
| 658 659 | 1.51976 I. 51745 | 432964 43428 I | 284890312 286191179 | $\begin{aligned} & 25.6515 \\ & 25.6710 \end{aligned}$ | 8.6978 8.7022 | 2.81823 <br> 2.81889 |
| 659 | 1.51745 | 43428I | 28619179 | 25.6710 | 8.7022 | 2.81889 |

VALUES OF RECIPROCALS, SQUARES, CUBES, SOUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 660 | 1.51515 | 435600 | 287496000 | 25.6905 | 8.7066 | 2.81954 |
| 661 | 1.51286 | 436921 | 288804781 | 25.7099 | 8.7110 | 2.82020 |
| 662 | 1.51057 | 438244 | 290177528 | 25.7294 | 8.7154 | 2.82086 |
| 663 | 1.50830 | 439569 | 291434247 | 25.7488 | 8.7198 | 2.82151 |
| 664 | 1.50602 | 440896 | 292754944 | 25.7682 | 8.7241 | 2.82217 |
| 665 | 1. 50376 | 442225 | 294079625 | 25.7876 | 8.7285 | 2.82282 |
| 666 | 1.50150 | 443556 | 295408296 | 25.8070 | 8.7329 | 2.82347 |
| 667 | 1.49925 | 444889 | 296740963 | 25.8263 | 8.7373 | 2.82413 |
| 668 | 1.49701 | 446224 | 298077632 | 25.8457 | 8.7416 | 2.82478 |
| 669 | 1.49477 | 44756I | 299418309 | 25.8650 | 8.7460 | 2.82543 |
| 670 | 1.49254 | 448900 | 300763000 | 25.8844 | 8.7503 | 2.82607 |
| 671 | 1.49831 | 450241 | 302111711 | 25.9037 | 8.7547 | 2.82672 |
| 672 | 1.48810 | 451584 | 303464448 | 25.9230 | 8.7590 | 2.82737 |
| 673 | 1.48588 | 452929 | 304821217 | 25.9422 | 8.7634 | 2.82802 |
| 674 | I. 48368 | 454276 | 306182024 | 25.9615 | 8.7677 | 2.82866 |
| 675 | 1.48148 | 455625 | 307546875 | 25.9808 | 8.7721 | 2.82930 |
| 676 | 1.47929 | 456976 | 308915776 | 26.0000 | 8.7764 | 2.82995 |
| 677 | 1.47710 | 458329 | 310288733 | 26.0192 | 8.7807 | 2.83059 |
| 678 | I. 47493 | 459684 | 311665752 | 26.0384 | 8.7850 | 2.83123 |
| 679 | 1.47275 | 461041 | 313046839 | 26.0576 | 8.7893 | 2.83187 |
| 680 | 1.47059 | 462400 | 314432000 | 26.0768 | 8.7937 | 2.83251 |
| 681 | 1.46843 | 463761 | 315821241 | 26.0960 | 8.7980 | 2.83315 |
| 682 | 1.46628 | 465124 | 317214568 | 26.1151 | 8.8023 | 2.83378 |
| 683 | 1.46413 | 466489 | 318611987 | 26.1343 | 8.8066 | 2.83442 |
| 684 | 1.46199 | 467856 | 320013504 | 26.1534 | 8.8108 | 2.83506 |
| 685 | 1.45985 | 469225 | 321419125 | 26.1725 | 8.8152 | 2.83569 |
| 686 | 1.45773 | 470596 | 322828856 | 26.1916 | 8.8194 | 2.83632 |
| 687 | 1.45560 | 471969 | 324242703 | 26.2107 | 8.8237 | 2.83696 |
| 688 | 1.45349 | 473344 | 325660672 | 26.2298 | 8.8280 | 2.83759 |
| 689 | 1.45138 | 474721 | 327082769 | 26.2488 | 8.8323 | 2.83822 |
| 690 | 1. 44928 | 476100 | 328509000 | 26.2679 | 8.8366 |  |
| 691 | 1.44718 | 47748I | 329939371 | 26.2869 | 8.8408 | $2.83948$ |
| 692 | 1.44509 | 478864 | 331573888 | 26.3059 | 8.845 I | 2.84011 |
| 693 694 | 1.44300 | 480249 | 332812557 | 26.3249 | 8.8493 | 2.84073 |
| 694 | 1.44092 | 481636 | 334255384 | 26.3439 | 8.8536 | 2.8456 |
| 695 | 1.43885 | 483025 | 335702375 | 26.3629 | 8.8578 |  |
| 696 | 1.43678 | 484416 | 337153536 | 26.3818 | 8.8621 | $2.8426 \mathrm{I}$ |
| 697 | I. 43472 | 485809 | 338608873 | 26.4008 | 8.8663 | 2.84323 |
| 698 | 1. 43266 | 487204 | 340068392 | 26.4197 | 8.8706 | 2.84386 |
| 699 | I. 43062 | 488601 | 341532099 | 26.4386 | 8.8748 | 2.84448 |
| 700 | 1.42857 | 490000 | 343000000 | 26.4575 | 8.8790 | 2.84510 |
| 701 | 1.42653 | 491401 | 344472101 | 26.4764 | 8.8833 | 2.84572 |
| 702 | I. 42450 | 492804 | 345948408 | 26.4953 | 8.8875 | 2.84634 |
| 703 | I. 42248 | 494209 | 347428927 | 26.5141 | 8.8917 | 2.84696 |
| 704 | 1.42045 | 495616 | 348913664 | 26.5330 | 8.8959 | 2.84757 |
| 705 | 1.41844 | 497025 |  | 26.5518 | 8.9001 |  |
| 706 | 1.41643 | 498436 | 351895816 | 26.5707 | 8.9043 | 2.84880 |
| 707 | 1.41443 | 499849 | 353393243 | 26.5895 | 8.9085 | 2.84942 |
| 708 709 | 1.41243 1.41044 | 501264 | 354894912 | 26.6083 | 8.9127 | $2.85003$ |
| 709 | 1.41044 | 502681 | 356400829 | 26.6271 | 8.9169 | 2.85065 |
| 710 | 1. 40845 | 504100 | 357911000 | 26.6458 | 8.921 I | 2.85126 |
| 711 | I. 40647 | 505521 | 359425431 | 26.6646 | 8.9253 | 2.85187 |
| 712 | 1.40449 | 506944 | 360944128 | 26.6833 | 8.9295 | 2.85248 |
| 713 | 1.40252 1.40056 | 508369 | 362467097 | 26.7021 | 8.9337 | 2.85309 |
| 714 | 1.40056 | 509796 | 363994344 | 26.7208 | 8.9378 | 2.85370 |

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 715 | 1.39860 | 511225 | 365525875 | 26.7395 | 8.9420 | 2.85431 |
| 716 | 1.39665 | 512656 | 367061696 | 26.7582 | 8.9462 | 2.85491 |
| 717 | I. 39470 | 514089 | 368601813 | 26.7769 | 8.9503 | 2.85552 |
| 718 | 1.39276 | 515524 | 370146232 | 26.7955 | 8.9545 | 2.85612 |
| 719 | 1.39082 | 516961 | 371694959 | 26.8142 | 8.9587 | 2.85673 |
| 720 | 1.38889 | 518400 | 373248000 | 26.8328 | 8.9628 | 2.85733 |
| 721 | 1.38696 | 519841 | 374805361 | 26.8514 | 8.9670 | 2.85794 |
| 722 | I. 38504 | 521284 | 376367048 | 26.8701 | 8.9711 | 2.85854 |
| 723 | 1.38313 | 522729 | 377933067 | 26.8887 | 8.9752 | 2.85914 |
| 724 | 1.38122 | 524176 | 379503424 | 26.9072 | 8.9794 | 2.85974 |
| 725 | 1.3793 I | 525625 | 381078125 | 26.9258 | 8.9835 | 2.86034 |
| 726 | 1.37741 | 527076 | 382657176 | 26.9444 | 8.9876 | 2.86094 |
| 727 | 1.37552 | 528529 | 384240583 | 26.9629 | 8.9918 | 2.86153 |
| 728 | 1.37363 | 529984 | 385828352 | 26.9815 | 8.9959 | 2.86213 |
| 729 | 1.37174 | 53 I 44 I | 387420489 | 27.0000 | 9.0000 | 2.86273 |
| 730 | 1.36986 | 532900 | 389017000 | 27.0185 | 9.004 I | 2.86332 |
| 731 | I. 36799 | 53436 I | 390617891 | 27.0370 | 9.0082 | 2.86392 |
| 732 | 1.36612 | 535824 | 392223168 | 27.0555 | 9.0123 | 2.8645 I |
| 733 | I. 36426 | 537289 | 393832837 | 27.0740 | 9.0164 | 2.86510 |
| 734 | 1.36240 | 538756 | 395446904 | 27.0924 | 9.0205 | 2.86570 |
| 735 | 1.36054 | 540225 | 397065375 | 27.1109 | 9.0246 | 2.86629 |
| 736 | 1.35870 | 541696 | 398688256 | 27.1293 | 9.0287 | 2.86688 |
| 737 | I.35685 | 543169 | 400315553 | 27.1477 | 9.0328 | 2.86747 |
| 738 | 1.35501 | 544644 | 401947272 | 27.1662 | 9.0369 | 2.86806 |
| 739 | 1.35318 | 546121 | 403583419 | 27.1846 | 9.0410 | 2.86864 |
| 740 | 1.35135 | 547600 | 405224000 | 27.2029 | 9.0450 | 2.86923 |
| 741 | 1.34953 | 549081 | 406869021 | 27.2213 | 9.0491 | 2.86982 |
| 742 | 1.34771 | 550564 | 408518488 | 27.2397 | 9.0532 | 2.87040 |
| 743 | 1.34590 | 552049 | 410172407 | 27.2580 | 9.0572 | 2.87099 |
| 744 | 1.34409 | 553536 | 411830784 | 27.2764 | 9.0613 | 2.87157 |
| 745 | 1.34228 | 555025 | 413493625 | 27.2947 | 9.0654 | 2.87216 |
| 746 | I. 34048 | 556516 | 415160936 | $27.313^{\circ}$ | 9.0694 | 2.87274 |
| 747 | I. 33869 | 558009 | 416832723 | 27.3313 | 9.0735 | 2.87332 |
| 748 | 1.33690 | 559504 | 418508992 | 27.3496 | 9.0775 | 2.87390 |
| 749 | 1.33511 | 561001 | 420189749 | 27.3679 | 9.0816 | 2.87448 |
| 750 | 1.33333 | 562500 | 421875000 | 27.3861 | 9.0856 |  |
| 751 | 1.33156 | 564001 | 423564751 | 27.4044 | 9.0896 | $2.87564$ |
| 752 | I. 32979 | 565504 | 425259008 | 27.4226 | 9.0937 | 2.87622 |
| 753 | 1.32802 | 567009 | 426957777 | 27.4408 | 9.0977 | 2.87679 |
| 754 | 1.32626 | 568516 | 428661064 | 27.4591 | 9.1017 | 2.87737 |
| 755 | 1.32450 | 570025 | 430368875 | 27.4773 | 9.1057 |  |
| 756 | I. 32275 | 571536 | 432081216 | 27.4955 | 9.1098 | $2.8785^{2}$ |
| 757 | 1.32100 | 573049 | 433798093 | 27.5136 | 9.1138 | 2.87910 |
| 758 | I. 31926 | 574564 | 435519512 | 27.5318 | 9.1178 | 2.87967 |
| 759 | 1.31752 | 576081 | 437245479 | 27.5500 | 9.1218 | 2.88024 |
| 760 |  | 577600 | 438976000 | 27.5681 | 9.1258 | 2.8808 I |
| 761 | 1.31406 | 579121 | 440711081 | 27.5862 | 9.1298 | 2.88 I 38 |
| 762 | 1.31234 | 580644 | 442450728 | 27.6043 | 9.1338 | 2.88195 |
| 763 | I. 31062 | 582169 | 444194947 | 27.6225 | 9.1378 9.1418 | 2.88252 2.88309 |
| 764 | 1.30890 | 583696 | 445943744 | 27.6405 | 9.1418 | 2.88309 |
| 765 | 1.30719 | 585225. | 447697125 | 27.6586 | 9.1458 | 2.88366 |
| 766 | 1.30548 | 586756 | 449455096 | 27.6767 | 9.1498 | 2.88423 |
| 767 | 1.30378 | 588289 | 451217663 | 27.6948 | 9.1537 | 2.88480 |
| 768 | 1. 30208 | 589824 | 452984832 | 27.7128 | 9.1577 | 2.88536 |
| 769 | 1.30039 | 591361 | 454756609 | 27.7308 | 9.1617 | 2.88593 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $u^{2}$ | $n^{8}$ | $\sqrt{n}$ | $\sqrt[8]{n}$ | $\log \cdot n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 770 | 1.29870 | 592900 | 456533000 | 27.7489 | 9.1657 | 2.88649 |
| 771 | 1.29702 | 594441 | 458314011 | 27.7669 | 9.1696 | 2.88705 |
| 772 | 1.29534 | 595984 | 460099648 | 27.7849 | 9.1736 | 2.88762 |
| 773 | 1. 29366 | 597529 | 461889917 | 27.8029 | 9.1775 | 2.88818 |
| 774 | I. 29199 | 599076 | 463684824 | 27.8209 | 9.1815 | 2.88874 |
| 775 | 1.29032 | 600625 | 465484375 | 27.8388 | 9.1855 | 2.88930 |
| 776 | 1.28866 | 602176 | 467288576 | 27.8568 | 9.1894 | 2.88986 |
| 777 | 1.28700 | 603729 | 469097433 | 27.8747 | 9.1933 | 2.89042 |
| 778 | 1.28535 | 605284 | 470910952 | 27.8927 | 9.1973 | 2.89098 |
| 779 | I. 28370 | 606841 | 472729139 | 27.9106 | 9.2012 | 2.89154 |
| 780 | 1.28205 | 608400 | 474552000 | 27.9285 | 9.2052 | 2.89209 |
| 781 | 1.28041 | 609961 | 476379541 | 27.9464 | 9.2091 | 2.89265 |
| 782 | 1.27877 | 6II524 | 478211768 | 27.9643 | 9.2130 | 2.89321 |
| 783 | 1.27714 | 613089 | 480048687 | 27.9821 | 9.2170 | 2.89376 |
| 784 | 1.27551 | 614656 | 481890304 | 28.0000 | 9.2209 | 2.89432 |
| 785 | $1.273^{89}$ | 616225 | 483736625 | 28.0179 | 9.2248 | 2.89487 |
| 786 | 1.27226 | 617796 | 485587656 | 28.0357 | 9.2287 | 2.89542 |
| 787 | 1.27065 | 619369 | 487443403 | 28.0535 | 9.2326 | 2.89597 |
| 788 | 1. 26904 | 620944 | 489303872 | 28.0713 | 9.2365 | 2.89653 |
| 789 | I. 26743 | 622521 | 491169069 | 28.0891 | 9.2404 | 2.89708 |
| 790 | 1.26582 | 624100 | 493039000 | 28.1069 | 9.2443 | 2.89763 |
| 791 | 1.26422 | 625681 | 49491367 I | 28.1247 | 9.2482 | 2.89818 |
| 792 | 1.26263 | 627264 | 496793088 | 28.1425 | 9.2521 | 2.89873 |
| 793 | 1.26103 | 628849 | 498677257 | 28.1603 | 9.2560 | 2.89927 |
| 794 | I. 25945 | 630436 | 500566184 | 28.1780 | 9.2599 | 2.89982 |
| 795 | 1.25786 | 632025 | 502459875 | 28.1957 | 9.2638 | 2.90037 |
| 796 | 1.25628 | 633616 | 504358336 | 28.2135 | 9.2677 | 2.90091 |
| 797 | 1.25471 | 635209 | 50626 I 573 | 28.2312 | 9.2716 | 2.90146 |
| 798 | 1.25313 | 636804 | 508169592 | 28.2489 | 9.2754 | 2.90200 |
| 799 | 1.25156 | 638401 | 510082399 | 28.2666 | 9.2793 | 2.90255 |
| 800 | 1. 25000 | 640000 | 512000000 | 28.2843 | 9.2832 | 2.90309 |
| 801 | 1.24844 | 641601 | 513922401 | 28.3019 | 9.2870 | 2.90363 |
| 802 | 1.24688 | 643204 | 515849608 | 28.3196 | 9.2909 | 2.90417 |
| 803 | 1.24533 | 644809 | 517781627 | 28.3373 | 9.2948 | 2.90472 |
| 804 | 1. 24378 | 646416 | 519718464 | 28.3549 | 9.2986 | 2.90526 |
| 805 | 1.24224 | 648025 | 521660125 | 28.3725 | 9.3025 | $2.90580$ |
| 806 | 1. 24069 | 649636 | 523606616 | 28.3901 | 9.3063 | 2.90634 |
| 807 | 1.23916 | 651249 | 525557943 | 28.4077 | 9.3102 | 2.90687 |
| 808 | 1.23762 | 652864 | 5275 I 4 II 2 | 28.4253 | 9.3140 | 2.9074 I |
| 809 | 1.23609 | 65448 I | 529475129 | 28.4429 | 9.3179 | 2.90795 |
| 810 | I. 23457 | 656100 | 531441000 | 28.4605 | 9.3217 | 2.90849 |
| 811 | 1.23305 | 657721 | 533411731 | 28.4781 | 9.3255 | 2.96902 |
| 8 I 2 | 1.23153 | 659344 | 535387328 | 28.4956 | 9.3294 | 2.90956 |
| $8 \mathrm{8I} 3$ | 1.23001 | 660969 | 537367797 | 28.5132 | 9.3332 | 2.91009 |
| 814 | 1.22850 | 662596 | 539353144 | 28.5307 | 9.3370 | 2.91062 |
| 815 | 1. 22699 | 664225 |  | 28.5482 | 9.3408 | 2.91116 |
| 8 I 6 | 1. 22549 | 665856 | 543338496 | 28.5657 | $9 \cdot 3447$ | 2.91169 |
| 817 818 | 1. 22399 | 667489 | 545338513 | 28.5832 | 9.3485 | 2.91222 |
| 818 | I. 22249 | 669124 | 547343432 | 28.6007 | 9.3523 | 2.91275 |
| 819 | 1.22100 | 670761 | 549353259 | 28.6182 | 9.356 I | 2.91328 |
| 820 |  |  | 551368000 | 28.6356 | 9.3599 | $2.913^{81}$ |
| 821 | 1.21803 | 674041 | 553387661 | 28.6531 | 9.3637 | 2.91434 |
| 822 | 1.21655 | 675684 | 555412248 | 28.6705 | 9.3675 | 2.91487 |
| 823 | I. 21507 | 677329 | 557441767 | 28.6880 | 9.3713 | 2.91540 |
| 824 | I.21359 | 678976 | 559476224 | 28.7054 | 9.375 5 | 2.91593 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } 1$ | $\sqrt[8]{n}$ | $\log . x$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 825 | 1.21212 | 680625 | 561515625 | 28.7228 | 9.3789 | 2.91645 |
| 826 | 1.21065 | 682276 | 563559976 | 28.7402 | 9.3827 | 2.91698 |
| 827 | 1.20919 | 683929 | 565609283 | 28.7576 | 9.3865 | 2.91751 |
| 828 | 1.20773 | 685584 | 567663552 | 28.7750 | 9.3902 | 2.91803 |
| 829 | 1.20627 | 68724 I | 569722789 | 28.7924 | 9.3940 | 2.91855 |
| 830 | 1.20482 | 688900 | 571787000 | 28.8097 | 9.3978 | 2.91908 |
| 831 | 1.20337 | 690561 | 573856191 | 28.8271 | 9.4016 | 2.91960 |
| 832 | 1.20192 | 692224 | 575930368 | 28.8444 | 9.4053 | 2.92012 |
| 833 | I. 20048 | 693889 | 578009537 | 28.8617 | 9.4091 | 2.92065 |
| 834 | 1.19904 | 695556 | 580093704 | 28.879 I | 9.4129 | 2.92117 |
| 835 | 1.19760 | 697225 | 582182875 | 28.8964 | 9.4166 | 2.92169 |
| 836 | 1.19617 | 698896 | 584277056 | 28.9137 | 9.4204 | 2.9222 I |
| 837 | 1. 19474 | 700569 | 586376253 | 28.9310 | 9.4241 | 2.92273 |
| 838 | 1.19332 | 702244 | 588480472 | 28.9482 | 9.4279 | 2.92324 |
| 839 | 1.19190 | 703921 | 590589719 | 28.9655 | 9.4316 | 2.92376 |
| 840 | 1.19048 | 705600 | 592704000 | 28.9828 | 9.4354 | 2.92428 |
| 841 | 1.18906 | 707281 | 594823321 | 29.0000 | 9.4391 | 2.92480 |
| 842 | 1.18765 | 708964 | 596947688 | 29.0172 | 9.4429 | 2.92531 |
| 843 | 1.18624 | 710649 | 599077107 | 29.0345 | 9.4466 | 2.92583 |
| 844 | 1.18483 | 712336 | 601211584 | 29.0517 | 9.4503 | 2.92634 |
| 845 | I. 18343 | 714025 | 603351125 | 29.0689 | 9.454 I | 2.92686 |
| 846 | 1.18203 | 715716 | 605495736 | 29.0861 | 9.4578 | 2.92737 |
| 847 | 1.18064 | 717409 | 607645423 | 29.1033 | 9.4615 | 2.92788 |
| 848 849 | 1.17925 1.17786 | 719104 | 609800192 611960049 | 29.1204 | 9.4652 | 2.92840 |
| 849 | 1.17786 | 720801 | 611960049 | 29.1376 | 9.4690 | 2.92891 |
| 850 | 1.17647 | 722500 | 614125000 | 29.1548 | 9.4727 | 2.92942 |
| 851 | 1.17509 | 724201 | 616295051 | 29.1719 | 9.4764 | 2.92993 |
| 852 | 1.1737 I | 725904 | 618470208 | 29.1890 | 9.4801 | 2.93044 |
| 853 | 1.17233 | 727609 | 620650477 | 29.2062 | 9.4838 | $2.93095$ |
| 854 | 1.17096 | 729316 | 622835864 | 29.2233 | 9.4875 | 2.93146 |
| 855 | 1.16959 | 731025 | 625026375 | 29.2404 | 9.4912 | 2.93197 |
| 856 | 1.16822 | 732736 | 627222016 | 29.2575 | 9.4949 | 2.93247 |
| 857 | 1.16686 | 734449 | 629422793 | 29.2746 | 9.4986 | 2.93298 |
| 858 | 1.16550 | 736164 | 631628712 | 29.2916 | 9.5023 | 2.93349 |
| 859 | 1.16414 | 737881 | 633839779 | 29.3087 | 9.5060 | 2.93399 |
| 860 | 1.16279 | 739600 | 636056000 | 29.3258 | 9.5097 | 2.93450 |
| 861 | 1.16144 | 741321 | 638277381 | 29.3428 | 9.51 34 | 2.93500 |
| 862 | 1.16009 | 743044 | 640503928 | 29.3598 | 9.5171 | 2.93551 |
| 863 | 1.15875 | 744769 | 642735647 | 29.3769 | 9. 5207 | 2.93601 |
| 864 | 1.15741 | 746496 | 644972544 | 29.3939 | 9.5244 | 2.93651 |
| 865 | 1.15607 | 748225 |  | 29.4109 | 9.528I | 2.93702 |
| 866 | 1.15473 | 749956 | 649461896 | 29.4279 | 9.5317 | 2.93752 |
| 867 | I.15340 | 751689 | 651714363 | 29.4449 | 9.5354 . | 2.93802 |
| 868 | 1.15207 | 753424 | 653972032 | 29.4618 29.4788 | 9.5391 9.5427 | 2.93852 |
| 869 | 1.15075 | 755161 | 656234909 | 29.4788 | 9.5427 | 2.93902 |
| 870 | 1.14943 | 756900 | 658503000 | 29.4958 | 9.5464 | 2.93952 |
| 871 | 1.14811 | 758641 | 660776311 | 29.5127 | 9.5501 | 2.94002 |
| 872 | 1.14679 | 760384 762129 | 663054848 66533817 | 29.5296 29.5466 | 9.5537 | 2.94052 2.94101 |
| 873 874 | 1.14548 1.14416 | 762129 763876 | 665338617 667627624 | 29.5466 29.5635 | 9.5574 9.5610 | $\begin{aligned} & 2.94 \mathrm{IOI} \\ & 2.94 \mathrm{I} 51 \end{aligned}$ |
| 874 | 1.14416 | 763876 | 667627624 | 29.5635 | 9.5610 | 2.94151 |
| 875 | 1.14286 |  |  | $29.5804$ |  |  |
| 876 877 | 1.14155 1.14025 | 767376 769129 | 672221376 674526133 | $\begin{array}{r} 29.5973 \\ 29.6142 \end{array}$ | $\begin{aligned} & 9.5683 \\ & 9.5719 \end{aligned}$ | $\begin{aligned} & 2.94250 \\ & 2.94300 \end{aligned}$ |
| 877 878 | 1.14025 1.13895 | 769129 770884 | 674526133 676836152 | 29.6142 29.6311 | 9.5719 9.5756 | 2.94300 2.94349 |
| 879 | 1.13766 | 772641 | 679151439 | 29.6479 | 9.5792 | 2.94399 |

Table 3.
VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON'LOGARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{\pi}$ | $n^{2}$ | $n^{3}$ | $\sqrt{ } \times$ | $8 \sqrt{n}$ | $\log . n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 880 | 1.13636 | 774400 | 681472000 | 29.6648 | 9.5828 | 2.94448 |
| 881 | 1. 13507 | 776161 | 683797841 | 29.6816 | 9.5865 | 2.94498 |
| 882 | 1.13379 | 777924 | 686128968 | 29.6985 | 9.5901 | 2.94547 |
| 883 | I.13250 | 779689 | 688465387 | 29.7153 | 9.5937 | 2.94596 |
| 884 | I. 13122 | 781456 | 690807104 | 29.732 I | 9.5973 | 2.94645 |
| 885 | I.I2994 | 783225 | 693154125 | 29.7489 | 9.6010 | 2.94694 |
| 886 | 1.12867 | 784996 | 695506456 | 29.7658 | 9.6046 | 2.94743 |
| 887 | 1.12740 | 786769 | 697864103 | 29.7825 | 9.6082 | 2.94792 |
| 888 | I.12613 | 788544 | 700227072 | 29.7993 | 9.6118 | 2.94841 |
| 889 | I.I2486 | 79032 I | 702595369 | 29.816 I | 9.6154 | 2.94890 |
| 890 | 1.12360 | 792100 | 704969000 | 29.8329 | 9.6190 | 2.94939 |
| 891 | 1.12233 | 79388 I | 707347971 | 29.8496 | 9.6226 | 2.94988 |
| 892 | 1.12108 | 795664 | 709732288 | 29.8664 | 9.6262 | 2.95036 |
| 893 | I.11982 | 797449 | 712121957 | 29.8831 | 9.6298 | 2.95085 |
| 894 | I.I1857 | 799236 | 714516984 | 29.8998 | 9.6334 | 2.95134 |
| 895 | I.II732 | 801025 | 716917375 | 29.9166 | 9.6370 | 2.95182 |
| 896 | 1.11607 | 802816 | 719323136 | 29.9333 | 9.6406 | 2.95231 |
| 897 | I.11483 | 804609 | 721734273 | 29.9500 | 9.6442 | 2.95279 |
| 898 | 1.11359 | 806404 | 724150792 | 29.9666 | 9.6477 | 2.95328 |
| 899 | 1.11235 | 808201 | 726572699 | 29.9833 | 9.6513 | 2.95376 |
| 900 | 1.11111 | 810000 | 729000000 | 30.0000 | 9.6549 | 2.95424 |
| 901 | 1.10988 | 811801 | 731432701 | 30.0167 | 9.6585 | 2.95472 |
| 902 | 1.10865 | 813604 | 733870808 | 30.0333 | 9.6620 | 2.95521 |
| 903 | 1.10742 | 815409 | 736314327 | 30.0500 | 9.6656 | 2.95569 |
| 904 | 1.10619 | 817216 | 738763264 | 30.0666 | 9.6692 | 2.95617 |
| 905 | 1.10497 | 819025 | 741217625 | 30.0832 | 9.6727 | 2.95665 |
| 906 | 1.10375 | 820836 | 743677416 | 30.0998 | 9.6763 | 2.95713 |
| 907 | 1.10254 | 822649 | 746142643 | 30.1164 | 9.6799 | 2.95761 |
| 908 | 1.10132 | 824464 | 748613312 | 30.1330 | 9.6834 |  |
| 909 | 1.10011 | 82628I | 751089429 | 30.1496 | 9.6870 | 2.95856 |
| 910 | 1.09890 | 828100 | 753571000 | 30.1662 | 9.6905 |  |
| 911 | 1.09769 | 829921 | 756058031 | 30.1828 | 9.694 I | 2.95952 |
| 912 | 1.09649 | 831744 | 758550528 | 30.1993 | 9.6976 | 2.95999 |
| 913 | 1.09529 | 833569 | 761048497 | 30.2159 | 9.7012 | 2.96047 |
| 914 | 1.09409 | 835396 | 763551944 | 30.2324 | 9.7047 | 2.96095 |
| 915 | 1.09290 |  | 766060875 | 30.2490 | 9.7082 |  |
| 916 | 1.09170 | 839056 | 768575296 | 30.2655 | 9.7118 | $2.96190$ |
| 917 | 1.09051 | 840889 | 771095213 | 30.2820 | 9.7153 | 2.96237 |
| 918 | 1.08932 | 842724 | 773620632 | 30.2985 | 9.7188 | 2.96284 |
| 919 | 1.08814 | 844561 | 776151559 | 30.3150 | 9.7224 | 2.96332 |
| 920 | 1. 08696 | 846400 | 778688000 |  | 9.7259 |  |
| 921 | 1.08578 | 848241 | 78122996r | 30.3480 | $9.7294$ | $2.96426$ |
| 922 | 1.08460 | 850084 | 783777448 | 30.3645 | 9.7329 | 2.96473 |
| 923 | 1.08342 | 851929 853776 | 786330467 | 30.3809 | 9.7364 | 2.96530 |
| 924 | 1.08225 | 853776 | 788889024 | 30.3974 | 9.7400 | 2.96567 |
|  | 1.08r08 |  |  | 30.4138 |  |  |
| 926 | 1.07991 | 857476 | 794022776 | 30.4302 | 9.7470 | 2.96661 |
| 927 | 1.07875 | 859329 | 796597983 | 30.4467 | 9.7505 | 2.96708 |
| 928 | 1.07759 | 861184 | 799178752 | 30.4631 | 9.7540 | $2.96755$ |
| 929 | 1.07643 | 863041 | 801765089 | 30.4795 | 9.7575 | 2.96802 |
| 930 | 1.07527 | 864900 | 804357000 | 30.4959 | 9.7610 | 2.96848 |
| 931 | 1.07411 | 866761 | 806954491 | 30.5123 | 9.7645 | 2.96895 |
| 932 | 1.07296 | 868624 | 809557568 | 30.5287 | 9.7680 | 2.96942 |
| 933 | 1.07181 | 870489 87235 | 812166237 | 30.5450 | 9.7715 | 2.96988 |
| 934 | 1.07066 | 872356 | 814780504 | 30.5614 | 9.7750 | 2.97035 |

Smithsonian Tables. ROOTS, AND COMMON' LOCARITH'MS OF NATURAL NUMBERS.

| $n$ | $1000 \cdot \frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | 8 n | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 935 | 1.06952 | 874225 | 817400375 | 30.5778 | 9.7785 | 2.97081 |
| 936 | 1.06838 | 876096 | 820025856 | 30.5941 | 9.7819 | 2.97128 |
| 937 | 1.06724 | 877969 | 822656953 | 30.6105 | 9.7854 | 2.97174 |
| 938 | 1.06610 | 879844 | 825293672 | 30.6268 | 9.7889 | 2.97220 |
| 939 | 1.06496 | 881721 | 827936019 | 30.643 I | 9.7924 | 2.97267 |
| 940 | 1.06383 | 883600 | 830584000 | 30.6594 | 9.7959 | 2.97313 |
| 941 | 1.06270 | 885481 | 833237621 | 30.6757 | 9.7993 | 2.97359 |
| 942 | I.061 57 | 887364 | 835896888 | 30.6920 | 9.8028 | 2.97405 |
| 943 | 1.06045 | 889249 | 838561807 | 30.7083 | 9.8063 | 2.9745 I |
| 944 | 1.05932 | 891136 | 841532384 | 30.7246 | 9.8097 | 2.97497 |
| 945 | 1.05820 | 893025 | 843908625 | 30.7409 | 9.8132 | 2.97543 |
| 946 | 1.05708 | 894916 | 84.6590536 | 30.757 I | 9.8167 | 2.97589 |
| 947 | 1.05597 | 896809 | 849278123 | 30.7734 | 9.8201 | 2.97635 |
| 948 | 1.05485 | 898704 | 851971392 | 30.7896 | 9.8236 | 2.9768 I |
| 949 | 1.05374 | 900601 | 854670349 | 30.8058 | 9.8270 | 2.97727 |
| 950 | 1.05263 | 902500 | 857375000 | 30.8221 | 9.8305 | 2.97772 |
| 951 | 1.05152 | 904401 | 860085351 | 30.8383 | 9.8339 | 2.97818 |
| 952 | 1.05042 | 906304 | 862801408 | 30.8545 | 9.8374 | 2.97864 |
| 953 | 1.04932 | 908209 | 865523177 | 30.8707 | 9.8408 | 2.97909 |
| 954 | 1.04822 | 910116 | 868250664 | 30.8869 | 9.8443 | 2.97955 |
| 955 | 1.04712 | 912025 | 870983875 | 30.9031 | 9.8477 | 2.98000 |
| 956 | 1.04603 | 913936 | 873722816 | 30.9192 | 9.851 I | 2.98046 |
| 957 | 1.04493 | 915849 | 876467493 | 30.9354 | 9.8546 | 2.98091 |
| 958 | 1.04384 | 917764 | 879217912 | 30.9516 | 9.8580 | 2.98137 |
| 959 | 1.04275 | 919681 | 881974079 | 30.9677 | 9.8614 | 2.98182 |
| 960 | 1. 04167 | 921600 | 884736000 | 30.9839 | 9.8648 | 2.98227 |
| 961 | 1.04058 | 923521 | 88750368 I | 31.0000 | 9.8683 | 2.98272 |
| 962 | 1.03950 | 925444 | 890277128 | 31.0161 | 9.8717 | 2.98318 |
| 963 | 1.03842 | 927369 | 893056347 | 31.0322 | 9.8751 | $2.98363$ |
| 964 | 1.03734 | 929296 | 895841344 | 31.0483 | 9.8785 | 2.98408 |
|  | 1.03627 I. 03520 |  |  |  |  |  |
| 966 967 | 1.03520 1.03413 | 933156 935089 | 901428696 904231063 | 31.0805 31.0966 | 9.8854 9.8888 | $\begin{aligned} & 2.98498 \\ & 2.98543 \end{aligned}$ |
| 968 | 1.03306 | 937024 | 907039232 | 31.1127 | 9.8922 | 2.98588 |
| 969 | 1.03199 | 938961 | 909853209 | 31.1288 | 9.8956 | 2.98632 |
| 970 | 1.03093 | 940900 | 912673000 | 31.1448 | 9.8990 | 2.98677 |
| 971 | 1.02987 | 942841 | 915498611 | 31.1609 | 9.9024 | 2.98722 |
| 972 | I. 02881 | 944784 | 918330048 | 31.1769 | 9.9058 | 2.98767 |
| 973 | 1.02775 1.02669 | 946729 948676 | 921167317 924010424 | 31.1929 31.2090 | 9.9092 9.9126 | 2.9881 I |
| 974 | 1.02669 | 948676 | 924010424 | $3^{1.2090}$ | 9.9126 | 2.98856 |
| 975 | 1.02564 | 950625 | 926859375 | 31.2250 | 9.9160 |  |
| 976 | I. 02459 | 952576 | 929714176 | 3 3 .24 .10 | 9.9194 | 2.98945 |
| 977 | 1.02354 | 954529 | 932574833 | 3 S .2570 | 9.9227 | 2.98989 |
| 978 | 1.02249 | 956484 | 935441352 | 3 3 .2730 | 9.9261 | 2.99034 2.99078 |
| 979 | 1.02145 | 95844 I | 938313739 | 31.2890 | 9.9295 | 2.99078 |
| 980 | 1.02041 | 960400 | 941192000 | 3 T .3050 | 9.9329 | 2.99123 |
| 98 I | 1.01937 | 962361 | 944076141 | 31.3209 | 9.9363 | 2.99167 |
| 982 | I. 01833 | 964324 | 946966168 | 3 L .3369 | 9.9396 | 2.99211 |
| 983 | 1.01729 | 966289 | 949862087 |  | 9.9430 |  |
| 984 | 1.01626 | 968256 | 952763904 | 31.3688 | 9.9464 | 2.99300 |
| 985 | 1.O1 523 | 970225 | 955671625 | 31.3847 | 9.9497 | 2.99344 |
| 986 | 1.01420 | 972196 | 958585256 | 31.4006 | 9.9531 | 2.99388 |
| 987 | 1.01317 | 974169 | 961504803 | 31.4166 | 9.9565 | 2.99432 |
| 988 | 1.01215 | 976144 | 964430272 | 31.4325 | 9.9598 | 2.99476 |
| 989 | 1.01112 | 978121 | 967361669 | $3^{1} \cdot 4484$ | 9.9632 | 2.99520 |

## Table 3.

VALUES OF RECIPROCALS, SQUARES, CUBES, SQUARE ROOTS, CUBE ROOTS, AND COMMON LOCARITHMS OF NATURAL NUMBERS.

| $n$ | 1000. $\frac{1}{n}$ | $n^{2}$ | $n^{8}$ | $\sqrt{ } \times$ | $\sqrt[8]{n}$ | log. $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 990 | 1.01010 | 980100 | 970299000 | 31.4643 | 9.9666 | 2.99564 |
| 991 | 1.00908 | 982081 | 97324227 I | 3 3 .4802 | 9.9699 | 2.99607 |
| 992 | 1.00806 | 984064 | 976191488 | 31.4960 | 9.9733 | 2.99651 |
| 993 | 1.00705 | 986049 | 979146657 | 31.5119 | 9.9766 | 2.99695 |
| 994 | 1.00604 | 988036 | 982107784 | 31.5278 | 9.9800 | 2.99739 |
| 995 | 1.00503 | 990025 | 985074875 | 31.5436 | 9.9833 |  |
| 996 | 1.00402 | 992016 | 988047936 | 31.5595 | 9.9866 | 2.99826 |
| 997 | r.00301 | 994009 | 991026973 | 31.5753 | 9.9900 | 2.99870 |
| 998 | 1.00200 | 996004 | 994011992 | 31.5911 | 9.9933 | 2.99913 |
| 999 | 1.00100 | 998001 | 997002999 | 31.6070 | 9.9967 | 2.99957 |
| 1000 | 1.00000 | 1000000 | 1000000000 | 31.6228 | 10.0000 | 3.00000 |

CIRCUMFERENCE AND AREA OF CIRCLE IN TERMS OF DIAMETER $a$.

| $d$ | $\pi d$ | $\frac{1}{4} \pi d^{2}$ | $d$ | $\pi d$ | $\frac{1}{4} \pi d^{2}$ | $d$ | $\pi d$ | $\frac{1}{4} \pi d^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 31.416 | 78.5398 | 40 | 125.66 | 1256.64 | 70 | 219.91 | 3848.45 |
| II | 34.558 | 95.0332 | 41 | I 28.81 | 1320.25 | 71 | 223.05 | 3959.19 |
| 12 | 37.699 | 113.097 | 42 | 13 I .95 | I 385.44 | 72 | 226.19 | 4071.50 |
| 13 | 40.841 | 132.732 | 43 | 135.09 | 1452.20 | 73 | 229.34 | 4185.39 |
| 14 | 43.982 | 153.938 | 44 | 138.23 | 1520.53 | 74 | 232.48 | 4300.84 |
| 15 | 47.124 | 176.715 | 45 | 141.37 | I 590.43 | 75 | 235.62 | 4417.86 |
| 16 | 50.265 | 201.062 | 46 | 144.51 | 1661.90 | 76 | 238.76 | 4536.46 |
| 17 | 53.407 | 226.980 | 47 | 147.65 | 1734.94 | 77 | 241.90 | 4656.63 |
| 18 | 56.549 | 254.469 | 48 | 150.80 | 1809.56 | 78 | 245.04 | 4778.36 |
| 19 | 59.690 | 283.529 | 49 | 153.94 | 1885.74 | 79 | 248.19 | 4901.67 |
| 20 | 62.832 | 314.159 | 50 | 157.08 | 1963.50 | 80 | 251.33 | 5026.55 |
| 2 I | 65.973 | 346.361 | 51 | 160.22 | 2042.82 | 8 I | 254.47 | 5153.00 |
| 22 | 69.115 | 380.133 | 52 | 163.36 | 2123.72 | 82 | 257.61 | 528 I .02 |
| 23 | 72.257 | 415.476 | 53 | 166.50 | 2206.18 | 83 | 260.75 | 5410.61 |
| 24 | $75 \cdot 398$ | 452.389 | 54 | 169.65 | 2290.22 | 84 | 263.89 | 5541.77 |
|  | 78.540 81.681 | 490.874 530.929 |  | 172.79 175.93 | 2375.83 | 85 86 | 267.04 270.18 | 5674.50 5808.80 |
| 26 27 | 81.68 I 84.823 | 530.929 | 56 | 175.93 | 2463.01 | 86 87 | 270.18 | 5808.80 |
|  | 84.823 | 572.555 | 57 | I 79.07 | 2551.76 | 87 | 273.32 | 5944.68 |
| 28 | 87.965 | 615.752 | 58 | 182.21 | 2642.08 | 88 | 276.46 | 6082.12 |
| 29 | 9 T .106 | 66.520 | 59 | 185.35 | 2733.97 | 89 | 279.60 | 6221.14 |
| 30 | 94.248 | 706.858 | 60 | 188.50 | 2827.43 | 90 | 282.74 | 6361.73 |
| 31 | 97.389 | 754.768 | 61 | 191. 64 | 2922.47 | 9 I | 285.88 | 6503.88 |
| 32 | 100.53 | 804.248 | 62 | 194.78 | 3019.07 | 92 | 289.03 | 6647.61 |
| 33 | 103.67 | 855.299 | 63 | 197.92 | 3117.25 | $93{ }^{\circ}$ | 292.17 | 6792.91 |
| 34 | 106.81 | 907.920 | 64 | 201.06 | 3216.99 | 94 | 295.31 | 6939.78 |
| 35 | 109.96 | 962.113 | 65 | 204.20 | 3318.31 | 95 | 298.45 | 7088.22 |
| 36 | 113.10 | IOT 7.88 | 66 | 207.35 | 342 I. 19 | 96 | 301.59 | 7238.23 |
| 37 | 116.24 | 1075.21 | 67 | 210.49 | 3525.65 | 97 | 304.73 | 7389.81 |
| 38 39 | 119.38 | 1134.11 | 68 | 213.63 | 3631.68 | 98 | 307.88 | 7542.96 |
| 39 | 122.52 | I 194.59 | 69 | 216.77 | 3739.28 | 99 | 311.02 | 7697.69 |

Smithsonian Tables.


Smithsonian Tables.

LOGARITHMS OF NUMBERS.


Smithsonian Tables.


Smithsonian Tables.

ANTILOGARITHMS.


Smithsonian Tables.

Table 7.

Natural Sines.

| Angle. | $0^{\prime}$ | $10^{\prime}$ | 20' | $30^{\prime}$ | $40^{\prime}$ | 50' | $60^{\prime}$ | Angle. | Proy. Parts for 1 . |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | . 000000 | . 002909 | . 005818 | . 008727 | . 011635 | . 014544 | . $01745^{2}$ | $89^{\circ}$ | 2.9 |
| 1 | . 017452 | . 02036 | . 02327 | .026I 8 | . 02908 | . 03199 | . 03490 | 88 | 2.9 |
| 2 | . 03490 | . 03781 | . 04071 | . 04362 | . 04653 | . 04943 | . 05234 | 87 | 2.9 |
| 3 | . 05234 | . 05524 | .05814 | .0610 5 | . 06395 | . 06685 | . 06976 | 86 | 2.9 |
| 4 | . 06976 | . 07266 | . 07556 | . 07846 | .08136 | . 08426 | . 08716 | 85 | 2.9 |
| 5 | .087r 6 | . 09005 | . 09295 | . 09585 | . 09874 | .ror 64 | .10453 | 84 | 2.9 |
| 6 | . 10453 | . 10742 | . 1103 I | .11320 | . 11609 | .11898 | . 12187 | 83 | 2.9 |
| 7 | .12187 | . 12476 | .12764 | . 13053 | . 334 | . 1363 | . 1392 | 82 | 2.9 |
| 8 | . 1392 | .1421 | . 1449 | .1478 | . 1507 | . 1536 | . 1564 | 8 r | 2.9 |
| 9 | .I 564 | .I 593 | . 1622 | . 1650 | .1679 | . 1708 | . 1736 | 80 | 2.9 |
| 10 | . 1736 | . 1765 | . 1794 | . 1822 | . 1851 | . 1880 | . 1908 | 79 | 2.9 |
| 11 | .1908 | . 1937 | .1965 | . 1994 | . 2022 | . 2051 | . 2079 | 78 | 2.9 |
| 12 | . 2079 | . 2108 | . 2136 | . 2164 | . 2193 | . 2221 | . 2250 | 77 | 2.8 |
| 13 | . 2250 | . 2278 | . 2306 | . 2334 | . 2363 | . 2391 | . 2419 | 76 | 2.8 |
| 14 | .2419 | . 2447 | . 2476 | . 2504 | .2532 | :2560 | . 2588 | 75 | 2.8 |
| 15 | . 2588 | . 2616 | . 2644 | . 2672 | . 2700 | . 2728 | . 2756 | 74 | 2.8 |
| 16 | . 2756 | . 2784 | .2812 | . 2840 | . 2868 | . 2896 | . 2924 | 73 | 2.8 |
| 17 | . 2924 | . 2952 | . 2979 | -3007 | . 3035 | . 3062 | -3090 | 72 | 2.8 |
| 18 | . 3090 | -3118 | .3145 | . 3173 | . 3201 | . 3228 | . 3256 | 71 | 2.8 |
| 19 | .3256 | .3283 | .331 1 | . 3338 | . 3365 | . 3393 | - 3420 | 70 | 2.7 |
| 20 | . 3420 | . 3448 | -3475 | . 3502 | . 3529 | . 3557 | . 3584 | 69 | 2.7 |
| 21 | . 3584 | . 3611 | . 3638 | . 3665 | . 3692 | . 3719 | . 3746 | 68 | 2.7 |
| 22 | . 3746 | . 3773 | . 3800 | . 3827 | . 3854 | . 3881 | . 3907 | 67 | 2.7 |
| 23 | . 3907 | . 3934 | -3961 | . 3987 | . 4014 | .4041 | . 4067 | 66 | 2.7 |
| 24 | . 4067 | . 4094 | . 4120 | . 4147 | -4173 | . 4200 | . 4226 | 65 | 2.7 |
| 25 | . 4226 | .4253 | . 4279 | . 4305 | .4331 | . 4358 | .4384 | 64 | 2.6 |
| 26 | . 4384 | .4410 | . 4436 | . 4462 | . 4488 | . 4514 | . 4540 | 63 | 2.6 |
| 27 | -4540 | . 4566 | . 4592 | - 4617 | . 4643 | . 4669 | . 4695 | 62 | 2.6 |
| 28 | . 4695 | . 4720 | . 4746 | . 4772 | . 4797 | . 4823 | . 4848 | 61 | 2.6 |
| 29 | . 4848 | . 4874 | . 4899 | . 4924 | . 4950 | . 4975 | . 5000 | 60 | 2.5 |
| 30 | . 5000 | . 5025 | . 5050 | . 5075 | . 5100 | . 5125 | . 5150 | 59 | 2.5 |
| 31 | . 5150 | . 5175 | . 5200 | . 5225 | . 5250 | . 5275 | . 5299 | . 58 | 2.5 |
| 32 | . 5299 | . 5324 | . 5348 | . 5373 | . 5398 | . 5422 | . 5446 | 57 | 2.5 |
| 33 | . 5446 | . 54771 | . 5495 | .5519 .5664 | . 5544 | . 5568 | . 5592 | 56 | 2.4 |
| 34 | - 5592 | -56r6 | . 5640 | . 5664 | . 5688 | . 5712 | . 5736 | 55 | 2.4 |
| 35 | . 5736 | . 5760 | .5783 | . 5807 | . 5831 | . 5854 | . 5878 | 54 | 2.4 |
| 36 | . 5878 | . 5901 | . 5925 | . 5948 | . 5972 | . 5995 | . 6013 | 53 | 2.3 |
| 37 | . 6018 | . 604 I | . 6065 | . 6088 | .6111 | . 6134 | . 6157 | 52 | 2.3 |
| 38 | .6157 6293 | . 6180 | . 6202 | . 6225 | . 6248 | . 6271 | . 6293 | 51 | 2.3 |
| 39 | . 6293 | . 6316 | . 6338 | .636I | . 6383 | . 6406 | . 6428 | 50 | 2.3 |
| 40 | .. 6428 |  | . 6472 |  |  |  |  | 49 | 2.2 |
| 41 | . 6561 | . 6583 | . 6604 | . 6626 | . 6648 | . 6670 | . 6691 | 48 | 2.2 |
| 42 | ..6691 | . 6713 | . 6734 | . 6756 | . 6777 | . 6799 | . 6820 | 47 | 2.2 |
| - 43 | . 6820 | . 6841 | . 6862 | . 6884 | . 6905 | . 6926 | . 6947 | 46 | 2.1 |
| - 44 | .. 6947 | . 6967 | . 6988 | .7009 | . 7030 | .7050 | .7071 | 45 | 2.1 |
|  | 60' | $50^{\prime}$ | $40^{\prime}$ | $30^{\prime}$ | $20^{\prime}$ | $10^{\prime}$ | $0^{\prime}$ | Angle. |  |

Smithsonian Tables.

NATURAL SINES AND COSINES.
Natural Sines.

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline Angle. \& \(0^{\prime}\) \& 10' \& \(20^{\prime}\) \& \(30^{\prime}\) \& 40' \& \(50^{\prime}\) \& 60' \& Angle. \& \[
\left\lvert\, \begin{aligned}
\& \text { Prop, } \\
\& \text { Parto } \\
\& \text { tar 1. }
\end{aligned}\right.
\] \\
\hline \(45^{\circ}\) \& .7071 \& . 7092 \& 7112 \& 7133 \& . 7153 \& . 7173 \& . 7193 \& \(44^{\circ}\) \& 2.0 \\
\hline 46 \& .7193 \& . 7214 \& . 7234 \& . 7254 \& . 7274 \& . 7294 \& . 7314 \& 43 \& 2.0 \\
\hline 47 \& . 7314 \& . 7333 \& . 7353 \& . 7373 \& . 7392 \& . 7412 \& .743I \& 42 \& 2.0 \\
\hline 48 \& .743r \& . 77515 \& . 7478 \& . 7490 \& . 7509 \& . 7528 \& . 7547 \& \(4{ }^{1}\) \& 1.9 \\
\hline 49 \& .7547 \& . 7566 \& . 758 \& . 7604 \& . 7623 \& . 7642 \& . 7660 \& 40 \& 1.9 \\
\hline 50 \& . 7660 \& . 7679 \& . 7698 \& . 7716 \& . 7735 \& . 7753 \& .7771 \& 39 \& 1.9 \\
\hline 51 \& .7771 \& . 7790 \& .7808 \& . 7826 \& . 7844 \& . 7868 \& . 7880 \& 38 \& 1.8 \\
\hline \begin{tabular}{l}
52 \\
53 \\
\hline
\end{tabular} \& \& .7898 \& .7916
.8021 \& . 7934 \& . 7951 \& . 7969 \& .7986 \& 37
36 \& I. 8
I .7 \\
\hline 54 \& . 8090 \& . 8107 \& . 8 I 24 \& . 8141 \& . 8158 \& . 8175 \& . 8192 \& 35 \& \(\begin{array}{r}1.7 \\ \mathrm{r} \\ \hline\end{array}\) \\
\hline 55 \& . 8192 \& . 8208 \& . 8225 \& .8241 \& . 8258 \& . 8274 \& . 8290 \& 34 \& r. 6 \\
\hline 56 \& .8290 \& . 8307 \& . 8323 \& . 8339 \& . 8355 \& . 8371 \& . 8387 \& 33 \& 1.6 \\
\hline 57
58 \& . 8388 \& . 8403 \& . 8418 \& . 8434 \& . 8450 \& . 8465 \& . 8480 \& 32 \& 1. 6 \\
\hline 58 \& . 8480 \& .8496 \& . 8511 II \& . 8526 \& . 8542 \& . 8555 \& . 8572 \& 31 \& 1.5 \\
\hline 59 \& . 8572 \& . 8587 \& \& \& .8631 \& . 8646 \& . 8660 \& 30 \& 1.5 \\
\hline 60 \& . 8660 \& . 8675 \& . 8689 \& . 8704 \& . 8718 \& . 8732 \& . 8746 \& 29 \& 1.4 \\
\hline 6 F \& . 8746 \& . 8760 \& . 8774 \& . 8788 \& . 8802 \& . 8816 \& . 8829 \& 28 \& 1.4 \\
\hline 62 \& . 8829 \& . 8843 \& . 8857 \& . 8870 \& . 8884 \& . 8897 \& . 8910 \& 27 \& - 1.4 \\
\hline 63 \& . 89810 \& . 8923 \& . 8936 \& . 8949 \& . 8962 \& . 8975 \& . 8998 \& 26 \& I. 3 \\
\hline 64 \& . 8988 \& . 9001 \& . 9013 \& . 9026 \& . 9038 \& . 9051 \& . 9063 \& 25 \& 1.3 \\
\hline 65 \& . 9063 \& . 9075 \& . 9088 \& . 9100 \& . 9112 \& .9124 \& .9135 \& 24 \& 1.2 \\
\hline 66 \& . 9135 \& . 9147 \& .9159 \& . 9171 \& -9182 \& . 9194 \& .9205 \& 23 \& I. 2 \\
\hline 67 \& . 9205 \& . 9216 \& . 9228 \& . 9239 \& . 9250 \& .926I \& . 9272 \& 22 \& 1.1 \\
\hline 68 \& . 9272 \& . 9283 \& . 9293 \& . 9304 \& . 9315 \& . 9325 \& . 9336 \& 21 \& \({ }^{1.1}\) \\
\hline 69 \& . 9336 \& . 9346 \& . 9356 \& . 9367 \& . 9377 \& . 9388 \& -9397 \& 20 \& I. 0 \\
\hline 70 \& . 9397 \& . 9407 \& .9417 \& . 9426 \& . 9436 \& . 9446 \& . 9455 \& 19 \& 1.0 \\
\hline 71 \& . 9455 \& . 9465 \& . 9474 \& . 9483 \& . 9492 \& . 9502 \& . 9511 \& 18 \& 0.9 \\
\hline 72 \& .9511 \& -9520 \& . 9528 \& . 95387 \& . 9546 \& . 9555 \& . 9563 \& 17 \& -0.9 \\
\hline 73
74 \& .9563 \& . 95972 \& . 95680 \& . 95688 \& . 9596 \& . 9605 \& . 9613 \& \({ }_{15}^{15}\) \& 0.8
0.8 \\
\hline 74 \& . 9613 \& . 9621 \& . 9628 \& . 9636 \& .9644 \& . 9652 \& . 9659 \& 15 \& 0.8 \\
\hline 75 \& . 9659 \& . 9667 \& . 9674 \& .9681 \& . 9689 \& . 9696 \& . 9703 \& 14 \& 0.7 \\
\hline 76 \& . 9703 \& . 97710 \& .9717 \& . 9724 \& . 9730 \& . 9737 \& . 9744 \& I3 \& 0.7 \\
\hline 77
78 \& . 9744 \& .9750
.9787 \& . 97575 \& . 9763 \& . 9769 \& . 97715 \& .978 r
.9816 \& \begin{tabular}{l}
12 \\
11 \\
\hline 1
\end{tabular} \& 0.6
0.6 \\
\hline 78
79 \& .97816 \& .9787
.9822 \& . 97893 \& . 97893 \& . 98838 \& . 98813 \& .9816
.9848 \& İ
IO \& 0.6
0.5 \\
\hline 80 \& . 9848 \& . 9853 \& . 9858 \& . 9863 \& . 9868 \& . 9872 \& . 9877 \& 9 \& 0.5 \\
\hline \(8 \mathrm{8r}\) \& . 9877 \& . 988 I \& . 9886 \& . 9890 \& . 9894 \& . 9899 \& . 9903 \& 8 \& 0.4 \\
\hline 82 \& . 9903 \& . 9907 \& . 9911 \& . 9914 \& . 9918 \& . 9922 \& . 9925 \& 7 \& 0.4 \\
\hline 8 \& . 9925 \& . 9929 \& . 9932 \& . 9936 \& -9939 \& . 9942 \& -9945 \& 6 \& 0.3 \\
\hline 84 \& . 9945 \& . 9948 \& . 9951 \& . 9954 \& . 9957 \& . 9959 \& . 9962 \& 5 \& 0.3 \\
\hline 85 \& . 9962 \& . 9964 \& . 9967 \& . 9969 \& . 9971 \& . 9974 \& . 9976 \& 4 \& 0.2 \\
\hline 86 \& . 9976 \& . 9978 \& . 9980 \& . 9985 \& . 9983 \& . 9985 \& . 9986 \& 3 \& 0.2 \\
\hline 87 \& . 9986 \& . 9988 \& . 9989 \& . 9990 \& . 9992 \& -9993 \& . 9994 \& 2 \& 0.1 \\
\hline 89 \& . 9994 \& . 99999 \& . 99996 \& \(\begin{array}{r}.9997 \\ \hline 1.0000\end{array}\) \& r. 9997

1.000 \& r. \& $\begin{array}{r}\text { r } \\ \hline\end{array}$ \& $\stackrel{1}{\circ}$ \& 0.1
0.0 <br>
\hline \& $60^{\circ}$ \& 50' \& $40^{\prime}$ \& $30^{\prime}$ \& $20^{\prime}$ \& 10' \& $0^{\prime}$ \& Anglo. \& <br>
\hline
\end{tabular}

Emithsonian Tables.
Natural Cosines.

NATURAL TANGENTS AND COTANGENTS.
Natural Tangents.

| Angle. | $0^{\prime}$ | 10' | $20^{\prime}$ | $30^{\prime}$ | $40^{\prime}$ | 50' | $60^{\prime}$ | Angle. | Prop. Parts for 1". |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | . 00000 | . 0029 I | . 00582 | . 00873 | . 01164 | . 01455 | .01746 | $89^{\circ}$ | 2.9 |
| 1 | . 01746 | . 02036 | . 02328 | . 02619 | .02910 | . 03201 | . 03492 | 88 | 2.9 |
| 2 | .03492 | . 0378 | . 04075 | . 04366 | . 04658 | . 04949 | . 05241 | 87 | 2.9 |
| 3 | . 05241 | . 05533 | . 05824 | .06II 6 | . 06408 | . 06700 | . 06993 | 86 | 2.9 |
| 4 | . 06993 | . 07285 | . 07578 | . 07870 | . 08163 | .08456 | . 08749 | 85 | 2.9 |
| 5 | . 08749 | . 09042 | . 09335 | .09629 | .09923 | .1021 6 | .10510 | 84 | 2.9 |
| 6 | . 10510 | . 10805 | . 11099 | .II39 4 | . 11688 | . 11983 | . 12278 | 83 | 2.9 |
| 7 | . 12278 | . 12574 | . 12869 | .13165 | . 1346 | . 1376 | . 1405 | 82 | 3.0 |
| 8 | . 1405 | .1435 | .1465 | . 1495 | . 1524 | . 1554 | . 1584 | $8 \mathrm{8r}$ | 3.0 |
| 9 | . 1584 | .16I4 | . 1644 | .1673 | .1703 | . 1733 | . 1763 | 80 | 3.0 |
| 10 | . 1763 | . 1793 | . 1823 | . 1853 | . 1883 | . 1914 | . 1944 | 79 | 3.0 |
| 11 | . 1944 | . 1974 | . 2004 | . 2035 | . 2065 | . 2095 | . 2126 | 78 | 3.0 |
| 12 | .2126 | .2156 | . 2186 | .2217 | . 2247 | . 2278 | .2309 | 77 | 3.1 |
| 13 | . 2309 | . 2339 | . 2370 | . 2401 | . 2432 | . 2462 | . 2493 | 76 | 3.1 |
| 14 | . 2493 | . 2524 | . 2555 | . 2586 | . 2617 | . 2648 | . 2679 | 75 | 3.1 |
| 15 | . 2679 | . 2711 | . 2742 | . 2773 | . 2805 | . 2836 | . 2867 | 74 | $3 \cdot 1$ |
| 16 | . 2867 | . 2899 | . 2931 | . 2962 | . 2994 | . 3026 | . 3057 | 73 | 3.2 |
| 17 | . 3057 | . 3089 | -3121 | . 3153 | . 3185 | -3217 | . 3249 | 72 | 3.2 |
| 18 | . 3249 | . 328 r | .3314 | . 3346 | -3378 | . 3411 | . 3443 | 71 | 3.2 |
| 19 | . 3443 | . 3476 | . 3508 | . 3541 | -3574 | $\cdot 3607$ | - 3640 | 70 | $3 \cdot 3$ |
| 20 | . 3640 | . 3673 | .3706 | . 3739 | . 3772 | .3805 | .3839 | 69 | $3 \cdot 3$ |
| 21 | . 3839 | .3872 | . 3906 | .3939 | . 3973 | . 4006 | . 4040 | 68 | 3.4 |
| 22 | . 4040 | . 4074 | -4108 | . 4142 | .4176 | .4210 | . 4245 | 67 | 3.4 |
| 23 | . 4245 | . 4279 | . 4314 | . 4348 | . 4383 | . 4417 | . 4452 | 66 | $3 \cdot 5$ |
| 24 | . $445^{2}$ | . 4487 | . $45^{22}$ | . 4557 | . 4592 | . 4628 | .4663 | 65 | 3.5 |
| 25 | . 4663 | . 4699 | . 4734 | . 4770 | . 4806 | . 4841 | . 4877 | 64 | 3.6 |
| 26 | . 4877 | . 4913 | . 4950 | . 4986 | . 5022 | . 5059 | . 5095 | 63 | 3.6 |
| 27 | . 5095 | -51 $3^{2}$ | . 5169 | . 5206 | . 5243 | . 5280 | . 5317 | 62 | 3.7 |
| 28 | . 5317 | - 5354 | . 5392 | . 5430 | . 5467 | . 5505 | . 5543 | 61 | 3.8 |
| 29 | . 5543 | -558I | . 5619 | . 5658 | . 5696 | . 5735 | . 5774 | 60 | 3.8 |
| 30 | . 5774 | . 5812 | . 5851 | . 5890 | . 5938 | . 5969 | . 6009 | 59 | 3.9 |
| 31 | . 6009 | . 6048 | . 6088 | . 6128 | . 6168 | . 6208 | . 6249 | 58 | 4.0 |
| 32 | . 6249 | . 6289 | . 6330 | . 6371 | . 6412 | . 6453 | . 6494 | 57 | 4.1 |
| 33 | . 6494 | . 6536 | . 6577 | . 6619 | . 6661 | . 6703 | . 6745 | 56 | 4.2 |
| 34 | . 6745 | . 6787 | . 6830 | . 6873 | . 6916 | . 6959 | . 7002 | 55 | $4 \cdot 3$ |
| 35 | . 7002 | . 7046 | . 7089 | .7133 | . 7177 | . 7221 | $.7265^{\circ}$ | 54 | 4.4 |
| 36 | .7265 | .7310 | .7355 .7627 | .7400 .7673 | . 7445 | . 7490 | . 7536 | 53 | 4.5 |
| 37 | .7536 .7813 | .7581 .7860 | .7627 .7907 | .7673 .7954 | .7720 | . 7766 | .7813 | 52 | 4.6 4.7 |
| 39 | . 8098 | .8146 | . 8195 | . 8243 | . 8292 | . 8342 | . 8391 | 50 | 4.7 |
| 40 | .8391 | . 8441 | . 8491 | . 8541 | . 8591 | . 8642 | . 8693 | 49 | 5.0 |
| 41 | . 8693 | . 8744 | . 8796 | . 8847 | . 8899 | . 8952 | . 9004 | 48 | 5.2 |
| 42 | . 9004 | . 9057 | .9110 | . 9163 | . 9217 | . 9271 | . 9325 | 47 | $5 \cdot 4$ |
| 43 | . 9325 | . 9380 | . 9435 | . 9490 | . 9545 | .9601 | . 9657 | 46 | $5 \cdot 5$ |
| 44 | . 9657 | .9713 | . 9770 | . 9827 | . 9884 | . 9942 | 1.0000 | 45 | 5.7 |
|  | 60' | 50' | $40^{\circ}$ | $30^{\prime}$ | 20' | $10^{\prime}$ | $0^{\prime}$ | Angle. |  |

Smithsonian Tables.
Natural Cotangents.

Natural Tangents.

| Angis, | $0^{\prime}$ | $10^{\prime}$ | $20^{\prime}$ | $30^{\prime}$ | $40^{\prime}$ | $50^{\prime}$ | $60^{\prime}$ | Angle. | Prop. Parts for $1^{\prime}$. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $45^{\circ}$ | 1.0000 | 1.0058 | 1.0117 | 1.0176 | 1.0235 | 1. 0295 | 1.0355 | $44^{\circ}$ | $5 \cdot 9$ |
| 46 | 1.0355 | 1.0416 | 1.0477 | 1.0538 | 1.0599 | 1.066 I | 1.0724 | 43 | 6.1 |
| 47 | 1.0724 | 1.0786 | 1.0850 | 1.0913 | 1.0977 | 1.104 | 1.1106 | 42 | 6.4 |
| 48 | 1.1106 | 1.1171 | 1.1237 | 1.1303 | 1.1369 | 1.1436 | 1.1504 | 41 | 6.6 |
| 49 | 1.1504 | 1.1571 | 1.1640 | 1.1708 | 1.1778 | I.I847 | 1.1918 | 40 | 6.9 |
| 50 | 1.1918 | 1.1988 | 1.2059 | 1.2131 | 1.2203 | 1.2276 | 1.2349 | 39 | 7.2 |
| 51 | 1.2349 | 1.2423 | 1.2497 | 1.2572 | 1.2647 | 1.2723 | 1. 2799 | 38 | 7.5 |
| 52 | I. 2799 | 1.2876 | 1.2954 | I. 3032 | I.3111 | 1.3190 | 1.3270 | 37 | 7.9 |
| 53 | I. 3270 | 1.3351 | 1.3432 | 1.3514 | I. 3597 | I. 3680 | 1.3764 | 36 | 8.2 |
| 54 | 1.3764 | 1.3848 | 1.3934 | 1.4019 | 1.4106 | 1.4193 | I.428r | 35 | 8.6 |
| 55 | 1.428r | 1.4370 | 1.4460 | 1.4550 | 1.464 I | I. 4733 | 1.4826 | 34 | 9.1 |
| 56 | 1.4826 | 1.4919 | 1.5013 | 1.5108 | 1. 5204 | 1.5301 | I. 5399 | 33 | 9.6 |
| 57 | 1.5399 | 1.5497 | 1.5597 | I. 5697 | 1.5798 | 1.5900 | 1.6003 | 32 | 10.1 |
| 58 | 1.6003 | 1.6107 | 1.6212 | 1.6319 | 1.6426 | 1.6534 | $\pm .6643$ | 31 | 10.7 |
| 59 | 1.6643 | 1.6753 | 1. 6864 | ェ. 6977 | 1.7090 | 1.7205 | 1.732 I | 30 | 11.3 |
| 60 | 1.7321 | 1.7437 | 1.7556 | 1. 7675 | 1.7796 | 1.7917 | 1.8040 | 29 | 12.0 |
| 61 | 1.8040 | 1.8165 | 1.8291 | 1.8418 | 1.8546 | 1.8676 | 1.8807 | 28 | 12.8 |
| 62 | 1.8807 | 1.8940 | 1.9074 | 1.9210 | 1.9347 | 1.9486 | 1.9626 | 27 | 13.6 |
| 63 | 1.9626 | 1.9768 | 1.9912 | 2.0057 | 2.0204 | 2.0353 | 2.0503 | 26 | 14.6 |
| 64 | 2.0503 | 2.0655 | 2.0809 | 2.0965 | 2.1123 | 2.1283 | 2.1445 | 25 | 15.7 |
| 65 | 2.1445 | 2.1609 | 2.1775 | 2.1943 | 2.2113 | 2.2286 | 2.2460 | 24 | 16.9 |
| 66 | 2.2460 | 2.2637 | 2.2817 | 2.2998 | 2.3183 | 2.3369 | 2.3559 | 23 | 18.3 |
| 67 | 2.3559 | 2.3750 | 2.3945 | 2.4142 | 2.4342 | 2.4545 | 2.4751 | 22 | 19.9 |
| 68 | 2.4751 | 2.4960 | 2.5172 | 2.5386 | 2.5605 | 2.5826 | 2.6051 | 21 | 21.7 |
| 69 | 2.6051 | 2.6279 | 2.6511 | 2.6746 | 2.6985 | 2.7228 | 2.7475 | 20 | 23.7 |
| 70 | 2.7475 | 2.7725 | 2.7980 | 2.8239 | 2.8502 | 2.8770 | 2.9042 | 19 |  |
| 71 | 2.9042 | 2.9319 | 2.9600 | 2.9887 | 3.0178 | 3.0475 | 3.0777 | 18 |  |
| 72 | 3.0777 | 3.1084 | 3.1397 | 3.1716 | 3.2041 | 3.237 I | 3.2709 | 17 |  |
| 73 | 3.2709 | $3.305^{2}$ | 3.3402 | 3.3759 | 3.4124 | 3.4495 | 3.4874 | 16 |  |
| 74 | $3 \cdot 4874$ | $3 \cdot 526 \mathrm{t}$ | $3 \cdot 5656$ | 3.6059 | 3.6470 | 3.689 I | 3.7321 | 15 |  |
| 75 | 3.7321 | 3.7760 | 3.8208 | 3.8667 | 3.9136 | 3.9617 | 4.0108 | 14 |  |
| 76 | 4.0108 | 4.0611 | 4.1126 | 4.1653 | 4.2193 | 4.2747 | 4.3315 | 13 |  |
| 77 | 4.3315 | $4 \cdot 3897$ | 4.4494 | 4.5107 | $4.573^{6}$ | 4.6382 | 4.7046 | 12 |  |
| 78 | 4.7046 | 4.7729 | 4.8430 | 4.9152 | 4.9894 | 5.0658 | 5.1446 | 11 |  |
| 79 | 5.1446 | 5.2257 | $5 \cdot 3093$ | $5 \cdot 3955$ | 5.4845 | $5 \cdot 5764$ | 5.6713 | Io |  |
| 130 | 5.6713 | 5.7694 | 5.8708 | 5.9758 | 6.0844 | 6.1970 | $6.313^{8}$ | 9 |  |
| 81 | 6.3138 | 6.4348 | 6.5606 | 6.6912 | 6.8269 | 6.9682 | 7.1154 | 8 |  |
| 82 | 7.1154 | 7.2687 | 7.4287 | 7.5958 | 7.7704 | 7.9530 | 8.1443 | 7 |  |
| 83 | 8.1443 | 8.3450 | 8.5555 | 8.7769 | 9.0098 | 9.2553 | 9.5144 | 6 |  |
| 84 | 9.5144 | 9.7882 | 10.0780 | 10.3854 | 10.7119 | 11.0594 | II 1.4301 | 5 |  |
| 85 | I 1.4301 | 11.8262 | 12.2505 | 12.7062 | 13.1969 | 13.7267 | 14.3007 | 4 |  |
| 86 | 14.3007 | 14.9244 | 15.6048 | 16.3499 | 17.1693 | 18.0750 | 19.0811 |  |  |
| 87 | 19.08II | 20.2056 | 21.4704 | 22.9038 | 24.5418 | 26.4316 | 28.6363 | 2 |  |
| 88 | 28.6363 | 31.2416 | 34.3678 | 38.1885 | 42.964 I | 49.1039 | 57.2900 | 1 |  |
| 89 | 57.2900 | 69.7501 | 85.9398 | I 14.5887 | 171.8854 | 343.7737 | $\infty$ | 0 |  |
|  | 60' | 50' | 40' | $30^{\prime}$ | 20' | $10^{\prime}$ | $0^{\prime}$ | Angle. |  |

Smithsonian Tables.
Natural Cotangents.

DIFFERENCES OF LATITUDE AND DEPARTURE．

|  |  | $0^{\circ}$ |  | $1{ }^{\circ}$ |  | $2^{\circ}$ |  |  | $\begin{aligned} & \dot{0} \\ & \stackrel{\rightharpoonup}{y} \\ & \underset{y y y}{\mid c} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 1.00000 | 0.00000 | 0.99984 | 0.01745 | 0.99939 | 0.03490 | 1 |  |
|  | 2 | 2.00000 | 0.00000 | 1.99969 | 0.03490 | 1.99878 | 0.06980 | 2 |  |
|  | 3 | 3.00000 | 0.00000 | 2.99954 | 0.05235 | 2.99817 | 0.10470 | 3 |  |
|  | 4 | 4.00000 | 0.00000 | 3.99939 | 0.06980 | 3.99756 | 0.13960 | 4 |  |
|  | 5 | 5.00000 | 0.00000 | 4.99923 | 0.08726 | 4.99695 | 0.17450 | 5 | 60 |
|  | 6 | 6.00000 | 0.00000 | 5.99908 | 0.10471 | 5.99634 | 0.20940 | 6 |  |
|  | 7 | 7.00000 | 0.00000 | 6.99893 | 0.12216 | 6.99573 | 0.24430 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 8.00000 | 0.00000 | 7.99875 | 0.13961 | 7.99512 | 0.27920 | $8$ |  |
|  | 9 | 9.00000 | 0.00000 | 8.99862 | 0.15707 | 8.99451 | 0.31410 | 9 |  |
| 15 | 1 | 0.99999 | 0.00436 | 0.99976 | 0.0218 I | 0.99922 | 0.03925 | 1 |  |
|  | 2 | 1.99998 | 0.00872 | 1.99952 | 0.04363 | 1.99845 | 0.0785 I | 2 |  |
|  | 3 | 2.99997 | 0.01308 | 2.99923 | 0.06544 | 2.99768 | 0.11777 | 3 |  |
|  | 4 | 3.99995 | 0.01745 | 3.99904 | 0.08725 | 3.99591 | 0.15703 | 4 |  |
|  | 5 | 4.99995 | 0.02181 | 4.9988 I | 0.10907 | 4.99614 | 0． 19629 | 5 | 45 |
|  | 6 | 5.99994 | 0.02617 | 5.99857 | 0.13089 | 5.99537 | 0.23555 | 6 |  |
|  | 7 | 6.99993 | 0.03054 | 6.99833 | 0.15270 | 6.99460 | 0.27481 | $7$ |  |
|  | 8 | 7.99992 | 0.03490 | 7.99809 | 0.17452 | 7.99383 | 0.31407 | $8$ |  |
|  | 9 | 8.99991 | 0.03926 | 8.99785 | 0.19633 | 8.99306 | 0.35333 | 9 |  |
| 30 | 1 | 0.99996 | 0.00872 | 0.99965 | 0.02617 | 0.99904 | 0.04361 | 1 |  |
|  | 2 | 1.99992 | 0.01745 | 1.99931 | 0.05235 | 1.99809 | $0.08723^{-}$ | 2 |  |
|  | 3 | 2.99988 | 0.02617 | 2.99897 | 0.07853 | 2.99714 | －． 13085 | 3 |  |
|  | 4 | 3.99984 | 0.03490 | 3.99862 | 0.10470 | 3.99619 | 0.17447 | 4 |  |
|  |  | 4.99981 | 0.04363 | 4.99828 | 0.13088 | 4.99524 | 0.21809 | 5 | 30 |
|  | 6 | 5.99977 | 0.05235 | 5.99794 | 0.15706 | 5．99428 | 0.26171 | 6 |  |
|  | 7 | 6.99973 | 0.06108 | 6.99760 | 0.18323 | 6.99333 |  | 7 |  |
|  | 8 | 7.99969 | 0.06981 | 7.99725 | 0.2094 I | 7.99238 | $0.34895$ | 8 |  |
|  | 9 | 8.99965 | 0.07853 | 8.99691 | 0.23559 | 8.99143 | $0.39257$ | 9 |  |
| 45 | 1 | 0.99991 | 0.01308 |  | 0.03053 | 0.99884 |  | 1 |  |
|  | 2 | I． 99982 | 0.02617 | 1.99906 | 0.06107 | 1.99769 | 0.09595 | 2 |  |
|  | 3 | 2.99974 | 003926 | 2.99860 | 0.09161 | 2.99654 | 0.14393 | 3 |  |
|  | 4 | 3.99955 | 0.05235 | 3.99813 | 0．12215 | $3.99539$ | －．19191 | 4 |  |
|  | 5 | 4.99957 | 0.06544 | 4.99766 | 0.15269 | 4.99424 | $0.239^{\circ} 9^{\prime}$ | 5 | 15 |
|  | 6 | 5.99948 | 0.07853 | 5.99720 | 0.18323 | 5.99309 | 0.28786 | 6 |  |
|  | 7 | 6.99940 | 0.09162 | 6.99673 | 0.21376 | 6.99193 | 0.33584 | 7 |  |
|  | 8 | 7.9993 ［ | 0.10471 | 7.99626 | 0.24430 | 7.99078 | 0.38382 | 8 |  |
|  | 9 | 8.99922 | 0.11780 | 8.99580 | 0.27484 | 8.98963 | 0.43180 | 9 |  |
| 条 <br> 景 |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． |  |  |
|  | ¢ | $89^{\circ}$ |  | $88^{\circ}$ |  | $87^{\circ}$ |  | $\stackrel{8}{8}$ | 雚 |

Smithsonian Tables．

DIFFERENCES OF LATITUDE AND DEPARTURE．－CONTINUED．

|  | $$ | $3^{\circ}$ |  | $4^{\circ}$ |  | $5^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.99863 | 0.05233 | 0.99756 | 0.06975 | 0．996i9 | 0.08715 | 1 |  |
|  | 2 | 1.99726 | 0．10467 | 1.99512 | 0.13951 | 1.99238 | 0．17431 | 2 |  |
|  | 3 | 2.99589 | 0.15700 | 2.99269 | 0.20926 | $2.9885^{8}$ | 0.26146 | 3 |  |
|  | 4 | 3.99452 | 0.20934 | 3.99025 | 0.27902 | 3.98477 | 0.34862 | 4 |  |
|  | 5 | 4.99315 | 0.26168 | 4.98782 | 0.34878 | 4.98097 | 0.43577 | 5 | 60 |
|  | 6 | 5.99178 | 0.31401 | 5.98538 | 0.41853 | 5.97716 | 0.52293 | 6 |  |
|  | 8 | 6.99041 | 0.36635 | 6.98294 | 0.48829 | 6.97336 | 0.61008 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.98904 | 0.41868 | 7.98051 | 0.55805 | 7.96955 | 0.69724 | $8$ |  |
|  | 9 | 8.98767 | 0.47102 | 8.97807 | 0.62780 | 8.96575 | 0.78440 | 9 |  |
| 15 | 1 | 0.99839 | 0.05669 | 0.99725 | 0.07410 | 0.99580 | 0.09150 | 1 |  |
|  | 2 | 1．99678 | 0.11338 | 1.99450 | 0.14821 | 1.99160 | 0.18300 | 2 |  |
|  | 3 | 2.99517 | 0.17007 | 2.99175 | 0.22232 | 2.98741 | 0.27450 | 3 |  |
|  | 4 | 3.99356 | 0.22677 | 3.98900 | 0.29643 | 3.98321 | 0.36600 | 4 |  |
|  | 5 | 4.99195 | 0.28346 | 4.98625 | 0.37054 | 4.97902 | 0.45750 | 5 | 45 |
|  | 6 | 5.99035 | 0.34015 | 5.98350 | 0.44465 | 5.97482 | 0.54900 | 6 |  |
|  | 7 | 6.98874 | 0.39684 | 6.98075 | 0.51875 | 6.97063 | 0.64051 | $7$ |  |
|  | 8 | 7.98713 | 0.45354 | 7.97800 | 0.59286 | 7.96643 | 0.73201 | 8 |  |
|  | 9 | 8.98552 | 0.51023 | 8.97525 | 0.66697 | 8.96224 | 0.82351 | 9 |  |
| 30 | 1 | 0.99813 | 0.06104 | 0.99691 | 0.07845 | 0.99539 |  | 1 |  |
|  | 2 | 1.99626 | 0.12209 | 1.99383 | 0.15691 | 1.99079 | 0.19169 | 2 |  |
|  | 3 | 2.99440 | 0.18314 | 2.99075 | 0.23537 | 2.98618 | 0.28753 | 3 |  |
|  | 4 | 3.99253 | 0.24419 | 3.98766 | 0.31383 | 3.98158 | 0.38338 | 4 |  |
|  | 5 | 4.99067 | 0.30524 | 4.98458 | 0.39229 | 4.97698 | 0.47922 | ． 5 | 30 |
|  | 6 | 5.98880 | 0.36629 | 5.98150 | 0.47075 | 5.97237 | 0.57507 | 6 |  |
|  | 8 | 6.98694 | 0.42733 | 6.97842 | 0.54921 | 6.96777 | 0.67092 | $7$ |  |
|  | 8 | 7.98507 | 0.48838 | 7.97533 | 0.62767 |  | $0.76676$ | 8 |  |
|  | 9 | 8.9832 I | 0.54943 | 8.97225 | 0.70613 | 8.95856 | 0.8626 I | 9 |  |
| 45 | 1 | 0.99785 | 0.06540 | 0.99656 | 0.08280 | 0.99496 | 0.10018 | 1 |  |
|  | 2 | 1.9957 I | 0.13080 | 1.99313 | 0.16561 | 1.98993 | 0.20037 | 2 |  |
|  | 3 | 2.99357 | 0.19620 | 2.98969 | 0.24842 | 2.98490 | 0.30056 | 3 |  |
|  | 4 | 3.99143 | 0.26161 | 3.98626 | 0.33123 | 3.97987 | 0.40075 | $4$ |  |
|  | 5 | 4.98929 | 0.32701 | 4.98282 | 0.41404 | 4.97484 | 0.50094 | $\begin{aligned} & 5 \\ & 5 \end{aligned}$ | 15 |
|  | 6 | 5.98715 | 0.39241 | 5.97939 | 0.49684 | 5.96981 | 0.60112 | 6 |  |
|  | 7 | 6.98501 | 0.45782 | 6.97595 | 0． 57965 | 6.96477 | 0.70131 | 7 |  |
|  | 8 | 7.98287 | 0.52322 | 7.97252 | 0.66246 | 7.95974 | 0.80150 | 8 |  |
|  | 9 | 8.98073 | 0． 58862 | 8.96908 | 0.74527 | 8.95471 | 0.90169 | 9 |  |
| $\begin{aligned} & \text { 总 } \\ & \text { 号 } \\ & \text { ? } \end{aligned}$ |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． |  | 各 |
|  |  | $86^{\circ}$ |  | $85^{\circ}$ |  | $84^{\circ}$ |  |  |  |

Smithsonian Tables．

TRAVERSE TABLE，
DIFFERENCES OF LATITUDE AND DEPARTURE．－CONTINUED．

| $\begin{aligned} & \dot{\mathbf{y}} \\ & \text { 岂 } \\ & \text { 穻 } \end{aligned}$ | $\begin{aligned} & \text { 若 } \\ & \text { H } \\ & \text { 忽 } \end{aligned}$ | $6^{\circ}$ |  | $7{ }^{\circ}$ |  | $8^{\circ}$ |  |  | $\begin{aligned} & \text { 发 } \\ & \underbrace{E}_{B} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| $\bigcirc$ | 1 | 0.99452 | 0.10452 | 0.99254 | 0.12186 | 0.99026 | 0.13917 | 1 |  |
|  | 2 | 1.98904 | 0.20905 | 1.98509 | 0.24373 | 1．98053 | 0.27834 | 2 |  |
|  | 3 | 2.98356 | $0.3135^{8}$ | 2.97763 | 0.36560 | 2.97080 | 0.41751 | 3 |  |
|  | 4 | 3.97808 | 0.41811 | 3.97018 | 0.48747 | 3.96107 | 0.55669 | 4 |  |
|  | 5 | 4.97261 | 0.52264 | 4.96273 | 0.60934 | 4.95134 | 0.69586 | $5$ | 60 |
|  | 6 | 5.96713 | 0.62717 | 5.95519 | 0.73121 | 5.94160 | 0.83503 | $6$ |  |
|  | 7 | 6.96165 | 0.73169 | 6.94782 | 0.85308 | 6.93187 | 0.97421 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.95617 | 0.83622 | 7.94038 | 0.97495 | 7.92214 | 1.11338 | $8$ |  |
|  | 9 | 8.95069 | 0.94075 | 8.93291 | 1.09682 | 8.91241 | I． 25255 | 9 |  |
| 15 | 1 | 0.99405 | 0.10886 | 0.99200 | 0.12619 | 0.98965 | 0.14349 | 1 |  |
|  | 2 | 1．98811 | 0.21773 | 1.98400 | 0.25239 | 1.97930 | 0.28698 | 2 |  |
|  | 3 | 2.98216 | 0.32660 | 2.97601 | 0.37859 | 2.96895 | 0.43047 | 3 |  |
|  | 4 | 3.97622 | 0.43546 | 3.96801 | 0.50479 | 3.95860 | 0.57397 | 4 |  |
|  | 5 | 4.97028 | 0.54433 | 4.96002 | 0.63099 | 4.94825 | 0.71746 | $5$ | 45 |
|  | 6 | 5.96433 | 0.65320 | 5.95202 | 0.75719 | 5.93790 | 0.86095 | $\stackrel{\breve{6}}{ }$ |  |
|  | 7 | 6.95839 | 0.76206 | 6.94403 | 0.88339 | 6.92755 | 1.00444 | $7$ |  |
|  | 8 | 7.95245 | 0.87093 | 7.93603 | 1.00959 | 7.91721 | 1．14794 | 8 |  |
|  | 9 | 8.94650 | 0.97980 | 8.92804 | 1．1 3579 | 8.90686 | 1.29143 | 9 |  |
| 30 | 1 | 0.99357 | 0.11320 | 0.99144 | 0.13052 | 0.98901 | 0.14780 | 1 |  |
|  | 2 | 1.98714 | 0.22640 | 1.98288 | 0.26105 | 1.97803 | 0.29561 | 2 |  |
|  | 3 | 2.9807 I | 0.33960 | 2.97433 | 0.39157 | 2.96704 | 0.44342 | 3 |  |
|  | 4 | 3.97428 | 0.45281 | 3.96577 | 0.52210 | 3.95606 | 0.59123 | 4 |  |
|  | 5 | 4.96786 | 0.56601 | 4.95722 | 0.65263 | 4.94508 |  | $5$ | 30 |
|  | 6 | 5.96143 | 0.67921 | 5.94866 | 0.78315 | 5．93409 | $0.88685$ | $6$ |  |
|  | 7 | 6.95500 | 0.79242 | 6.94011 | 0.91368 | 6.92311 | $1.03466$ | $7$ |  |
|  | 8 | 7.94857 | 0.90562 | 7.93155 | 1.04420 | 7.91212 | $\text { I. } 18247$ | $8$ |  |
|  | 9 | 8.94214 | 1.01882 | 8.92300 | 1.17473 | 8.90114 | $1.33028$ | 9 |  |
| 45 | 1 | 0.99306 | 0.11753 | 0.99086 | 0.13485 | 0.98836 |  | 1 |  |
|  | 2 | 1.98613 | 0.23507 | 1.98173 | 0.26970 | 1．97672 | 0.30424 | 2 |  |
|  | 3 | 2.97920 | 0.35261 | 2.97259 | 0.40455 | 2.96508 | $0.45637$ | 3 |  |
|  | 4 | 3.97227 | 0.47014 | 3.96346 | 0.53940 | 3.95344 | 0.60849 | 4 |  |
|  | 5 | 4.96534 | 0.58768 | 4.95432 | 0.67425 | 4.94180 | 0.76061 | 5 | 15 |
|  | 6 | 5.95841 | 0.70522 | 5.94519 | 0.80910 | 5.93016 | 0.91274 | 6 |  |
|  | 7 | 6.95147 | 0.82276 | 6.93606 | 0.94395 | 6.91853 | 1． 06486 | $7$ |  |
|  | 8 | 7.94454 | 0.94029 | 7.92692 | 1.07880 | 7.90689 | 1． 21698 | 8 |  |
|  | 9 | 8.93761 | 1.05783 | 8.91779 | I． 21365 | 8.89525 | 1．36911 | 9 |  |
| 条落菏 |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． |  | 录 |
|  |  | $83^{\circ}$ |  | $82^{\circ}$ |  | $81^{\circ}$ |  |  |  |

Smithsonian Tables．

TRAVERSE TABLE．
DIFFERENGES OF LATITUDE AND DEPARTURE．－Continued．

| $\begin{aligned} & \text { 駕 } \\ & \text { 号 } \end{aligned}$ |  | $9^{\circ}$ |  | $10^{\circ}$ |  | $11^{\circ}$ |  |  | $\begin{aligned} & \text { 霛 } \\ & \stackrel{y y y y}{\mid c} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.98768 | 0.15643 | 0.98480 | 0.17364 | 0.98162 | 0.19081 | 1 |  |
|  | 2 | 1.97537 | 0.31286 | 1.96961 | 0.34729 | 1.96325 | 0.38162 | 2 |  |
|  | 3 | 2.96306 | 0.46930 | 2.95442 | 0.52094 | 2.94488 | 0． 57243 | 3 |  |
|  | 4 | 3.95075 | 0.62573 | 3.93923 | 0.69459 | 3.92650 | 0.76324 | 4 |  |
|  | 5 | 4.93844 | 0.78217 | 4.92403 | 0.86824 | 4.90813 | 0.95405 | 5 | 60 |
|  | 6 | 5.92612 | 0.93860 | 5.90884 | 1.04188 | 5.88976 | I． 14486 | 6 |  |
|  | 7 | 6.9138 I | 1.09504 | 6.89365 | 1.21553 | 6.87 I 39 | 1.33566 | 7 |  |
|  | 8 | 7.90150 | 1.25147 | 7.87846 | 1.38918 | 7.85301 | 1.52648 | 8 |  |
|  | 9 | 8.88919 | 1．40791 | ．8．86327 | 1.56283 | 8.83464 |  | 9 |  |
| 15 | 1 | 0.98699 | 0.16074 | 0.98404 | 0.17794 | 0.98078 | 0.19509 | 1 |  |
|  | 2 | I． 97399 | 0.32148 | r． 96808 | 0.35588 | 1．96157 | 0.39018 | 2 |  |
|  | 3 | 2.96098 | 0.48222 | 2.95212 | 0.53383 | 2.94235 | 0.58527 | 3 |  |
|  | 4 | 3.94798 | 0.64297 | 3.93616 | 0.71177 | 3.92314 | 0.78036 | 4 |  |
|  | 5 | 4.93498 | 0.8037 I | 4.92020 | 0.88971 | 4.90392 | 0.97545 | 5 | 45 |
|  | 6 | 5.92197 | 0.96445 | 5.90424 | 1.06766 | 5.8847 I | I． 17054 | 6 |  |
|  | 7 | 6.90897 | 1.12519 | 6.88828 | 1.24560 | 6.86549 | I． 36563 | 7 |  |
|  | 8 | 7.89597 | 1.28594 | 7.87232 | I． 42354 | 7.84628 | 1．56072 | 8 |  |
|  | 9 | 8.88296 | I． 44668 | 8.85636 | I．60149 | 8.82706 | 1.75581 | 9 |  |
| 30 | 1 | 0.98628 | 0.16504 | 0.98325 | 0.18223 | 0.97992 | 0.19936 | 1 |  |
|  | 2 | 1． 97257 | 0.33009 | 1.96650 | 0.36447 | r． 95984 | 0.39873 | 2 |  |
|  | 3 | 2.95885 | 0.49514 | 2.94976 | 0.54670 | 2.93977 | 0.59810 | 3 |  |
|  | 4 | 3.94514 | 0.66019 | 3.93301 | 0.72894 | 3.91969 | 0.79747 | 4 |  |
|  | 5 | 4.93142 | 0.82523 | 4.91627 | 0.91117 | 4.89962 | 0.99683 | 5 | 30 |
|  | 6 | 5.91771 | 0.99028 | 5.89952 | 1.09341 | 5.87954 | 1.19620 | 6 |  |
|  | 7 | 6.90399 | I． 55533 | 6.88278 | 1.27564 | 6.85947 | 1． 39557 | 7 |  |
|  | 8 | 7.89028 | 1． 32038 | 7.86603 | 1．45788 | 7.83939 | 1.59494 | 8 |  |
|  | 9 | 8.87657 | 1.48542 | 8.84929 | I． 6401 I | 8．81932 | 1.79431 | 9 |  |
| 45 | 1 |  |  | 0.98245 | 0.18652 | 0.97904 | 0.20364 | 1 |  |
|  | 1 | 1．97 III | 0．33870 | 1.96490 | 0.37304 | 1.95809 | 0.40728 | 2 |  |
|  | 3 | 2.95666 | 0.50805 | 2.94735 | 0.55957 | 2.93713 | 0.61092 | 3 |  |
|  | 4 | 3.94222 | 0.67740 | 3.92980 | 0.74609 | 3.91618 | 0.81456 | 4 |  |
|  | 5 | 4.92778 | 0.84675 | 4.91225 | 0.93262 | 4.89522 | 1.01820 | 5 | 15 |
|  | 6 | 5.91333 | 1.01610 | 5.89470 | I．II914 | 5.87427 | I． 22185 | 6 |  |
|  | 7 | 6.89889 | 1.18545 | 6.87715 | 1． 30566 | 6.85331 | 1.42549 | 7 |  |
|  | 8 | 7.88444 | 1.35480 | 7.85960 | 1． 49219 | 7.83236 | 1．62913 | 8 |  |
|  | 9 | 8.87000 | 1．52415 | 8.84205 | 1.67871 | 8.81140 | 1．83277 | 9 |  |
| 条 | $\begin{aligned} & \underset{0}{6} \\ & \text { W } \\ & \text { H } \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． |  | 鸪 |
|  |  | $80^{\circ}$ |  | $79^{\circ}$ |  | $78^{\circ}$ |  |  |  |

Smithsonian Tables．

Table 9.
TRAVERSE TABLE.
DIFFERENCES OF LATITUDE AND DEPARTURE.-CONTINUED.

|  |  | $12^{\circ}$ |  | $13^{\circ}$ |  | $14^{\circ}$ |  |  | $\begin{aligned} & \dot{0} \\ & \stackrel{y}{U} \\ & \text { B } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. |  |  |
| $\bigcirc$ | 1 | 0.97814 | 0.20791 | 0.97437 | 0.22495 | 0.97029 | 0.24192 | 1 |  |
|  | 2 | 1.95629 | 0.41582 | I. 94874 | 0.44990 | 1.94059 | 0.48384 | 2 |  |
|  | 3 | 2.93444 | 0.62373 | 2.92311 | 0.67485 | 2.91088 | 0.72576 | 3 |  |
|  | 4 | 3.91259 | 0.83164 | 3.89748 | 0.89980 | 3.88118 | 0.96768 | 4 |  |
|  | 5 | 4.89073 | I. 03955 | 4.87185 | 1.12475 | 4.85147 | 1.20961 | 5 | 60 |
|  | 6 | 5.86888 | I. 24747 | 5.84622 | 1.34970 | 5.82177 | 1.45153 | 6 |  |
|  | 7 | 6.84703 | 1. 45538 | 6.82059 | 1. 57465 | 6.79206 | 1.69345 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.82518 | 1.66329 | 7.79496 | 1.79960 | 7.76236 | 1.93537 | $8$ |  |
|  | 9 | $8.8033^{2}$ | 1.87120 | 8.76933 | 2.02455 | 8.73266 | 2.17729 | 9 |  |
| 15 | 1 | 0.97723 | 0.21217 | 0.97337 | 0.22920 | 0.96923 | 0.24615 | 1 |  |
|  | 2 | I.95446 | 0.42435 | I. 94675 | 0.45840 | 1.93846 | 0.49230 | 2 |  |
|  | 3 | 2.93169 | 0.63653 | 2.92013 | 0.68760 | 2.90769 | 0.73845 | 3 |  |
|  | 4 | 3.90892 | 0.84871 | 3.89351 | 0.91680 | 3.87692 | 0.98461 | 4 |  |
|  | 5 | 4.88615 | 1.06088 | 4.86689 | 1.14600 | 4.84615 | $1.23076$ | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 45 |
|  | 6 | $5.8633^{8}$ | 1.27306 | 5.84027 | 1.37520 | 5.81538 | $1.47691$ | 6 |  |
|  | 7 | 6.84061 | 1.48524 | 6.81365 | 1.60440 | 6.7846 I | 1.72307 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.81784 | 1. 69742 | 7.78703 | 1. 83360 | 7.75384 | I. 96922 | $8$ |  |
|  | 9 | 8.79507 | 1.90959 | 8.7604 I | 2.06280 | 8.72307 | 2.21537 | 9 |  |
| 30 | 1 | 0.97629 | 0.21644 | 0.97237 | 0.23344 | 0.96814 | 0.25038 | 1 |  |
|  | 2 | 1.95259 | 0.43288 | 1.94474 | 0.46689 | 1.93629 | 0.50076 | 2 |  |
|  | 3 | 2.92888 | 0.64932 | 2.917 II | 0.70033 | 2.90444 | 0.75114 | $3$ |  |
|  | 4 | 3.90518 | 0.86576 | 3.88948 | 0.93378 | 3.87259 | I. 00152 | $4$ |  |
|  |  | 4.88148 | 1.08220 | 4.86185 | I.16722 | 4.84073 | 1.25190 | $5$ | 30 |
|  | 6 | 5.85777 | 1.29864 | 5.83422 | 1. 40067 | 5.80888 | 1.50228 | 6 |  |
|  | 7 | 6.83407 | 1.51508 | 6.80659 | I. 6341 I | 6.77703 | 1.75266 | $7$ |  |
|  | 8 | 7.81036 | 1.73152 | 7.77896 | 1. 86756 | 7.74518 | 2.00304 | $8$ |  |
|  | 9 | 8.78666 | 1. 94796 | 8.75133 | 2.10100 | 8.71332 | 2.25342 | 9 |  |
| 45 | 1 | 0.97534 | 0.22069 | 0.97134 | 0.23768 | 0.96704 | 0.25460 | 1 |  |
|  | 2 | 1.95068 | 0.44139 | 1.94268 | 0.47537 | 1.93409 | 0.50920 | 2 |  |
|  | 3 | 2.92602 | 0.66209 | 2.91402 | 0.71305 | 2.90113 | 0.76380 | $3$ |  |
|  | 4 | 3.90136 | 0.88278 | 3.88536 | 0.95074 | 3.86818 | 1.01840 | $4$ |  |
|  | 5 | 4.87671 | I. 10348 | 4.85671 | I. 18843 | 4.83523 | 1.27301 | 5 | 15 |
|  | 6 | 5.85205 | 1.32418 | 5.82805 | I. 42611 | 5.80227 | 1.52761 | 6 |  |
|  | 7 | 6.82739 | 1. 54488 | 6.79939 | 1.66380 | 6.76932 | 1.78221 | 7 |  |
|  | 8 | 7.80273 | 1. 76557 | 7.77073 | I.90148 | 7.73636 | 2.03681 | 8 |  |
|  | 9 | 8.77808 | I. 98627 | 8.74207 | 2.13917 | 8.70341 | 2.29141 | 9 |  |
|  | $\begin{aligned} & \underset{W}{\theta} \\ & \text { 品 } \\ & 0 \end{aligned}$ | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. |  | 客 |
|  |  | $77^{\circ}$ |  | $76^{\circ}$ |  | $75^{\circ}$ |  |  |  |

TRAVERSE TABLE，
Table 9.
DIFFERENCES OF LATITUDE AND DEPARTURE．－CONTINUED．

| $\begin{aligned} & \text { 要 } \\ & \text { H. } \\ & \text { Hix } \end{aligned}$ |  | $15^{\circ}$ |  | $16^{\circ}$ |  | $17^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| $\bigcirc$ | 1 | 0.96592 | 0.25881 | 0.96126 | 0.27563 | 0.95630 | 0.29237 | 1 |  |
|  | 2 | 1.93185 | 0.51763 | 1．92252 | 0.55127 | 1.91260 | 0.58474 | 2 |  |
|  | 3 | 2.89777 | 0.77645 | 2.88378 | 0.82691 | 2.86891 | 0.877 II | 3 |  |
|  | 4 | 3.86370 | 1.03527 | 3.84504 | 1．10254 | 3.8252 I | I．16948 | 4 |  |
|  | 5 | 4.82962 | 1.29409 | 4.80630 | I． 37818 | 4.78 I 52 | I． 46185 | $5$ | 60 |
|  | 6 | 5.79555 | 1．5529I | 5.76757 | 1.65382 | 5.73782 | 1.75423 | $6$ |  |
|  | 7 | 6.76148 | 1.81173 | 6.72883 | 1.92946 | 6.69413 | 2.04660 | 7 |  |
|  | 8 | 7.72740 | 2.07055 | 7.69009 | 2.20509 | 7.65043 | 2.33897 | $8$ |  |
|  | 9 | 8.69333 | 2.32937 | 8.65135 | 2.48073 | 8.60674 | 2.63 I 34 | 9 |  |
| 15 | 1 | 0.96478 | 0.26303 | 0.96005 | 0.27982 | 0.95502 | 0.29654 | 1 |  |
|  | 2 | 1.92957 | 0.52606 | 1.92010 | 0.55965 | 1.91004 | 0.59308 | 2 |  |
|  | 3 | 2.89436 | 0.78909 | 2.88015 | 0.83948 | 2.86506 | $0.88 \mathrm{g62}$ | 3 |  |
|  | 4 | 3.85914 | 1.05212 | 3.84020 | 1.11931 | 3.82008 | 1.18616 | 4 |  |
|  | 5 | 4.82393 | I．3I5I5 | 4.80025 | 1.39914 | 4.77510 | 1． 48270 | $5$ | 45 |
|  | 6 | 5.78872 | 1．57818 | 5.76030 | 1.67897 | 5.73012 | 1.77924 | 6 |  |
|  | 7 | 6.75351 | 1.84121 | 6.72035 | 1.95880 | 6.68514 | 2.07579 | 7 |  |
|  | 8 | 7.71829 | 2.10424 | 7.68040 | 2.23863 | 7.64016 | 2.37233 | 8 |  |
|  | 9 | 8.68308 | 2.36728 | 8.64045 | 2.51846 | 8.59518 | 2.66887 | 9 |  |
| 30 | 1 | 0.96363 | 0.26723 | 0.95882 | 0.28401 | 0.95371 |  | 1 |  |
|  | 2 | 1.92726 | 0.53447 | 1.91764 | 0.56803 | 1.90743 | 0.60141 | 2 |  |
|  | 3 | 2.89089 | 0.80171 | 2.87646 | 0.85204 | 2.86115 | 0.90211 | 3 |  |
|  | 4 | $3.8545^{2}$ | 1.06895 | 3.83528 | I． 13606 | 3．81486 | 1.20282 | 4 |  |
|  |  | 4.81815 | 1． 33619 | 4.79410 | I． 42007 | 4.76858 |  | $5$ | 30 |
|  | 6 | 5.78178 | 1． 60343 | 5.75292 | 1.70409 | 5.72230 | 1.80423 | 6 |  |
|  | 7 | 6.7454 I | 1． 87066 | 6.71174 | 1．98810 | 6.67601 | 2.10494 | 7 |  |
|  | 8 | 7.70904 | 2．13790 | 7.67056 | 2.27212 | 7.62973 |  | 8 |  |
|  | 9 | 8.67267 | 2.40514 | 8.62938 | 2.55613 | 8.58345 | $2.70635$ | 9 |  |
| 45 | 1 | 0.96245 | 0.27144 | 0.95757 | 0.28819 | 0.95239 | 0.30486 | 1 |  |
|  | 2 | 1.9249 I | 0.54288 | 1.91514 | 0.57639 | 1.90479 | 0.60972 | 2 |  |
|  | 3 | 2.88736 | 0.81432 | 2.87271 | 0.86458 | 2.85718 | 0.91459 | 3 |  |
|  | 4 | 3.84982 | 1.08576 | 3.83028 | 1．15278 | 3.80958 | 1．21945 | 4 |  |
|  |  | 4.81227 | 1.35720 | 4.78785 | 1． 44098 | 4.76197 | 1.52432 | 5 | 15 |
|  | 6 | 5.77473 | 1.62864 | 5.74542 | 1． 72917 | 5.71437 | 1.82918 | 6 |  |
|  | 7 | 6.73718 | 1.90008 | 6.70299 | 2.01737 | 6.66677 | 2.13405 | 7 |  |
|  | 8 | 7.69964 | 2.17152 | 7.66057 | 2.30557 | 7.61916 | 2.43891 | 8 |  |
|  | 9 | 8.66209 | 2.44296 | 8．61814 | 2.59376 | 8.57156 | 2.74377 | 9 |  |
| 条薷 |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． | $\begin{aligned} & \text { 苞 } \\ & \text { B } \\ & \stackrel{\rightharpoonup}{0} \end{aligned}$ | 苞 |
|  |  | $74^{\circ}$ |  | $73^{\circ}$ |  | $72^{\circ}$ |  |  |  |

Smithsonian Tables．

Table 9.
TRAVERSE TABLE.
DIFFERENCES OF LATITUDE AND DEPARTURE.-CONTINUED.

|  |  | $18^{\circ}$ |  | $19^{\circ}$ |  | $20^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. |  |  |
| - | 1 | 0.95105 | 0.30901 | 0.94551 | 0.32556 | 0.93969 | 0.34202 |  |  |
|  | 2 | 1.90211 | 0.61803 | 1.89103 | 0.65113 | 1.87938 | 0.68404 | 2 |  |
|  | 3 | 2.85316 | 0.92705 | 2.83655 3.78207 | 0.97670 | 2.81907 | I. I I. 368008 | 4 |  |
|  | 4 | 3.80422 4.75528 | I. 23606 I. 54508 | 3.78207 4.72759 | 1.30227 1.62784 | 3.75877 4.69846 | 1.36808 1.71010 | $4$ | 60 |
|  | 6 | 5.70633 | I.85410 | 5.67311 | 1.95340 | 5.63815 | 2.05212 | 6 |  |
|  | 7 | 6.65739 | 2.16311 | 6.61863 | 2.27897 | 6.57784 | 2.39414 2.73616 | 7 |  |
|  | 8 | 7.60845 | 2.47213 | 7.56414 8.50966 | 2.60454 2.93011 | 7.51754 8.45723 | $\begin{aligned} & 2.73616 \\ & 3.07818 \end{aligned}$ | $\begin{aligned} & 8 \\ & 9 \end{aligned}$ |  |
|  | 9 | 8.55950 | 2.78115 | 8.50966 | 2.93011 | 8.45723 |  |  |  |
| 15 | 1 | 0.94969 | 0.31316 | 0.94408 | 0.32969 | 0.93819 | 0.34611 | 1 |  |
|  | 2 | 1. 89939 | 0.62632 | 1.88817 | 0.65938 | I. 876388 | 0.69223 | 2 |  |
|  | 3 | 2.84909 | 0.93949 | 2.83226 | 0.98907 | 2.81457 3.7576 | 1.03835 I. 38446 | 3 |  |
|  |  | 3.79879 4.74849 | 1.25265 1.56581 | 3.77635 <br> 4.72044 | 1.31876 I. 64845 | 3.75276 4.69095 | 1.38446 1.73058 | 4 5 5 | 45 |
|  | 6 | 4.69819 | 1. 8.7898 | 5.66453 | 1.97814 | 5.62914 | 2.07670 | 6 | 4 |
|  | 7 | 6.64789 | 2.19214 | 6.60862 | 2.30783 | 6.56733 | 2.44281 | 7 |  |
|  | 8 | 7.59759 8.54729 | 2.50531 2.81847 | 7.55271 8.49680 | 2.63752 2.96721 | 7.50553 8.44372 | 2.76893 3.11505 | $\begin{aligned} & 8 \\ & 9 \end{aligned}$ |  |
|  | 9 | 8.54729 | 2.81847 | 8.49680 | 2.96721 | $8.4437^{2}$ | 3.11505 |  |  |
| 30 | 1 | 0.94832 | 0.31730 | 0.94264 | 0.33380 | 0.93667 | 0.35020 | 1 |  |
|  | 2 | 1.89664 | 0.63460 | 1.88528 | 0.66761 | 1.87334 | 0.70041 | 2 |  |
|  | 3 | 2.84497 | 0.95191 | 2.82792 377056 | 1.00142 | ${ }^{2} 8.81001$ | 1.05062 T.40082 | 3 |  |
|  | 4 | 3.79329 | 1.26921 | 3.77056 | 1.33522 | 3.74668 | 1.40082 | 4 |  |
|  | 5 | ${ }^{4.74161}$ |  |  |  | 4.68336 |  | 5 | 30 |
|  | 6 |  | 1.90382 2.22113 | 5.65584 6.59849 | 2.00284 2.3364 | ${ }^{5.62003}$ | 2.10124 2.45145 | $6$ |  |
|  | 7 | 6.63826 7.58658 | 2.22113 | 6.59849 | 2.33664 | 6.55670 | 2.45145 2.80165 | 7 |  |
|  | 9 | 7.58658 8.53491 | 2.53843 2.85574 | 7.54113 8.48377 | 2.67045 3.00426 | 8.43004 | 3.15186 | 9 |  |
| 45 |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | 1 |  |
|  | 2 | 1.89386 | 0.64287 | 1.88235 | 0.37583 | 1.87027 <br> 2.8054 | 0.70858 | 3 |  |
|  | 3 4 4 | 2.84079 3.78772 | 0.96431 1.28575 | 2.82352 3.76470 | 1.01375 1. 35166 | 2.80540 3.74054 | 1.06287 1.41716 | 3 4 4 |  |
|  | 5 | 4.73465 | 1.60719 | 4.70588 | т. 68958 | 4.67567 | 1.77145 | 5 | 15 |
|  | 6. | 5.68158 | 1.92863 | 5.64705 | 2.02750 | 5.61081 | 2.12574 | 6 |  |
|  | 7 | 6.62851 | 2.25007 | 6.58823 | 2.36541 |  | 2.48003 | 7 |  |
|  | 8 |  | 2.57151 2.89295 | 7.52940 8.47058 | 2.70333 3.04125 | 7.48108 | 2.83432 3.18865 | 8 |  |
|  |  | 8.52237 | 2.89295 | 8.47058 | 3.04125 | 8.41621 | 3.18861 | 9 |  |
|  | 불:000 | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. |  | 当 |
|  |  | $71^{\circ}$ |  | $70^{\circ}$ |  | $69^{\circ}$ |  |  |  |

Smithsonian Tables.

TRAVERSE TABLE.
DIFFERENCES OF LATITUDE AND DEPARTURE.-CONTINUED.

|  |  | $21^{\circ}$ |  | $22^{\circ}$ |  | $23^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. |  |  |
| 0 | 1 | 0.93358 | 0.35836 | 0.92718 | 0.37460 | 0.92050 | 0.39073 | 1 |  |
|  | 2 | 1.86716 | 0.71673 | 1. 85436 | 0.74921 | 1.84100 | 0.78146 | 2 |  |
|  | 3 | 2.80074 | 1.07510 | 2.78155 | 1.12381 | 2.76151 | 1.17219 | 3 |  |
|  | 4 | 3.73432 | 1.43347 | 3.70873 | 1.49842 | 3.68201 | 1.56292 | 4 |  |
|  | 5 | 4.66790 | 1.79183 | 4.63591 | 1.87303 | 4.60252 | 1.95365 | $5$ | 60 |
|  | 6 | 5.60148 | 2.15020 | 5.56310 | 2.24763 | 5.52302 | 2.34438 | $6$ |  |
|  | 7 | 6.53506 | 2.50857 | 6.49028 | 2.62224 | 6.44353 | 2.73511 | $7$ |  |
|  | 8 | $7 \cdot 46864$ | 2.86694 | 7.41747 | 2.99685 | $7 \cdot 36403$ | $3.12584$ | $8$ |  |
|  | 9 | 8.40222 | 3.22531 | 8.34465 | 3.37145 | 8.28454 | $3.51657$ | 9 |  |
| 15 | 1 | 0.93200 | 0.36243 | 0.92554 | 0.37864 | 0.91879 |  | 1 |  |
|  | 2 | 1.86401 | 0.72487 | 1.85108 | 0.75729 | 1.83758 | 0.78948 | 2 |  |
|  | 3 | 2.79602 | 1.08731 | 2.77662 | 1.13594 | 2.75637 | 1.18423 | 3 |  |
|  | 4 | 3.72803 | 1.44975 | 3.70216 | I. 51459 | 3.67516 | 1. 57897 | 4 |  |
|  | 5 | 4.66004 | 1.81219 | 4.62770 | 1.89324 | 4.59395 | 1. 97372 | $5$ | 45 |
|  | 6 | 5.59204 | 2.17462 | 5.55324 | 2.27189 | 5.51274 | 2.36846 | $6$ |  |
|  | 7 | 6.52405 | 2.53706 | 6.47878 | 2.65054 | 6.43153 | 2.76320 | $7$ |  |
|  | 8 | 7.45606 | 2.89950 |  | 3.02918 | 7.35032 | $3.15795$ | $8$ |  |
|  | 9 | 8.38807 | 3.26194 | 8.32986 | 3.40783 | 8.26912 | $3 \cdot 55269$ | 9 |  |
| 30 | 1 | 0.93041 | 0.36650 | 0.92388 | 0.38268 | 0.91706 | 0.39874 | 1 |  |
|  | 2 | I. 86083 | 0.73300 | 1.84776 | 0.76536 | 1.83412 | 0.79749 | 2 |  |
|  | 3 | 2.79125 | 1.09950 | 2.77164 | 1.14805 | 2.75118 | I. 19624 | 3 |  |
|  | 4 | 3.72167 | 1.46600 | 3.69552 | 1. 53073 | 3.66824 | 1. 59499 | 4 |  |
|  | 5 | 4.65208 | 1.83250 | 4.61940 | 1.91341 | 4.58530 | 1.99374 | $\frac{5}{6}$ | 30 |
|  | 6 | 5.58250 | 2.19900 | 5.54328 | 2.29610 | $5 \cdot 502.36$ | 2.39249 | 6 |  |
|  | 7 | 6.51292 | 2.56550 | 6.46716 | 2.67878 | 6.41942 | 2.79124 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.44334 | 2.93200 | 7.39104 | 3.06146 | $7 \cdot 33648$ | 3.18999 | $8$ |  |
|  | 9 | 8.37375 | 3.29851 | 8.31492 | 3.44415 | 8.25354 | 3.58874 | 9 |  |
| 45 | 1 | 0.92881 | 0.37055 | 0.92220 | 0.38671 | 0.91531 | 0.40274 | 1 |  |
|  | 2 | 1.85762 | 0.74111 | 1. 84440 | 0.77342 | 1.83062 | 0.80549 | 2 |  |
|  | 3 | 2.78643 | 1.11167 | 2.76660 | 1.16013 | 2.74593 | 1.20824 | 3 |  |
|  | 4 | 3.71524 | I. 48222 | 3.68880 | 1. 54684 | 3.66124 | 1.61098 | $4$ |  |
|  | 5 | 4.64405 | 1. 85278 | 4.61100 | I. 93355 | 4.57655 | 2.01373 | 5 | I 5 |
|  | 6 | 5.57286 | 2.22334 | 5.53320 | 2.32026 | 5.49 I 86 | 2.41648 | 6 |  |
|  | 7 | 6.50167 | 2.59390 | 6.45540 | 2.70697 | 6.40718 | 2.81922 | 7 |  |
|  | 8 | 7.43048 | 2.96445 | 7.37760 | 3.09368 | 7.32249 | 3.22197 | 8 |  |
|  | 9 | 8.35929 | 3.33501 | 8.29980 | 3.48039 | 8.23780 | 3.62472 | 9 |  |
| 3E苞 |  | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. |  | 录 |
|  |  | $68^{\circ}$ |  | $67^{\circ}$ |  | $66^{\circ}$ |  |  |  |

8mithsonian Tables.

TABLE 9.
TRAVERSE TABLE．
DIFFERENCES OF LATITUDE AND DEPARTURE，－CONTINUED．

| $\begin{aligned} & \dot{8} \\ & \stackrel{y}{t} \\ & \text { 足 } \end{aligned}$ | $\begin{aligned} & \dot{\text { U }} \\ & \text { Hy } \\ & \text { H. } \\ & \text { H. } \end{aligned}$ | $24^{\circ}$ |  | $25^{\circ}$ |  | $26^{\circ}$ |  |  | $\frac{\dot{\mathscr{y}}}{\underset{y}{y}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.91354 | 0.40673 | 0.90630 | 0.42261 | 0.89879 | 0.43837 | 1 |  |
|  | 2 | 1.82709 | 0.81347 | 1.81261 | 0.84523 | 1.79758 | 0.87674 | 2 |  |
|  | 3 | 2.74063 | 1.22020 | 2.71892 | 1.26785 | 2.69638 | 1.31511 | 3 |  |
|  | 4 | 3.65418 | 1． 62694 | 3.62523 | 1.69047 | 3.59517 | 1.75348 | 4 |  |
|  | 5 | 4.56772 | 2.03368 | 4.53153 | 2.11309 | 4.49397 | 2.19185 | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 60 |
|  | 6 | 5.48127 | 2.44041 | 5.43784 | 2.53570 | $5 \cdot 39276$ | 2.63022 | $6$ |  |
|  | 7 | 6.39481 | 2.84715 | 6.34415 | 2.95832 | 6.29155 | 3.06859 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | $7 \cdot 30836$ | 3.25389 | 7.25046 | 3.38094 | 7.19035 | 3.50696 | $8$ |  |
|  | 9 | 8.22190 | 3.66062 | 8．15677 | 3.80356 | 8.08914 | 3.94533 | 9 |  |
| 15 | 1 | 0.91176 | 0.41071 | 0.90445 | 0.42656 | 0.89687 | 0.44228 | 1 |  |
|  | 2 | r． 82352 | 0.82143 | 1.80891 | 0.85313 | 1.79374 | 0.88457 | 3 |  |
|  | 3 | 2.73528 | 1.23215 | 2.71336 | 1.27970 | 2.69061 | 1.32686 | 3 |  |
|  | 4 | 3.64704 | 1.64287 | 3.61782 | 1.70627 | 3.58749 | 1.76915 | 4 |  |
|  | 5 | 4.5588 I | 2.05359 | 4.52227 | 2.13284 | 4.48436 | 2.21144 | $\frac{5}{6}$ | 45 |
|  | 6 | $5 \cdot 47057$ | 2.46431 | 5.42673 | 2.55941 | 5．38123 | 2.65373 | $6$ |  |
|  | 7 | 6.38233 | 2.87503 | 6.33118 | 2.98598 | 6.27810 | 3.09602 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.29409 | 3.28575 | 7.23564 | 3.41254 | 7.17498 8.07185 | $3.53830$ | $\begin{aligned} & 8 \\ & 0 \end{aligned}$ |  |
|  | 9 | 8.20585 | 3.69647 | 8.14009 | 3.83911 | 8.07185 |  | 9 |  |
| 30 | 1 | 0.90996 | 0.41469 | 0.90258 | 0.43051 | 0.89493 | 0.44619 | 1 |  |
|  | 2 | 1.81992 | 0.82938 | 1.80517 | 0.86102 | 1.78986 | 0.89239 | 2 |  |
|  | 3 | 2.72988 | 1.24407 | 2.70775 | 1.29153 | 2.68480 | 1.33859 | 3 |  |
|  | 4 | 3.63984 | 1.65877 | 3.61034 | 1.72204 | 3.57973 | 1.78479 | 4 |  |
|  |  | 4.54980 | 2.07346 | 4.51292 | 2.15255 | 4.47467 |  | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 30 |
|  | 6 | 5.45976 | 2.48815 | 5.41551 | 2.58306 | 5．36960 | $2.67718$ | $\overline{6}$ |  |
|  | 7 | 6.36972 | 2.90285 | 6.31809 | 3.01357 | 6.26454 | 3.12338 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 7.27969 | 3.31754 | 7.22068 | 3.44408 | 7.15947 | 3．56958 | $8$ |  |
|  | 9 | 8.18965 | 3.73223 | 8.12326 | 3.87459 | 8.05440 | 4.01578 | 9 |  |
| 45 | 1 | 0.90814 | 0.41866 | 0.90069 | 0.43444 | 0.89297 | 0.45009 | 1 |  |
|  | 2 | 1.81628 | 0.83732 | 1．801 39 | 0.86889 | 1.78595 | 0.90019 | 2 |  |
|  | 3 | 2.72442 | 1.25598 | 2.70209 | 1.30333 | 2.67893 | 1.35029 | $3$ |  |
|  | 4 | 3.63257 | 1． 67464 | 3.60279 | 1.73778 | 3.57191 | 1.80039 | 4 |  |
|  | 5 | 4.54071 | 2.09330 | 4.50349 | 2.17222 | 4.46489 | 2.25049 | 5 | 15 |
|  | 6 | 5.44885 | 2.51196 | 5.40418 | 2.60667 | 5.35787 | 2.70059 | 6 |  |
|  | 7 | 6.35700 | 2.93062 | 6.30488 | 3.04111 | 6.25085 | 3.15068 | 7 |  |
|  | 8 | 7.26514 | 3.34928 | 7.20558 | 3.47556 | 7.14383 | 3.60078 | 8 |  |
|  | 9 | 8.17328 | 3.76794 | 8.10628 | 3.91000 | 8.0368 I | 4.05088 | 9 |  |
|  | $\begin{aligned} & \underset{W}{H} \\ & \text { O } \\ & \text { Hen } \\ & 0 \end{aligned}$ | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． | 式 | 军 |
|  |  | $65^{\circ}$ |  | $64^{\circ}$ |  | $63^{\circ}$ |  |  |  |

Smithsonian Tables．
differences of latitude and departure. - Continued.

|  |  | $27^{\circ}$ |  | $28^{\circ}$ |  | $29^{\circ}$ |  | 烒 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. |  |  |
| 0 | 1 | 0.89100 | 0.45399 | 0.88294 | 0.46947 | 0.87462 | 0.4848 I | 1 |  |
|  | 2 | 1.78201 | 0.90798 | 1.76589 | 0.93894 | 1.74924 | 0.96962 | 2 |  |
|  | 3 | 2.67301 | 1.36197 | 2.64884 | 1.40841 | 2.62386 | I. 45443 | 3 |  |
|  | 4 | 3.56402 | 1.81596 | 3.53179 | 1.87788 | 3.49848 | I. 93924 | 4 |  |
|  | 5 | 4.45503 5.34603 | 2.26995 2.72394 | 4.41473 <br> 5.29768 | 2.34735 <br> 2.81682 | +4.37310 <br> 5.24772 | 2.42405 2.90886 | 5 | 60 |
|  | 7 | 6.23704 | 3.17793 | 6.18063 | 3.28630 | 6.12234 | 3.39367 | 7 |  |
|  | 8 | 7.12805 | 3.63193 | 7.06358 | 3.75577 | 6.99695 (i | 3.87848 | 8 |  |
|  | 9 | 8.01905 | 4.08591 | 7.94652 | 4.22524 | 7.87156 | 4.36329 | 9 |  |
| 15 | 1 | 0.88901 | 0.45787 | 0.88089 | 0.47332 | 0.87249 | 0.48862 | 1 |  |
|  | 2 | 1.77803 | 0.91574 | 1.76178 | 0.94664 | 1.74499 | 0.97724 | 2 |  |
|  | 3 | 2.66705 | I. 37362 | 2.64267 | 1.41996 | 2.61748 | 1.46566 | 3 |  |
|  | 4 | 3.55606 | 1.83149 | 3.52356 | 1.89328 | 3.48998 | I. 95448 | 4 |  |
|  | 5 | 4.44508 5.33410 | 2.28937 2.74724 | 4.40445 5.28534 | 2.36660 2.83992 | 4.36248 5.23497 | 2.44310 2.93172 3.2 | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 45 |
|  | 7 7 | 5.33410 6.22311 | 2.74724 3.20511 | 5.28534 6.16623 | 2.83992 3.31324 | 5.23497 6.10747 | 2.93172 3.42034 | 7 |  |
|  | 8 | 7.11213 | 3.66299 | 7.04712 | 3.78656 | 6.97996 | 3.90896 | 8 |  |
|  | 9 | 8.00115 | 4.12086 | 7.92801 | 4.25988 | 7.85246 | 4.39759 | 9 |  |
| 30 | 1 | 0.88701 | 0.46174 | 0.87885 | 0.47715 | 0.87035 |  | 1 |  |
|  | 2 | 1.77402 | 0.92349 | 1.7.7563 | 0.95431 | 1.74071 | 0.98484 | 2 |  |
|  | 3 | 2.66103 | 1. 38524 | 2.63645 | 1.43147 | 2.61106 | 1.47727 | 3 |  |
|  | 4 | 3.54804 | 1. 84699 | 3.51526 | 1.90863 | 3.48142 | 1.96969 | 4 |  |
|  |  |  |  |  |  |  | 2.46211 2.95454 | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 30 |
|  | $\begin{aligned} & 6 \\ & 7 \end{aligned}$ | 5.32206 6.20907 | 2.77049 3.23224 | 5.27290 <br> 6.15171 | 2.86295 3.34011 3 | 5.22213 <br> 6.09248 | 2.95454 3.44696 | $\begin{aligned} & 6 \\ & 7 \end{aligned}$ |  |
|  | $7$ | 6.20907 7.09608 | 3.23224 3.69398 | 6.15171 7.03053 | 3.34011 3.81727 | 6.09248 6.96284 | 3.44696 <br> 3.93938 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | $\begin{aligned} & 8 \\ & 9 \end{aligned}$ | 7.09608 7.98309 | 3.69398 4.15573 | 7.03053 7.90935 | 3.81727 4.29442 | 6.98284 7.83320 | 3.9393818 4.431 | 9 |  |
| 45 | 1 | 0.88498 | 0.46561 | 0.87672 | 0.48098 | 0.86819 | 0.49621 | 1 |  |
|  | 2 | 1.76997 | 0.93122 | 1.75345 | 0.96197 | 1.73639 | 0.99243 | 2 |  |
|  | 3 | 2.65496 | I. 39684 | 2.63018 | I. 44296 | 2.60459 | I. 48864 | 3 |  |
|  | 4 | 3.53995 | 1.86245 | 3.50690 | I.92395 | 3.47279 | 1. 98486 $\mathbf{2} 48108$ | 4 |  |
|  |  | ${ }^{4.42493}$ | 2.32807 <br> 2.79368 | ${ }^{4.38363}$ |  |  | 2.48108 | 5 | 15 |
|  | 6 | 5.30992 <br> 6.19491 <br> 1.0799 | 2.79368 3.25930 3 | 5.26036 <br> 6.13708 <br> .8 | 2.88593 3.36692 | 5.20919 6.07739 | $\begin{aligned} & 2.97729 \\ & 3.47355 \end{aligned}$ | 6 |  |
|  | $7$ | 6.19491 7.07990 | 3.25930 3.7249 I | 6.13708 7.01381 | 3.36692 3.84791 | 6.07739 6.94599 | 3.4735 I 3.96973 | 7 |  |
|  | 9 | 7.96488 | 4.19053 | 7.89054 | 4.32889 | 7.81378 | 4.46594 | 9 |  |
|  |  | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. |  | 当 |
|  |  | $62^{\circ}$ |  | $61^{\circ}$ |  | $60^{\circ}$ |  |  |  |

Smithsonian Tables.

Table 9．
TRAVERSE TABLE，
DIFFERENCES OF LATITUDE AND DEPARTURE．－CONTINUED．

|  |  | $30^{\circ}$ |  | $31^{\circ}$ |  | $32^{\circ}$ |  |  | 気 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| $\bigcirc$ | 1 | 0.86602 | 0.50000 | 0.85716 | 0.51503 | 0.84804 | 0.52991 | 1 |  |
|  | 2 | 1.73205 | 1.00000 | 1.71433 | 1.03007 | 1.69609 | 1.05983 | 2 |  |
|  | 3 | 2.59807 | 1.50000 | 2.57150 | 1.54511 | 2.54414 | 1． 58975 | 3 |  |
|  | 4 | 3.46410 | 2.00000 | 3．42866 | 2.06015 | 3.39219 | 2.11967 | 4 |  |
|  | 5 | 4.33012 | 2.50000 | 4.28583 | 2.57519 | 4.24024 | 2.64959 | $5$ | 60 |
|  | 6 | 5.19615 | 3.00000 | 5.14300 | 3：09022 | 5.08828 | 3.17951 | $6$ |  |
|  | 7 | 6.06217 | 3.50000 | 6.00017 | 3.60526 | 5.93633 | 3.70943 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 6.92820 | 4.00000 | 6.85733 | 4.12030 | 6.78438 | 4.23935 | $8$ |  |
|  | 9 | 7.79422 | 4.50000 | 7.71450 | 4.63534 | 7.63243 | 4.76927 | 9 |  |
| 15 | 1 | 0.86383 | 0.50377 | 0.85491 | 0.51877 | 0.84572 | 0.53361 | 1 |  |
|  | 2 | 1.72767 | 1.00754 | 1.70982 | 1.03754 | 1.69145 | 1.06722 | 2 |  |
|  | 3 | 2.59150 | 1.51132 | 2.56473 | 1.55631 | 2.53718 | 1.60084 | 3 |  |
|  | 4 | 3.45534 | 2.01509 | 3.41964 | 2.07509 | 3.38291 | 2.13445 | 4 |  |
|  | 5 | 4.31917 | 2.51887 | 4.27456 | 2.59386 | 4.22863 | 2.66807 | $5$ | 45 |
|  | 6 | 5.18301 6.04684 | 3.02264 | 5.12947 | 3．11263 | 5.07436 | $3.20168$ | $6$ |  |
|  | 7 | 6.04684 | 3.52641 | 5.98438 | 3.63141 | 5.92009 | $3.7353^{\circ}$ | $7$ |  |
|  | 8 | 6.91068 | 4.03019 | 6.83929 | 4.15018 | 6.76582 | 4.26891 | 8 |  |
|  | 9 | 7.77451 | 4.53396 | 7.69420 | 4.66895 | 7．61 155 | 4.80253 | 9 |  |
| 30 | 1 | 0.86162 | 0.50753 | 0.85264 | 0.52249 | 0.84339 | $0.5373{ }^{\circ}$ | 1 |  |
|  | 2 | 1.72325 | 1.01507 | 1．70528 | 1.04499 | 1.68678 | 1.07460 | 2 |  |
|  | 3 | 2.58488 | 1．52261 | 2.55792 | 1.56749 | 2.53017 | 1.61190 | 3 |  |
|  | 4 | 3.44651 | 2.03015 | 3.41056 | 2.08999 | 3.37356 | 2.14920 | 4 |  |
|  | 5 | 4.30814 | 2.53769 | 4.26320 | 2.61249 | 4.21695 | $2.68650$ | $\begin{aligned} & 4 \\ & 5 \end{aligned}$ | 30 |
|  | 6 | 5.16977 | 3.04523 | 5.11584 | 3.13499 | 5.06034 | 3.22380 | $6$ |  |
|  | 8 | 6.03140 | 3.55276 | 5.96948 | 3.65749 | 5.90373 |  | $7$ |  |
|  | 8 | 6.89303 | 4.06030 | $6.82 \mathrm{II2}$ | 4.17998 | 6.74713 | 4.29840 | $8$ |  |
|  | 9 | 7.75466 | 4.56784 | 7.67376 | 4.70248 | 7.59052 | 4.83570 | 9 |  |
| 45 | 1 | 0.85940 | 0.51129 | 0.85035 | 0.52621 | 0.84103 | 0.54097 | 1 |  |
|  | 2 | 1.71881 | $1.0225^{8}$ | 1.70070 | 1.05242 | 1.68207 | 1.08194 | 2 |  |
|  | 3 | 2.57821 | 1.53387 | 2.55105 | 1.57864 | 2.52311 | 1.62292 | $3$ |  |
|  | 4 | 3.43762 | 2.04517 | 3.40140 | 2．10485 | 3.36415 | 2.16389 | $\begin{aligned} & 3 \\ & 4 \end{aligned}$ |  |
|  | 5 | 4.29703 | 2.55646 | 4.25176 | 2.63107 | 4.20519 | 2.70487 | $\begin{aligned} & 4 \\ & 5 \end{aligned}$ | 15 |
|  | 6 | 5.15643 | 3.06775 | 5.10211 | 3.15728 | 5.04623 | 3.24584 | 6 | 15 |
|  | 7 | 6.01584 | 3.57905 | 5.95246 | 3.68349 | 5.88827 | 3.78682 | $7$ |  |
|  | 8 | 6.87525 | 4.09034 | 6.8028 I | 4.20971 | 6.72831 | 4.32779 | 8 |  |
|  | 9 | 7.73465 | 4.60163 | 7.65316 | 4.73592 | 7.56935 | 4.86877 | 9 |  |
|  |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． | $\begin{aligned} & \text { V} \\ & \stackrel{0}{0} \\ & \ddot{H} \\ & \overparen{¢} \end{aligned}$ | $\begin{aligned} & \text { 条 } \\ & \text { 苞 } \\ & \text { 荡 } \end{aligned}$ |
|  |  | $59^{\circ}$ |  | $58^{\circ}$ |  | $57^{\circ}$ |  |  |  |

[^22]DIFFERENCES OF LATITUDE AND DEPARTURE，－CONTINUED．

|  | $\begin{aligned} & \dot{U} \\ & \text { Ü } \\ & \text { H } \\ & \stackrel{\mu}{n} \end{aligned}$ | $33^{\circ}$ |  | $34^{\circ}$ |  | $35^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.83867 | 0.54463 | 0.82903 | 0.55919 | 0.81915 | 0.57357 | 1 |  |
|  | 2 | 1.67734 | 1.08927 | 1.65807 | 1．11838 | 1.63830 | 1．14715 | 2 |  |
|  | 3 | 2.51601 | 1.63391 | 2.48711 | 1． 67757 | 2.45745 | 1.72072 | 3 |  |
|  | 4 | $3 \cdot 35468$ | 2.17855 | $3 \cdot 31615$ | 2.23677 | 3.27660 | 2.29430 | $4$ |  |
|  | 5 | 4.19335 | 2.72319 | 4．14518 | 2.79596 | 4.09576 | 2.86788 | $5$ | 60 |
|  | 6 | 5.03202 | 3.26783 | 4.97422 | 3.35515 | 4.91491 | 3.44145 | 6 |  |
|  | 7 | 5.87069 | 3．81247 | 5.80326 | 3.91435 | 5.73406 | 4.01503 | 7 |  |
|  | 8 | 6.70936 | 4.35711 | 6.63230 | 4.47354 | 6.5532 I | 4.58861 | 8 |  |
|  | 9 | $7 \cdot 54803$ | 4.90175 | 7．46I 33 | 5.03273 | $7 \cdot 37236$ | 5．16218 | 9 |  |
| 15 | 1 | 0.83628 | 0.54829 | 0.82659 | 0.56280 | 0.81664 | 0.57714 | 1 |  |
|  | 2 | 1.67257 | 1.09658 | 1.65318 | I．I 2560 | 1.63328 | I． 5429 | 2 |  |
|  | 3 | 2． 50885 | 1.64487 | 2.47977 | 1.68841 | 2.44992 | 1．73143 | 3 |  |
|  | 4 | $3 \cdot 34514$ | 2.19317 | 3.30636 | 2.25121 | 3.26656 | 2.30858 | $4$ |  |
|  | 5 | 4.18143 5.01771 | 2.74146 3.28975 | 4.13295 4.95954 | 2.81402 3.37682 | 4.08320 4.89984 | $2.88572$ $3.46287$ | $5$ | 45 |
|  | 6 | 5.01771 | 3.28975 | 4.95954 | 3.37682 | 4.89984 5.71649 | 3.46287 4.04001 | $\begin{aligned} & 6 \\ & 7 \end{aligned}$ |  |
|  | 7 | 5.85400 | 3.83805 | 5.78613 | 3.93963 | 5.71649 | 4.04001 | 7 |  |
|  | 8 | 6.69028 | 4.38634 | 6.61272 | 4.50243 | 6.533 I 3 | 4.61716 | 8 |  |
|  | 9 | $7 \cdot 52657$ | 4.93463 | $7 \cdot 43931$ | 5.06524 | 7.34977 | 5．19430 | 9 |  |
| 30 | 1 | 0.83388 | 0.55193 | 0.82412 | 0． 56640 | 0.81411 | 0.58070 | 1 |  |
|  | 2 | 1.66777 | 1.10387 | 1.64825 | 1.13281 | 1.62823 | 1．16140 | 2 |  |
|  | 3 | 2.50165 | 1.6558 r | 2.47237 | 1.69921 | 2.44234 | 1.74210 | 3 |  |
|  | 4 | $3 \cdot 33554$ | 2.20774 | 3.29650 | 2.26562 | 3.25646 | 2.3228 r | 4 |  |
|  | 5 | 4.16942 | 2.75968 | 4.12063 | 2.83203 | 4.07057 | $2.9035 \mathrm{I}$ | $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | 30 |
|  | 6 | 5.0033 I 5.83720 | $3.3 \mathrm{II62}$ 3.86355 | 4.94475 596888 | 3.39843 | 4.88469 | 3.4842 I | $6$ |  |
|  | 7 | 5.83720 | 3.86355 | 5.76888 | 3.96484 | 5.69880 | 4.06492 4.64562 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 8 | 6.67108 | 4.41549 | 6.59300 | 4.53124 | 6.51292 | 4.64562 | $8$ |  |
|  | 9 | $7 \cdot 50497$ | 4.96743 | 7－41713 | 5.09765 | 7.32703 | 5.22632 | 9 |  |
| 45 | 1 | 0.83147 | 0.55557 | 0.82164 | 5．56999 | 0.81157 | 0.58425 | 1 |  |
|  | 2 | 1． 66294 | 1.15114 | 1.64329 | I． 13999 | 1.62314 | I．16850 | 2 |  |
|  | 3 | 2.49441 | 1.6667 I | 2.46494 | I．70999 | 2.43472 | 1.75275 | 3 |  |
|  | 4 | $3 \cdot 32588$ | 2.22228 | 3.28658 | 2.27998 | 3.24629 | 2.33700 | 4 |  |
|  | 5 | 4.15735 | 2.77785 | 4.10823 | 2.84998 | 4.05787 | 2.92125 | 5 | 15 |
|  | 6 | 4.98882 | 3.33342 | 4.92988 | 3.41998 | 4.86944 | 3．50550 | 6 |  |
|  | 7 | 5.82029 | 3.88899 | 5.75152 | 3.98997 | 5．68101 | 4.08975 | 7 |  |
|  | 8 | 6.65176 | 4.44456 | 6.57317 | 4.55997 | 6.49260 | 4.67400 | 8 |  |
|  | 9 | 7.48323 | 5.00013 | 7.39482 | 5．12997 | 7．30416 | 5.25825 | 9 |  |
| 3易管 |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． |  | 急 |
|  |  | $56^{\circ}$ |  | $55^{\circ}$ |  | $54^{\circ}$ |  |  |  |

Smithsonian Tables．

Table 9.
TRAVERSE TABLE．
DIFFERENCES OF LATITUDE AND DEPARTURE．－CONTINUED．

| 密 |  | $36^{\circ}$ |  | $37^{\circ}$ |  | $38^{\circ}$ |  |  | $\begin{aligned} & \text { 氙 } \\ & \text { 品 } \\ & \text { 豆 } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.80901 | 0.58778 | 0.79863 | 0.6018 I | 0.78801 | 0.61566 | 1 |  |
|  | 2 | 1.61803 | 1.17557 | 1.59727 | 1.20363 | 1.57602 | 1.23 I 32 | 2 |  |
|  | 3 | 2.42705 | 1.76335 | 2.39590 | 1.80544 | 2.36403 | 1.84698 | 3 |  |
|  | 4 | 3.23606 | 2.35114 | 3.19454 | 2.40726 | 3.15204 | 2.46264 | 4 |  |
|  | 5 | 4.04508 | 2.93892 | 3.99317 | 3.00907 | 3.94005 | 3.07830 | $5$ | 60 |
|  | 6 | 4.85410 | $3 \cdot 52671$ | 4.79181 | 3.61089 | 4.72806 | 3.69396 | $6$ |  |
|  | 7 | 5.66311 | 4.11449 | 5.59044 | 4.21270 | $5 \cdot 51607$ | 4.30963 | $\begin{aligned} & 7 \\ & 8 \end{aligned}$ |  |
|  | 9 | 6.47213 7.2815 | 4.70228 5.29006 | 6.38908 7.18771 | 4.81452 5.41633 | 6.30408 7.09209 | 4.92529 5.54095 | $\begin{aligned} & 8 \\ & 9 \end{aligned}$ |  |
| 15 | 1 | 0.80644 | 0.59130 | 0.79600 | 0.60529 | 1.78531 | 0.61909 | 1 |  |
|  | 2 | 1.61288 | 1．1826ı | 1.59200 | 1.21058 | I． 57063 | 1.23818 | 2 |  |
|  | 3 | 2.41933 | 1.77392 | 2.38800 | r．81588 | 2.35595 | 1.85728 | 3 |  |
|  | 4 | 3.22577 | 2.36523 | 3.18400 | 2.42117 | 3.14126 | 2.47637 | 4 |  |
|  | 5 | 4.03222 | 2.95654 | 3.98001 | 3.02647 | 3.92658 | 3.09547 | $5$ | 45 |
|  | 6 | 4.83866 | 3.54785 | 4.77601 | 3.63176 | 4.71190 | 3.71456 | $6$ |  |
|  | 7 | 5.64511 | 4.13916 | 5．57201 | 4.23705 | 5.49721 | 4.33365 | $8$ |  |
|  | 8 | 6.45155 | 4.73047 | 6.36801 | 4.84235 | 6.28253 | 4.95275 | 8 |  |
|  | 9 | 7.25800 | $5 \cdot 32178$ | 7．16401 | $5 \cdot 44764$ | 7.06785 | 5．57184 | 9 |  |
| 30 | 1 | 0.80385 | 0.59482 | 0.79335 | 0.60876 | 0.78260 | 0.62251 | 1 |  |
|  | 2 | 1.6077 I | 1.18964 | 1.58670 | 1.21752 | I． 56521 | 1.24502 | 2 |  |
|  | 3 | 2.41 I 57 | 1.78446 | 2.38005 | 1．82628 | 2.34782 | 1.86754 | 3 |  |
|  | 4 | 3.21542 | 2.37929 | 3．1734 | 2.43504 | 3.13043 | 2.49005 | 4 |  |
|  | 5 | 4.81928 | 2.9741 I | 3.96676 | 3.04380 | 3.91304 | 3.11257 | 5 | $3^{\circ}$ |
|  | 6 | 4.82314 | 3.56893 | 4.76011 | 3.65256 | 4.69564 | 3.73508 | 6 |  |
|  |  | 5.62699 | 4.16375 | 5．55347 | 4.26 I 32 | 5.47825 | 4.35760 | $7$ |  |
|  | 8 | 6.43085 | 4.75858 | 6.34682 | 4.87009 | 6.26086 | 4.98011 | $8$ |  |
|  | 9 | 7．2347 1 | 5.35340 | 7．14017 | 5.47885 | 7.04347 | 5.60263 | 9 |  |
| 45 | 1 | 0.80125 | 0.59832 | 0.79068 | 0.61221 | 0.77988 | 0.62592 | 1 |  |
|  | 2 | 1.60250 | 1.19664 | 1．581 37 | 1.22443 | I． 55946 | 1.25184 | 2 |  |
|  | 3 | 2.40376 | 1.79497 | 2.37206 | 1.83665 | 2.33965 | 1.87777 | $3$ |  |
|  | 4 | 3.20501 | 2.39329 | 3.16275 | 2.44886 | 3.11953 | 2.50369 | $4$ |  |
|  |  | 4.00626 | 2.99162 | 3.95344 | 3.06108 | 3.89942 | 3.12961 | $5$ | 15 |
|  | 6 | 4.80752 | 3.58994 | 4.74413 | 3.67330 | 4.67930 | 3.75554 | 6 | 5 |
|  | 7 | 5.60877 | 4.18827 | 5.53482 | 4.28552 | 5.45919 | 4.38146 | $7$ |  |
|  | 8 | 6.41003 | 4.78659 | 6.32551 | 4.89773 | 6.23907 | 5.00738 | 8 |  |
|  | 9 | 7.21128 | $5 \cdot 38492$ | 7.11620 | 5．50995 | 7.01896 | 5.6333 I | 9 |  |
|  |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． | 苛 | 当 |
|  |  | $53^{\circ}$ |  | $52^{\circ}$ |  | $51^{\circ}$ |  |  |  |

Smithsonian Tables．

TRAVERSE TABLE.
DIFFERENCES OF LATITUDE AND DEPARTURE.-CONTINUED.

|  |  | $39^{\circ}$ |  | $40^{\circ}$ |  | $41^{\circ}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat. | Dep. | Lat. | Dep. | Lat. | Dep. |  |  |
| 0 | 1 | 0.77714 | 0.62932 | 0.76604 | 0.64278 | 0.75470 | 0.65605 | 1 |  |
|  | 2 | I. 55429 | 1.25864 | 1.53208 | 1.28557 | 1.50941 | 1.31211 | 2 |  |
|  | 3 | 2.33143 | 1.88796 | 2.29813 | I. 92836 | 2.26412 | 1.96817 | 3 |  |
|  | 4 | 3.10858 | 2.51728 | 3.06417 | 2.57115 | 3.01883 | 2.62423 | 4 |  |
|  | 5 | 3.88573 | 3.14660 | 3.83022 | 3.21393 | 3.77354 | 3.28029 | 5 | 60 |
|  | 6 | 4.66287 | 3.77592 | 4.59626 | 3.85672 | 4.52825 | 3.93635 | 6 |  |
|  | 7 | $5 \cdot 44002$ | 4.40524 | $5 \cdot 36231$ | 4.49951 | 5.28296 | 4.59241 | 7 |  |
|  | 8 | 6.21716 | 5.03456 | 6.12835 | 5.14230 | 6.03767 | 5.24847 | 8 |  |
|  | 9 | 6.9943 I | 5.66388 | 6.89439 | 5.78508 | 6.79238 | 5.90453 | 9 |  |
| 15 | 1 | 0.77439 | 0.63270 | 0.76323 | 0.64612 | 0.75184 | 0.65934 | 1 |  |
|  | 2 | 1.54878 | 1.26541 | 1. 52646 | 1.29224 | 1.50368 | 1.31869 | 2 |  |
|  | 3 | 2.32317 | 1.89811 | 2.28969 | 1.93837 | 2.25552 | 1.97803 | 3 |  |
|  | 4 | 3.09757 | 2.53082 | 3.05293 | 2.58449 | 3.00736 | 2.63738 | 4 |  |
|  | 5 | 3.87196 | 3.16352 | 3.81616 | 3.23062 | 3.75920 | 3.29672 | 5 | 45 |
|  | 6 | 4.64635 | 3.79623 | 4.57939 | 3.87674 | 4.51104 | 3.95607 | 6 |  |
|  | 7 | $5 \cdot 42074$ | 4.42893 | 5.34262 | 4.52286 | 5.26288 | 4.61542 | 7 |  |
|  | 8 | 6.19514 | 5.06164 | 6.10586 | 5.16899 | 6.01472 | 5.27476 | 8 |  |
|  | 9 | 6.96953 | 5.69434 | 6.86909 | 5.81511 | 6.76656 | $5 \cdot 93411$ | 9 |  |
| 30 | 1 | 0.77162 | 0.63607 | 0.76040 | 0.64944 | 0.74895 | 0.66262 | 1 |  |
|  | 2 | 1. 54324 | 1.27215 | 1.52081 | 1.29889 | 1.49791 | 1.32524 | 2 |  |
|  | 3 | 2.31487 | 1.90823 | 2.28121 | 1.94834 | 2.24686 | 1.98786 | 3 |  |
|  | 4 | 3.08649 | 2.54431 | 3.04162 | 2.59779 | 2.99582 | 2.65048 | 4 |  |
|  | 5 | 3.85812 | 3.18039 | 3.80203 | 3.24724 | 3.74477 |  | $5$ | 30 |
|  | 6 | 4.62974 | 3.81646 | 4.56243 | 3.89668 | 4.49373 | $3.97572$ | 6 |  |
|  | 7 | 5.40137 | 4.45254 | 5.32284 | 4.54613 | 5.24268 | 4.63834 | 7 |  |
|  | 8 | 6.17299 | 5.08862 | 6.08324 | 5.19558 | 5.99164 | $5 \cdot 30096$ | 8 |  |
|  | 9 | 6.94462 | 5.72470 | 6.84365 | 5.84503 | 6.74060 | $5 \cdot 96358$ | 9 |  |
| 45 | 1 | 0.76884 | 0.63943 | 0.75756 | 0.65276 | 0.74605 | 0.66588 | 1 |  |
|  | 1 | I. 53768 | 1.27887 | 1.51513 | 1.30552 | 1.49211 | 1.33176 | 2 |  |
|  | 3 | 2.30652 | 1.9183 I | 2.27269 | 1.95828 | 2.23817 | 1.99764 | 3 |  |
|  | 4 | 3.07536 | 2.55775 | 3.03026 | 2.61104 | 2.98422 | 2.66352 | 4 |  |
|  | 5 | 3.84420 | 3.19719 | 3.78782 | 2.26380 | 3.73028 | 3.32940 | 5 | 15 |
|  | 6 | 4.61305 | 3.83663 | 4.54539 | 3.91656 | $4 \cdot 47634$ | 3.99529 | 6 |  |
|  | 7 | 5.38189 | 4.47607 | 5.30295 | 4.56932 | 5.22240 | 4.66117 | 7 |  |
|  | 8 | 6.15073 | 5.11551 | 6.06052 | 5.22208 | 5.96845 | $5 \cdot 32705$ | 8 |  |
|  | 9 | 6.91957 | 5.75495 | 6.81808 | 5.87484 | 6.7145 I | 5.99293 | 9 |  |
| 3 <br> 3 <br> 苞 | $\begin{aligned} & \text { E.0. } \\ & \text {. } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Dep. | Lat. | Dep. | Lat. | Dep. | Lat. | $\underset{0}{0}$OO.© | $\begin{aligned} & \text { 客 } \\ & \stackrel{5}{7} \\ & \end{aligned}$ |
|  |  | $50^{\circ}$ |  | $49^{\circ}$ |  | $48^{\circ}$ |  |  |  |

Smithsonian Tables.

Table 9.
TRAVERSE TABLE．
DIFFERENCES OF LATITUDE AND DEPARTURE，－CDNTINUED．

|  |  | $42^{\circ}$ |  | $43^{\circ}$ |  | $44^{\circ}$ |  | $\begin{aligned} & \text { 荷 } \\ & \text { H } \\ & \text { H } \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Lat． | Dep． | Lat． | Dep． | Lat． | Dep． |  |  |
| 0 | 1 | 0.74314 | 0.66913 | 0.73135 | 0.68199 | 0.71933 | 0.69465 | 1 |  |
|  | 2 | I． 48628 | 1．33826 | 1.46270 | 1． 36399 | 1.43867 | 1.38931 | 1 |  |
|  | 3 | 2.22943 | 2.00739 | 2．19406 | 2.04599 | 2.15801 | 2.08397 | 3 |  |
|  | 4 | 2.97257 | 2.67652 | 2.92541 | 2.72799 | 2.87735 | 2.77863 | 4 |  |
|  | 5 | 3.71572 | 3.34565 | 3.65676 | 3.40999 | 3.59669 | 3.47329 | 5 | 60 |
|  | 6 | 4.45886 | 4.01478 | 4.38812 | 4.09199 | 4.31603 | 4.16795 | $6$ |  |
|  | 7 | 5.20201 | 4.6839 x | 5.11947 | 4.77398 | 5.03537 | 4.86260 | 7 |  |
|  | 8 | 5.94515 | 5.35304 | 5.85082 | 5.45598 | 5.75471 | 5．55726 | $8$ |  |
|  | 9 | 6.68830 | 6.02217 | 6.58218 | 6.13798 | 6.47405 | 6.25192 |  |  |
| 15 | 1 | 0.74021 | 0.67236 | 0.72837 | 0.68518 | 0.71630 | 0.69779 | 1 |  |
|  | 2 | 1.48043 | 1.34473 | 1.45674 | 1.37036 | 1.43260 | 1.39558 | 2 |  |
|  | 3 | 2.22065 | 2.01710 | 2.18511 | 2.05554 | 2.14890 | 2.09337 | 3 |  |
|  | 4 | 2.96087 | 2.68946 | 2.91348 | 2.74073 | 2.86520 | 2.79116 | 4 |  |
|  | 5 | 3.70109 | 3.36183 | 3.64185 | 3.42591 | 3.58151 | 3.48895 | 5 | 45 |
|  | 6 | 4.44130 | 4.03420 | 4.37022 | 4.11109 | 4.29781 | 4.18674 | 6 |  |
|  | 7 | 5.18152 | 4.70656 | 5.09859 | 4.79628 | 5.01411 | 4.88453 | 7 |  |
|  | 8 | 5.92174 | 5.37893 | 5.82696 | 5.48146 | 5.73041 | 5．58232 | 8 |  |
|  | 9 | 6.66796 | 6.05130 | 6.55533 | 6.16664 | 6.44671 | 6.2801 I | 9 |  |
| 30 | 1 | 0.73727 | 0.67559 | 0.72537 | 0.68835 | 0.71325 | 0.70090 | 1 |  |
|  | 2 | 1．47455 | 1.35118 | 1． 45074 | 1.37670 | 1.42650 | 1.40181 | 2 |  |
|  | 3 | 2.21183 | 2.02677 | 2.17612 | 2.06506 | 2.13975 | 2.10272 | 3 |  |
|  | 4 | 2.94910 | 2.70236 | 2.90149 | 2.75341 | 2.85300 | 2.80363 | 4 |  |
|  | 5 | 3.68638 | 3.37795 | 3.62687 | 3.44177 | 3.56625 | 3.50454 | $5$ | 30 |
|  | 6 | 4.42366 | 4.05354 | 4.35224 | 4.13012 | $4.2795^{\circ}$ | 4.20545 | $6$ |  |
|  | 7 | 5．16094 | 4.72913 | 5．07762 | 4.81848 | 4.99275 | 4.90636 | $7$ |  |
|  | 8 | 5.89821 | 5．40472 | 5.80299 | 5．50683 | 5.70600 | 5.60727 | 8 |  |
|  | 9 | 6.63549 | 6.08031 | 6.52836 | 6.19519 | 6.41925 | 6.30818 | 9 |  |
| 45 | 1 | 0.73432 | 0.67880 | 0.72236 | 0.69151 | 0.71018 | 0.70401 | 1 |  |
|  | 2 | 1.46864 ． | 1.35760 | 1.44472 | 1.38302 | 1.42037 | 1．40802 | 2 |  |
|  | 3 | 2.20296 | 2.03640 | 2.16709 | 2.07453 | 2.13055 | 2.11204 | 3 |  |
|  | 4 | 2.93729 | 2.71520 | 2.88945 | 2.76605 | 2.84074 | 2.81605 | 4 |  |
|  | 5 | 3.67161 | 3.39400 | 3.61182 | 3.45756 | 3.55092 | 3.52007 | 5 | 15 |
|  | 6 | $4 \cdot 40593$ | 4.07280 | 4．33418 | 4.14907 | 4．26III | 4.22408 | 6 |  |
|  |  | 5.14025 | 4.75160 | 5．05654 | 4.84059 | 4.97129 | 4.92810 | 7 |  |
|  | 8 | $5.8745^{8}$ | 5.43040 | 5.77891 | 5.532 IO | 5.68148 | 5.63211 | 8 |  |
|  | 9 | 6.60890 | 6.10920 | 6.50127 | 6.22361 | 6.39166 | 6.33613 | 9 |  |
| $\begin{aligned} & \text { 急 } \\ & \stackrel{\rightharpoonup}{0} \\ & \stackrel{0}{6} \end{aligned}$ |  | Dep． | Lat． | Dep． | Lat． | Dep． | Lat． | H | 条 |
|  | ¢ | $47^{\circ}$ |  | $46^{\circ}$ |  | $45^{\circ}$ |  | ？ | \％ |

Smithsonian Tableb．

DIFFERENCES OF LATITUDE AND DEPARTURE. -CONTINUED.


Smithsonian Tables.
[Derivation of table explained on p . xlv.]


Smithsonian Tables.
[Derivation of table explained on $p$. xlv.]

| Lat. | $1 \mathrm{I}^{0}$ | $12^{\circ}$ | $13^{\circ}$ | $14^{\circ}$ | $15^{\circ}$ | $16^{\circ}$ | $17^{\circ}$ | $18^{\circ}$ | $19^{\circ}$ | $20^{\circ}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\prime}$ | 7.317 | 7.317 | 7.317 | 7.317 | 7.318 | 7.318 | 7.318 | 7.318 | 7.318 | 7.318 | 4 |  |
|  | 8985 | 9285 | 9615 | 9960 | 0333 | 0730 | 1149 | 1591 | 2054 | 2539 |  |  |
| 1 | 8990 | 9290 | 9617 | 9966 | 0340 | $\bigcirc 737$ | 1156 | 1599 | 2062 | 2547 |  |  |
| ${ }^{2}$ | 8995 | 9296 | 9622 | 9972 | c346 | $\bigcirc 744$ | 1163 | 1606 | 2070 | 2556 |  |  |
| 3 | 8999 | 9301 | 9628 | 9978 | ${ }^{\circ} \mathrm{O} 53$ | ${ }^{0} 75^{\circ}$ | ${ }^{1171}$ | 1614 | 2078 | 2564 | 10 | .7 |
| 4 | 9004 | 9306 | 963.3 | 9984 | 0359 0366 | 0757 | 1178 | 1621 | 2086 | 2572 | 20 30 | 1.3 2.0 $\mathbf{2}$ |
| 6 | 9009 | 9312 | 9639 | 9990 | ${ }^{0366}$ | 0764 | 1185 | 1629 1637 | 2094 2102 | 2580 2589 | 40 | 2.8 2.7 |
|  | 9014 | 9317 | 9645 | 999 | ${ }^{0372}$ | ${ }^{271}$ | 1192 |  |  |  | 50 | 3.3 |
| 8 | 9019 9023 | 9322 9327 9 | 9650 9656 | $* 0002$ $* * 0008$ | 0372 037 0385 | 0778 0784 | 1199 <br> 1207 <br> 129 | 1644 <br> 1652 | 2110 2118 | 2597 2605 | 60 | 4.0 |
| 9 | 9028 | 9333 | 9661 | ${ }^{*} 0014$ | -392 | 0791 | 1214 | 1659 | 2126 | 2614 |  |  |
| 10 | 9033 | 9338 | 9667 | ${ }^{*} 0020$ | -398 | ${ }^{0} 798$ | 1221 | 1667 | 2134 | 2622 | 5 |  |
| 11 | 9038 | 9343 | 9673 | ${ }_{*}^{*} 0026$ | 0404 | 0805 | 1228 | 1675 | 2142 | 2630 |  |  |
| 12 | 9043 | 9349 | 9678 | ${ }_{*}^{*} 0032$ | 0411 | 0812 | 1236 | 1682 1690 | 2150 | 2639 2647 |  |  |
| 13 | 9048 | 9354 | 9684 | ${ }^{*} 0039$ | 0418 | 0819 | 1243 | 1690 | 2158 |  | 10 20 | . 8 |
| 14 | 9053 | 9359 | 9690 9696 | ${ }_{*}^{*} 0045$ |  |  |  | 1697 <br> 1705 | 2166 2174 2184 | 2655 2663 | 20 30 | 1.7 2.5 |
| 15 | 9058 9062 | 9365 9370 | 9696 9701 | ${ }_{*}^{*} \mathbf{0} \mathbf{0 0 5 1}$ | 0430 0437 | 0838 083 0839 | 1258 1265 | 1705 <br> $17 \times 3$ <br> 18 | 2174 $\mathbf{2 1 8 2}$ | 2663 2672 | 40 | 3.3 |
| 17 | 9067 | 9375 | 9707 | *0063 | 0443 | 0846 | 1272 | 1720 | 2190 | 2680 | 60 | 5.0 |
| 18 | 9072 | 9380 | 9713 | ${ }^{*} 0070$ | - 450 | 0853 | 1279 | ${ }^{1728}$ | 2198 | 2688 |  |  |
| 19 | 9077 | 9386 | 9718 | ${ }^{*} 0076$ | 0456 |  | 1287 | 1735 |  | 6697 | 8 |  |
| 20 | 9082 | 9391 | 9724 | *0082 | 0463 | 0867 | 1294 | 1743 | 2214 | 2705 |  |  |
| 21 | 9087 | 9396 | 9730 | ${ }^{*} 0088$ | 0470 | 0874 | ${ }^{1301}$ | ${ }^{1751}$ | 2222 | ${ }^{2713}$ |  |  |
| 22 | 9092 | 9402 | 9736 | ${ }^{*} 0094$ | 0476 | 0881 0888 088 | 1309 1316 | 1758 1766 | 2230 2238 | 2722 2730 |  |  |
| 23 | 9097 | 9407 | 9741 | *oror | 0483 |  | 1316 |  |  | ${ }^{2730}$ | 10 <br> 20 | 1.0 |
| 24 | 9102 | 9413 | 9747 | ${ }^{*}$ crio7 | -489 | 0895 | 1323 | 1774 | 2246 | ${ }^{2739}$ | 30 | 3.0 |
| 25 | 9107 | 9488 | 9753 | $*$ $*$ $*$ $*$ O11 | ${ }^{0} \mathbf{0} 496$ | O902 0909 | 1330 1338 138 | 1781 1789 | 2254 2262 | 2747 2755 | 40 | 4.0 |
|  | 9112 | 9423 | 9759 |  | ${ }^{\circ} 503$ | 0909 |  | 1789 |  | 2755 | 50 | 5.0 |
| 27 | 9157 | 9429 | 9765 | ${ }^{*} 0125$ | 0509 | 0916 | 1345 | 1797 | 2270 | 2764 | 60 | 6.0 |
| 28 | 9122 | 9434 | 9770 9776 | + ${ }_{*}^{*} \mathbf{0 1 3 2}$ | 0516 <br> 0522 <br> 05 | 0923 093 093 | $\begin{array}{r}1352 \\ 1360 \\ \hline\end{array}$ | 1805 1812 | 2278 <br> 2286 | 2772 2781 2788 |  |  |
| 30 |  |  |  |  |  |  |  |  |  |  | 7 |  |
|  | 9132 | 9445 | 9782 | *O144 | 0529 | 0937 | 1367 | 182 | 2294 | 2789 |  |  |
| 3 3 | 9 O 37 | 9450 | 9788 |  | ${ }^{0} 536$ | 0944 | 1374 | 1828 | 2302 |  |  |  |
| 32 33 3 | 9142 9147 | 9456 946 I | 9794 9800 | ${ }_{*}^{*}{ }^{0156} 5$ | 0542 0.549 0 | 0951 0958 0 | 1382 1389 | 1885 <br> 1843 | 2310 2318 | 2806 2814 | 10 | 1.2 |
| 33 | 9147 | 946I | 9800 | *0163 | 0549 | 0958 | 1389 |  | 2318 |  | 20 | 2.3 |
| 34 | 9152 | 9467 | 9806 | ${ }_{*}^{*}{ }_{*}^{*} 0169$ | 0555 | ${ }^{0965}$ | 1397 | 185 | 2326 | 2823 | 30 | 3.5 |
| 35 | 9357 | 9472 | 9812 9817 | *0175 | ${ }^{0} 9562$ | 0972 0979 | 1404 1411 | 1858 1866 | 2334 2343 | 2833 2840 | 40 | 4.7 |
| 36 | 9162 | 9477 | 9817 | ${ }^{* O 181}$ | ${ }^{\circ} 569$ | 0979 | 1411 |  | 2343 | 2840 | 50 | 5.8 |
| 37 | 9167 | 9483 | 9823 | ${ }_{*}^{*} 0187$ | $\bigcirc 575$ | 0986 | 1419 | 1874 | 2351 | 2848 | 60 | 7.0 |
| 38 39 | 9172 9177 | 9488 9494 | 98829 9855 | ${ }_{*}^{*}{ }_{*}^{*} \mathbf{0 2 0 0}$ | 0582 0588 0 | 0993 1000 | $\begin{array}{r}1426 \\ 1434 \\ \hline\end{array}$ | 1882 1889 | 2359 2367 | 2857 2865 |  |  |
| 40 | 9182 | 9499 | 9841 | *0206 | 0595 | 1007 | 1441 | 1897 | 2375 | 2874 | 8 |  |
| 41 | 9887 | 9505 | 9847 | *0212 | 0602 | 1014 | 1448 | 1905 | 2383 | 28822897 |  |  |
| 42 | 9192 | 9510 | 9853 | ${ }^{*} 02219$ | 0608 | 1021 | 1456 | 1913 | ${ }^{2391}$ |  |  |  |
| 43 | 9197 | 9516 | 9859 | *0225 | 0615 | 1028 | 1463 | 1920 | 2400 | 2899 | 10 | 1.3 2.7 |
| 44 | 9202 | 9523 | 9865 | ${ }^{*} 023 \mathrm{3I}$ | 0622 | 1035 | ${ }^{1471}$ | 1928 | 2408 | 2908 | 30 | 4.0 |
| 45 | 9207 | 9527 | 9871 | ${ }^{*} 0238$ | 0629 | 1042 | 1479 | 1936 | 2416 | 2916 | 40 | $5 \cdot 3$ |
| 46 | 9213 | 9533 | 9876 | *0244 | ${ }^{\circ} 665$ | 1050 | 1486 | 1944 | 2424 | 2925 | 50 | 6.7 |
| 47 | 92.8 | 9538 | 9882 | *0250 | ${ }^{0} 6{ }_{42}$ | 1057 | 1494 | 1952 | 2432 | 2933 | 60 | 8.0 |
| 48 | 9223 | 9544 | 9888 | ${ }^{*} 0256$ | ${ }^{\circ} 649$ | 1064 | 1501 | 1959 | ${ }^{2441}$ | 2942 |  |  |
| 49 | 9228 | 9549 | 9894 | ${ }^{*} 0263$ | 0655 | 1075 | 1509 | 1967 | 2449 | 2950 |  |  |
| 60 | 9233 | 9555 | 9900 | *0269 | 0662 | 1078 | 1516 | 1975 | 2457 | 2959 | 9 |  |
| 5 sr | 9238 | 9561 | 9906 | ${ }^{*} 0275$ |  | 1085 | 1524 | 1983 | 2465 | 2968 |  |  |
| 52 | 9243 | 9566 | 9912 | ${ }_{*}^{* 0282}$ | ${ }^{0676}$ | 1082 1099 | $\begin{array}{r}1531 \\ 1539 \\ \hline\end{array}$ | 1991 1999 | 2473 2482 | 2976 2985 | 10 | 1.5 |
| 53 | 9249 | 9572 | 9918 | *0288 | 0682 | 1099 | 1539 | 1999 | 2482 | 2985 | 20 | 3.0 |
| 54 | 9254 | 9577 | 9924 | ${ }_{*}^{*} 0295$ | 0689 | 1706 | 1546 | 2007 | 2490 | 2903 | 30 | 4.5 |
| 55 | 9259 | ${ }_{958}^{958}$ | 9936 | ${ }_{*}^{*} 0301$ | ${ }^{0696}$ | 1113 | 1554 | 2014 2022 | 2498 | 3002 | $4{ }^{4}$ | 6.0 |
| 56 | 9264 | 9589 | 9936 | *0307 | $\bigcirc 703$ | 1121 | 1561 | 2022 | 2506 | 3015 | 50 | 7.5 |
| 57 | 9269 | 9594 | 9942 | ${ }^{*} 0314$ |  | 1128 | ${ }^{1569}$ | 2030 | 2514 | 3019 |  | 9.0 |
| 58 | 9275 | 9600 | 9948 | ${ }^{*} \mathbf{0 3 2 0}$ | 0716 | 1135 | 1576 | 2038 | 2523 | 3028 |  |  |
| 59 | 9280 | 9605 | 9954 | *0327 | 0723 | 1142 | 1584 | 2046 | 2531 | 3036 |  |  |
| 80 | 9285 | 9673 | 9960 | ${ }^{*} 0333$ | -730 | 1749 | ${ }^{1591}$ | 2054 | 2539 | 3045 |  |  |

Table 10.
LOGARITHMS OF MERIDIAN RADIUS OF CURVATURE $\rho_{m}$ IN ENGLISH FEET.
[Derivation of table explained on p . xlv.]

[Derivation of table explained on p . xlv.]


Table 10.
LOGARITHMS OF MERIDIAN RADIUS OF CURVATURE $\rho_{m}$ IN ENGLISH
FEET.
[Derivation of table explained on p. xlv.]


## Table 10.

LOGARITHMS OF MERIDIAN RADIUS OF CURVATURE $\rho_{m}$ IN ENGLISH
FEET.
[Derivation of table explained on p. xlv.]


Smithsonian Tables.
[Derivation of table explained on $p$. xlv.]

| Lat. | $6 \mathrm{I}^{\circ}$ | $62^{\circ}$ | $63^{\circ}$ | $64^{\circ}$ | $65^{\circ}$ | $66^{\circ}$ | $67^{\circ}$ | $68^{\circ}$ | $69^{\circ}$ | . $70^{\circ}$ | P. P. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 7.321 | 11 |  |
|  | 1197 | 1845 | 2479 | 3097 | 3698 | 4282 | 4848 | 5396 | 5924 | 6432 |  |  |
| I | 1208 | ${ }^{1856}$ | 2489 | 3107 | 3708 | 4292 | 4857 | 5405 | 5933 | 6440 |  |  |
| 2 | 1219 | 1866 | 2500 | 3117 | 3718 | 4301 | 4867 | 5414 | 5941 | 6448 |  |  |
| 3 | 1230 | 1877 | 2510 | 3227 | 3728 | 4315 | 4876 | 5423 | 5950 | 6457 |  |  |
| 4 | 1241 | 1888 | 2521 | 3137 | 3738 | 4320 | 4885 | 5432 | 5958 | 6465 |  |  |
| 5 | ${ }^{1251}$ | 1898 | 2531 | 3147 | 3747 | 4330 | 4894 | 5440 | 5967 | 6473 |  |  |
|  | 5262 | 1909 | 2541 | 3158 | 3757 | 4340 | 4904 | 5449 | 5976 | 6481 |  |  |
| 7 | 1273 | 1920 | 2552 | 3168 | 3767 | 4349 | 4913 | 5458 | 5984 | 6488 | $\begin{array}{r} 10 \\ 20 \\ 30 \\ 40 \\ 50 \\ 60 \end{array}$ | $\begin{array}{r} 2.8 \\ 3.7 \\ 5.5 \\ 7.3 \\ 9.2 \\ 91.2 \end{array}$ |
| 8 | 1284 | 193 x | 2562 | 31788 | 3777 3787 | 4359 | 4922 | 5467 | 5993 | 6498 |  |  |
| 9 | 1295 | $194 \times$ | 2573 |  | 3787 | 4368 | 4932 |  | 6001 |  |  |  |
| 10 | 1306 | 1952 | 2583 | 3198 | 3797 | 4378 | $494{ }^{1}$ | 5485 | 6010 | 6514 |  |  |
| 12 | 1317 1328 138 | 1963 1973 | 2593 2604 | 3208 3218 | 3807 3817 | 4388 | $\begin{aligned} & 4950 \\ & 4959 \\ & 4969 \end{aligned}$ | $\begin{aligned} & 5494 \\ & 5503 \\ & 5512 \end{aligned}$ | $\begin{aligned} & 6018 \\ & 6027 \\ & 6035 \end{aligned}$ | $\begin{aligned} & 6522 \\ & 6530 \\ & 6539 \end{aligned}$ |  |  |
| 12 13 13 | 1328 1338 138 | 1973 1984 198 | 2604 2614 | 3218 3228 | 3817 3882 38 | 4397 4406 |  |  |  |  |  |  |
| 14 | ${ }^{1349}$ | $\underline{\mathrm{r}} 904$ | 2625 | 3238 | 3836 | $44 \times 6$ | 4978 | 5521 | 6044 | 6547 | 10 |  |
| 15 | 8360 | 2005 | 2635 | 3248 | 3846 | 4425 | 4987 | 5529 | 6052 | 6555 |  |  |  |
| 16 | ${ }^{1375}$ | 2016 | 2645 | 3259 | 3856 | 4435 | 4996 | 5538 | 6061 | 6563 |  |  |  |
| 17 | ${ }^{1382}$ | 2026 | 2656 | 3269 | 3866 | 4444 | 5005 | 5547 | 6069 | 6571 |  |  |  |
| 18 | 1392 | 2037 | 2666 | 3279 | 3875 | 4454 | 5015 | 5556 | 6078 | 6580 |  |  |  |
| 19 | 1403 | 2047 | 2677 | 3289 | 3885 | 4463 | 5024 | 5565 | 6086 | 6588 |  |  |  |
| 20 | 1414 | 2058 | 2687 | 3299 | 3895 | 4473 | 5033 | 5574 | 6095 | 6596 | $\begin{aligned} & 10 \\ & 20 \\ & 30 \\ & 40 \\ & 70 \\ & 60 \end{aligned}$ | $\begin{array}{r}1.7 \\ 3.3 \\ 5.0 \\ 6.7 \\ 8.3 \\ 10.0 \\ \hline\end{array}$ |
| 21 | 1425 | 2069 | 2697 | 3309 | 3905 | $44^{82}$ | 5042 | 558, | 6103 | 6604 |  |  |
| 22 | 1436 | 2079 | 2708 | 3319 | 3915 | 4492 | 5051 | 5592 | 6112 | 6612 |  |  |
| 23 | 1447 | 2090 | 27.8 | 3329 | 3924 | 4501 | 5060 | 5600 | 6120 | 6621 |  |  |
| 24 | 1458 | 2100 | 2728 | 3339 | 3934 | 4511 | 5069 | 5609 | $6 \times 29$ | 6629 |  |  |
| 25 | 1468 | 2111 | 2738 | 3349 | 3944 | 4520 | 5078 | 5618 | ${ }^{6137}$ | 6637 |  |  |
| 26 | 1479 | 2122 | 2749 | 3360 | 3954 | 4530 | 5088 | 5627 | 6846 | 6645 |  |  |
| 27 | 1490 | 2132 | 2759 | 3370 | 3964 | 4539 | 5097 | 5636 | 6154 | 6653 | 0 |  |
| 28 | 1501 | 2143 | 2769 | 3380 | 3973 | 4549 | 5106 | 5644 | 6163 | 6662 |  |  |  |
| 29 | 1512 | $2 \times 53$ | 2780 | 3390 | 3983 | 4558 | 5115 | 5653 | 6171 | 6670 |  |  |  |
| 30 | 1523 | 2164 | 2790 | 3400 | 3993 | 4568 | 5124 | 5662 | 6 6 80 | 6678 |  |  |  |
| 31 32 3 | 1534 <br> 1545 | 2175 2885 |  | 3410 3420 | 4003 4012 | 4577 4587 | 5133 <br> 5142 <br> 1 | ${ }_{5681}^{5671}$ | 6188 6197 | 6686 <br> 6694 6702 |  |  |  |
| 32 33 3 | 1545 1555 | 2185 2196 | 2812 2828 2831 | 3420 3430 | 4012 | 4587 4596 | 5152 | 5680 5688 | 6197 6205 |  |  |  |  |
| 34 | 1566 | 2206 | 2831 | 3440 | 4032 | 4606 | 5160 | 5697 | 6214 | 6710 | 10 | 1.53.0 |
| 35 | 1577 | 2217 | 2841 | 3450 | 4041 | 4615 | 5169 | 5706 | 6222 | 6718 | 20 |  |
| 36 | 1588 | 2228 | 2852 | 3460 | 4051 | 4624 | 5179 | 5715 | 6230 | 6727 | 3040 | 4.5 |
| 37 | 1599 | 2238 | 2862 | 3470 | 4061 | 4634 | 5188 |  |  |  |  |  |
| 38 | 1609 | 2249 | 2872 | 3480 | 4071 | 4643 | 5197 | 5732 | 6247 | 6743 | 5060 | 7.59.0 |
| 39 | 1620 | 2259 | 2883 | 3490 | 4080 | 4653 | 5206 | 574r | 6256 | 6751 |  |  |
| 40 | 1631 | 2270 | 2893 | 3500 | 4090 | 4662 | 5215 | 5750 | 6264 | 6759 | 8 |  |
| $4 x$ 42 4 | 1642 1652 162 | 2280 2291 | 2903 2983 | 3510 3520 | 4100 | 4675 | 5224 | 5759 | ${ }_{6}^{6272}$ | 6767 |  |  |  |
| 42 43 | 1642 1663 | 2291 2301 | 2913 <br> 2924 | 3520 3530 | 4109 4119 | 4685 | 5233 <br> 5242 | 5767 | $6_{6281}$ | 6775 |  |  |  |
| 44 |  |  |  |  |  | 4690 | 5242 | $577^{6}$ | 6289 | 678 |  |  |  |
| 45 | 1674 1684 | 2312 2322 | 2934 | 3540 | $4 \times 28$ | 4699 | 5251 | 5785 | 6298 | $\begin{aligned} & 6791 \\ & 6799 \\ & 6807 \end{aligned}$ |  |  |  |
| 46 | $\begin{array}{r}1684 \\ \\ \hline 695\end{array}$ | 2323 233 | 2944 | 3549 3559 | 4138 4148 | 4708 4718 | 5260 5270 | 5793 5802 | 6306 |  |  |  |  |
| 47 | 1706 | 2343 | 2964 | 3569 | 4157 | 4727 | 5279 | 58 II | 6323 |  | $\begin{aligned} & 10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \end{aligned}$ | $\begin{aligned} & 1.3 \\ & 2.6 \\ & 4.0 \\ & 5.3 \\ & 6.7 \\ & 8.0 \end{aligned}$ |
| 48 | 1717 | 2354 | 2975 | 3579 | 4167 | 4736 | 5288 | 5820 | 6331 | ${ }_{6823}$ |  |  |
| 49 | 1727 | 2364 | 2985 | 3589 | 4176 | 4746 | 5297 | 5828 | 6340 | 6831 |  |  |
| 50 | ${ }^{1738}$ | 2375 | 2995 | 3599 | 4186 | 4755 | 5306 | 5837 | 6348 | 6839 |  |  |
| 51 | 1749 | ${ }^{2385}$ | 3005 | 3609 | 4196 | 4764 | 5315 | 5846 | 6356 | $\begin{aligned} & 6847 \\ & 6855 \\ & 6863 \end{aligned}$ |  |  |
| 52 | 1759 | 2396 | 3015 | ${ }^{6619}$ | 4205 | 4774 | 5324 | 5854 | 6365 |  |  |  |
| 53 | 1770 | 2406 | 3026 | 3629 | 4215 | 4783 | 5333 | 5863 | 6373 |  |  |  |
| 54 | ${ }^{1788}$ | 2417 | 3036 | 3639 | 4224 | 4792 | 5342 | 5872 | 6382 | 6871 |  |  |
| 55 | 1791 1802 1820 | 2427 2437 | 3046 3056 | 3648 3658 | 4234 | 4801 | 5351 | 5880 | 6390 | 6879 |  |  |
| 56 | 1802 | 2437 | 3056 | 3658 | 4244 | $4^{811}$ | 5360 | 5889 | 6398 | 6887 |  |  |
| 57 | 1813 18 | 2448 | 3066 | 3668 |  | 4820 |  | 5898 |  |  |  |  |
| 58 | 1824 1834 | 2458 | 3077 | 3678 | 4263 | 4829 | 5378 | 5007 | 6445 | $6 \mathrm{6og} 3$ |  |  |
| 59 | 1834 | 2469 | 3087 | 3688 | 4272 | 4839 | 5387 | 5915 | 6424 | 6911 |  |  |
| 60 | ${ }^{2845}$ | 2479 | 3007 | 3698 | 4282 | 4848 | 5396 | 5924 | 6432 | 6919 |  |  |

[Derivation of table explained on p. xlv.]

| Lat. | $71^{\circ}$ | $72^{\circ}$ | $73^{\circ}$ | $74^{\circ}$ | $75^{\circ}$ | $76^{\circ}$ | $77^{\circ}$ | $78^{\circ}$ | $79^{\circ}$ | $80^{\circ}$ | P. P. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\prime}$ | $\begin{gathered} 7.321 \\ 6919 \end{gathered}$ | $\begin{gathered} 7.321 \\ 73^{85} \end{gathered}$ | $\begin{gathered} 7.321 \\ 7829 \end{gathered}$ | $\begin{gathered} 7.321 \\ 8251 \end{gathered}$ | $\begin{gathered} 7.321 \\ 8650 \end{gathered}$ | $\begin{gathered} 7.321 \\ 9025 \end{gathered}$ | $\begin{gathered} 7.321 \\ 9377 \end{gathered}$ | $\begin{gathered} 7.321 \\ 9704 \end{gathered}$ | 7.322 <br> 0007 | $\begin{gathered} 7.322 \\ 0284 \end{gathered}$ |  |  |
| 3 | $\begin{aligned} & 6927 \\ & 6935 \\ & 6943 \end{aligned}$ | $\begin{aligned} & 7392 \\ & 7400 \\ & 7407 \end{aligned}$ | 788 7836 7843 7855 785 | $\begin{aligned} & 8258 \\ & 8265 \\ & 827 x \end{aligned}$ | $\begin{aligned} & 8656 \\ & 8663 \\ & 8669 \end{aligned}$ | $\begin{aligned} & 903 x \\ & 9037 \\ & 9043 \end{aligned}$ | $\begin{aligned} & 9388 \\ & 9388 \\ & 9394 \end{aligned}$ | $\begin{aligned} & 9709 \\ & 9714 \\ & 9720 \end{aligned}$ | 0012 <br> 0017 <br> 0021 | $\begin{aligned} & 0288 \\ & 0293 \\ & 0297 \end{aligned}$ |  |  |
| $\begin{aligned} & 5 \\ & 6 \end{aligned}$ | $\begin{aligned} & 6955 \\ & 6958 \\ & 6966 \end{aligned}$ | $\begin{aligned} & 7415 \\ & 7422 \\ & 7430 \end{aligned}$ | $\begin{aligned} & 7858 \\ & 7865 \\ & 7872 \end{aligned}$ | $\begin{aligned} & 8278 \\ & 8285 \\ & 8292 \end{aligned}$ | $\begin{aligned} & 8676 \\ & 8682 \\ & 8688 \end{aligned}$ | $\begin{aligned} & 9049 \\ & 9055 \\ & 9061 \end{aligned}$ | $\begin{aligned} & 9399 \\ & 9405 \\ & 9411 \end{aligned}$ | $\begin{aligned} & 9725 \\ & 9730 \\ & 9735 \end{aligned}$ | 0026 <br> 0031 <br> 0036 | 0302 0306 0310 | 10 | 1.3 2.6 |
| 7 8 9 | $\begin{aligned} & 6974 \\ & 6982 \\ & 6990 \\ & \hline \end{aligned}$ | $\begin{aligned} & 7437 \\ & 7445 \\ & 7452 \end{aligned}$ | $\begin{aligned} & 7879 \\ & 7^{887} \\ & 7^{8994} \end{aligned}$ | $\begin{aligned} & 8299 \\ & 8395 \\ & 8312 \end{aligned}$ | $\begin{aligned} & 8695 \\ & 8751 \\ & 8708 \end{aligned}$ | $\begin{aligned} & 9067 \\ & 9073 \\ & 9079 \end{aligned}$ | $\begin{aligned} & 9416 \\ & 9422 \\ & \hline 0427 \end{aligned}$ | $\begin{aligned} & 9740 \\ & 9746 \\ & 9755^{2} \end{aligned}$ | $\begin{aligned} & 004 \mathrm{I} \\ & 0045 \\ & 0050 \end{aligned}$ | $\begin{aligned} & 0315 \\ & 0319 \\ & 0324 \\ & 0324 \end{aligned}$ | 30 40 50 60 | 4.0 5.3 6.7 8.0 |
| 10 | 6998 | 7460 | 7908 | 8319 | 8714 | 9085 | 9433 | 9756 | 0055 | ${ }^{0} 328$ | 7 |  |
| 11 12 13 | $\begin{aligned} & 7006 \\ & 7014 \\ & 7021 \end{aligned}$ | 7467 7475 7482 | 7908 7915 7922 | 8326 8332 8339 839 | 8720 8727 8733 | 9091 9097 9003 | 9438 9444 9449 | 976 I 9766 977 | 0060 <br> 0064 <br> 0069 | $\begin{aligned} & \mathbf{0 3 3 2} \\ & 0337 \\ & 0341 \end{aligned}$ |  |  |
| 14 | 7029 7037 | 7490 7497 | 7929 7936 | 8346 8353 8 | 8739 8745 | 9109 9115 | 9455 9460 | 9776 978 I | 0074 0078 008 | 0345 0349 |  |  |
| 16 | 7045 | 7505 | 7944 | 8359 | 8752 | 9121 | 9466 | 9787 | ${ }^{0083}$ | 0354 | 10 |  |
| 17 18 19 | 7053 <br> 7000 <br> 7068 | $\begin{aligned} & 7512 \\ & 7520 \\ & 7527 \end{aligned}$ | $\begin{aligned} & 7951 \\ & 7958 \\ & 7965 \end{aligned}$ | $\begin{aligned} & 8366 \\ & 8373 \\ & 8370 \end{aligned}$ | 8758 8764 8771 877 | $\begin{aligned} & 9 \times 27 \\ & 9133 \\ & 9 \times 39 \end{aligned}$ | $\begin{aligned} & 9471 \\ & 9477 \\ & 9482 \end{aligned}$ | 9792 9797 9802 | $\begin{aligned} & 0088 \\ & 0093 \\ & 0097 \end{aligned}$ | $\begin{aligned} & 0358 \\ & 0362 \\ & 036 \\ & 0367 \end{aligned}$ | 20 30 40 40 | 1.2 2.3 3.5 4.7 |
| 20 | 7076 | 7535 | 7972 | 8386 | 8777 | 9145 | 9488 | 9807 | 0102 | 0371 | 5 | 7.0 |
| 21 22 23 | 7084 7092 7099 | 7542 7550 7557 | 7979 7988 7993 | 8393 8399 8406 | 8783 8790 8796 | 9151 9157 9653 | 9493 9499 9594 | 9812 9887 9822 | Or07 O11 O16 | $\begin{aligned} & 0375 \\ & 0379 \\ & 037 \\ & 0384 \end{aligned}$ | 6 |  |
| 24 25 26 | 7107 7115 7123 | 7565 7572 7580 | 8000 8007 8014 | 8813 8413 8419 8426 | 8802 8808 8815 | 9169 9174 988 9880 | 9510 9515 9521 | $\begin{aligned} & 9827 \\ & 9832 \\ & 9838 \end{aligned}$ | $\begin{aligned} & 0120 \\ & 0125 \\ & 0130 \\ & 0130 \end{aligned}$ | ${ }_{0} 088$ <br> 0392 ${ }^{-396}$ |  |  |
| $\begin{aligned} & 27 \\ & 28 \\ & 29 \end{aligned}$ | $\begin{aligned} & 7131 \\ & 7188 \\ & 7146 \\ & \hline \end{aligned}$ | 7587 <br> 7595 <br> 7602 | 8021 <br> 8028 <br> 8035 | $\begin{aligned} & 8433 \\ & 8440 \\ & 8446 \end{aligned}$ | $\begin{aligned} & 882 \mathrm{x} \\ & 8827 \\ & 8834 \\ & \hline \end{aligned}$ | $\begin{aligned} & 9186 \\ & 9192 \\ & 9198 \\ & \hline \end{aligned}$ | $\begin{aligned} & 9526 \\ & 9532 \\ & 9537 \\ & \hline \end{aligned}$ | 9843 <br> 9848 <br> 9853 <br> 9853 | $\begin{array}{r} 0134 \\ \text { O1 } 39 \\ \text { O143 } \\ \hline \end{array}$ | $\begin{aligned} & 0400 \\ & 0405 \\ & 0499 \\ & \hline \end{aligned}$ | 10 20 30 | 1.0 2.0 3.0 |
| 30 | 7154 | 7610 | 8042 | 8453 | 8840 | 9204 | 9543 | 9858 | 0148 | 0413 | 40 50 | 4.0 5.0 |
| 31 32 33 | 7162 7170 7177 | 7617 765 7632 | 8049 8056 8063 | 8460 8466 8473 | 8846 8852 8859 | 9210 9216 $\mathbf{9 2 2 1}$ | 9548 9554 9559 | 9863 9868 9873 | $\begin{array}{r} 0153 \\ 0157 \\ 0150 \end{array}$ | $\begin{aligned} & 0417 \\ & 042! \end{aligned}$ | 60 | 6.0 |
| 33 34 34 35 36 | 7177 7185 7193 7201 | 7632 7639 7646 7654 | 8063 8070 8077 8084 | 8473 8479 8489 8493 | $\begin{aligned} & 8859 \\ & 8865 \\ & 887 \mathrm{I} \\ & 8877 \end{aligned}$ | $\mathbf{9 2 2 1}$ $\mathbf{9 2 2 7}$ $\mathbf{9 2 3 3}$ $\mathbf{9 2 3 9}$ | 9559 9565 9550 9575 | 9873 9878 9888 9888 988 | 0162 0166 0168 O17 O17 | 0426 0430 0434 0438 | 5 |  |
| $\begin{aligned} & 37 \\ & 38 \\ & 39 \end{aligned}$ | 7209 <br> 7216 <br> 7224 | 7661 7668 7676 | $\begin{aligned} & 809 \mathrm{x} \\ & 8098 \\ & 8 \times 05 \\ & \hline \end{aligned}$ | 8499 8506 8512 | $\begin{aligned} & 8883 \\ & 8890 \\ & 8896 \end{aligned}$ | $\begin{aligned} & 9245 \\ & 9250 \\ & 9256 \end{aligned}$ | $\begin{aligned} & 958 \mathrm{I} \\ & 9566 \\ & 9592 \end{aligned}$ | $\begin{aligned} & 9893 \\ & 9898 \\ & 9903 \end{aligned}$ | or8o 0185 or89 | 0442 <br> 0447 <br> 0451 <br> 045 | то | . 8 |
| 40 | 7232 | 7683 | 8112 | 8519 | 8902 | 9262 | 9597 | 9908 | 0194 | 0455 | 30 | 2.5 |
| 41 42 43 | 7240 7247 7255 | 7690 7798 7705 | 8112 8119 8126 8133 | 8526 8532 8539 | 8908 8984 8921 | $\mathbf{9 2 6 8}$ $\mathbf{9 2 7 4}$ $\mathbf{9 2 7 9}$ | $\begin{aligned} & 9602 \\ & 9608 \\ & 96 \times 3 \end{aligned}$ | 9913 9918 9988 9923 | 0199 0203 0208 0208 | 0459 0459 0463 0467 | 50 | 4.2 5.0 |
| $\begin{aligned} & 44 \\ & 45 \\ & 46 \end{aligned}$ | 7263 7270 7278 | 7712 7719 7727 | 818 8140 8147 8154 8154 8 | 8545 8552 855 8559 | $\begin{aligned} & 8927 \\ & 8933 \\ & 8939 \end{aligned}$ | 9285 9291 9297 98 | $\begin{aligned} & 9619 \\ & 9624 \\ & 9629 \end{aligned}$ | $\begin{aligned} & 9928 \\ & 9933 \\ & 9938 \end{aligned}$ | 0212 <br> 0217 <br> 0222 | $\begin{aligned} & 0471 \\ & 0475 \\ & 0480 \end{aligned}$ |  |  |
| $\begin{aligned} & 47 \\ & 48 \\ & 49 \end{aligned}$ | 7286 7224 7301 | $\begin{array}{r} 7734 \\ 7741 \\ .7749 \end{array}$ | $\begin{aligned} & 8161 \\ & 8168 \\ & 8175 \end{aligned}$ | $\begin{aligned} & 8565 \\ & 8572 \\ & 8578 \end{aligned}$ | $\begin{aligned} & 8945 \\ & 8952 \\ & 8958 \end{aligned}$ | 9303 9308 9354 | $\begin{aligned} & 9635 \\ & 9640 \\ & 9646 \end{aligned}$ | 9943 9948 9953 | $\begin{aligned} & 0226 \\ & 023 \mathrm{I} \\ & 0235 \\ & \hline 02 \end{aligned}$ | $\begin{aligned} & 0484 \\ & 0488 \\ & 0492 \end{aligned}$ | 4 |  |
| 50 | 7309 | 7756 | 8182 | 8585 | 8964 | 9320 | 9651 | 9958 | 0240 | 0496 | $\begin{aligned} & 10 \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \end{aligned}$ | .7 r. r |
| $\begin{aligned} & 51 \\ & 52 \\ & 53 \end{aligned}$ | 7317 7324 7332 | 7763 7771 7778 | $\begin{aligned} & 8189 \\ & 8196 \\ & 8203 \end{aligned}$ | 8591 8598 8604 | 8970 8976 8982 | 9326 9331 9337 | $\begin{aligned} & 9656 \\ & 9662 \\ & 9667 \end{aligned}$ | 9963 9968 9973 | 0244 024 0253 | $\begin{aligned} & 0500 \\ & 0504 \\ & 0508 \end{aligned}$ |  | 2.0 2.7 3.3 4.0 |
| 54 55 56 | $\begin{aligned} & 7339 \\ & 7347 \\ & 7355 \end{aligned}$ | 7785 7792 7800 | 8210 8216 8223 | $\begin{aligned} & 8611 \\ & 8617 \\ & 8624 \end{aligned}$ | 8988 <br> 8994 <br> 9001 | $\begin{aligned} & 9343 \\ & 9348 \\ & 9354 \end{aligned}$ | $\begin{aligned} & 9672 \\ & 9677 \\ & 9683 \end{aligned}$ | $\begin{aligned} & 9978 \\ & 9982 \\ & 9987 \end{aligned}$ | 0258 <br> 0262 <br> 0266 | $\begin{aligned} & 0512 \\ & 0516 \\ & 0520 \end{aligned}$ |  |  |
| $\begin{aligned} & 57 \\ & 58 \\ & 59 \end{aligned}$ | $\begin{aligned} & 7362 \\ & 7370 \\ & 7377 \\ & \hline \end{aligned}$ | $\begin{aligned} & 7807 \\ & 7_{814}^{814} \\ & 7822 \end{aligned}$ | $\begin{aligned} & 8230 \\ & 8237 \\ & 8244 \end{aligned}$ | $\begin{aligned} & 8630 \\ & 8637 \\ & 8643 \end{aligned}$ | $\begin{aligned} & 9007 \\ & 9013 \\ & \text { gorg } \end{aligned}$ | $\begin{aligned} & 9360 \\ & 9366 \\ & 937 \mathrm{I} \end{aligned}$ | $\begin{aligned} & 9688 \\ & 9693 \\ & 9699 \\ & \hline \end{aligned}$ | $\begin{array}{r} 9992 \\ \hline 9997 \\ { }^{990002} \\ \hline \end{array}$ | $\begin{aligned} & 027 x \\ & 0275 \\ & 0280 \end{aligned}$ | $\begin{array}{r} 0524 \\ 0528 \\ 0532 \end{array}$ |  |  |
| 60 | 7385 | 7829 | 8251 | 8650 | G025 | 9377 | 9704 | *0007 | 0284 | 0536 |  |  |

[Derivation of table explained on p. xlv.]

[Derivation of table explained on p. xlv.]


Table 11.
LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION
$\rho_{n}$ IN ENGLISH FEET.
[Derivation of table explained on p. xlv.]


Table 11 .
LOCARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION $\rho_{n}$ IN ENCLISH FEET.
[Derivation of table explained on $p$. xlv.]


Table 11.
LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION $\rho_{n}$ IN ENGLISH FEET.
[Derivation of table explained on p . xlv.]


Table 11 ,
LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION
$\rho_{n}$ IN ENGLISH FEET.
[Derivation of table explained on p. xlv.]


Table 11.
LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION $\rho_{n}$ IN ENGLISH FEET.
[Derivation of table explained on p. xlv.]


## LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION

 $\rho_{n}$ IN ENGLISH FEET.[Derivation of table explained on p . xlv.]


Table 11.
LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION $\rho_{n}$ IN ENGLISH FEET.
[Derivation of table explained on $p$. xlv.]


LOGARITHMS OF RADIUS OF CURVATURE OF NORMAL SECTION
$\rho_{n}$ IN ENCLISH FEET.
[Derivation of table explained on p. xlv.]


Table 12.
LOGARITHMS OF RADIUS OF CURVATURE $\rho_{a}$ (IN METRES) OF SECTION OF EARTH'S SURFACE INCLINED TO MERIDIAN AT AZIMUTH $a$.
[Formula for pa given on p. xlv.]

| Azimuth. | LATITUDE. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $22^{\circ}$ | $23^{\circ}$ | $24^{\circ}$ | $25^{\circ}$ | $26^{\circ}$ | $27^{\circ}$ | $28^{\circ}$ | $29^{\circ}$ | $30^{\circ}$ | $31^{\circ}$ |
| $0^{\circ}$ | 6.80237 | 6.80242 | 6.80248 | 6.80254 | 6.80260 | 6.80266 | 6.80272 | 6.80279 | 6.80285 | 6.80292 |
| 5 | 239 | 244 | 250 | 256 | 262 | 268 | 274 | 280 | 287 | 294 |
| 10 | 244 | 250 | 255 | 261 | 267 | 273 | 279 | 285 | 292 | 298 |
| 15 | 254 | 259 | 264 | 270 | 276 | 282 | 288 | 294 | 300 | 306 |
| 20 | 266 | 271 | 277 | 282 | 288 | 293 | 299 | 305 | 311 | 317 |
| 25 | 282 | 287 | 292 | 297 | 302 | 308 | 313 | 319 | 325 | 331 |
| 30 | 300 | 305 | 309 | 314 | 319 | 324 | 330 | 335 | 340 | 346 |
| 35 | 320 | 324 | 329 | 333 | 338 | 343 | 348 | 353 | 358 | 363 |
| 40 | 341 | 345 | 350 | 354 | 358 | 362 | 367 | 372 | 377 | 382 |
| 45 | 364 | 367 | 371 | 375 | 379 | 383 | 387 | 391 | 396 | 400 |
| 50 | 386 | 389 | 392 | 396 | 399 | 403 | 407 | 411 | 415 | 419 |
| 55 | 407 | 410 | 413 | 416 | 420 | 423 | 426 | 430 | 434 | 437 |
| 60 | 427 | 430 | 432 | 435 | $43^{8}$ | 442 | 445 | 448 | 451 | 455 |
| 65 | 445 | $44^{8}$ | 450 | 453 | 455 | 458 | 46 I | 464 | 467 | $47{ }^{\circ}$ |
| 70 | 46 r | 463 | 465 | 468 | 470 | 473 | 475 | 478 | 481 | 484 |
| 75 | 473 | 476 | 478 | 480 | 482 | 484 | 487 | 489 | 492 | 494 |
| 80 | 483 | 485 | 487 | 489 | 491 | 493 | 495 | 498 | 500 | 502 |
| 85 | 489 | 490 | 492 | 494 | 496 | 498 | 501 | 503 | 505 | 507 |
| 90 | 490 | 492 | 494 | 496 | 498 | 500 | 502 | 504 | 507 | 509 |
| Azimuth. | LATITUDE. |  |  |  |  |  |  |  |  |  |
|  | $32^{\circ}$ | $33^{\circ}$ | $34^{\circ}$ | $35^{\circ}$ | $36^{\circ}$ | $37^{\circ}$ | $3^{8}$ | $39^{\circ}$ | $40^{\circ}$ | $41^{\circ}$ |
| $0^{\circ}$ | 6.80299 | 6.80306 | 6.803 r 3 | 6.80320 | 6.80327 | 6.80335 | 6.80342 | 6.80350 | 6.80357 | 6.80365 |
| 5 | 300 | 307 | 314 | 322 | 329 | 336 | 344 | 351 | 359 | 366 |
| 10 | 305 | 312 | 319 | 326 | 333 | 340 | 348 | 355 | 363 | 370 |
| 15 | 313 | 320 | 326 | 333 | 340 | 348 | 355 | 362 | 369 | 376 |
| 20 | 324 | 330 | 337 | 343 | 350 | 357 | 364 | 371 | 378 | 385 |
| 25 | 337 | 343 | 349 | 355 | 362 | 368 | 375 | 382 | 388 | 395 |
| 30 | 352 | 358 | 364 | 370 | 376 | 382 | 388 | 394 | 401 | 407 |
| 35 | 369 | 374 | 380 | 385 | 391 | 397 | 402 | 408. | 414 | 420 |
| 40 | 386 | 392 | 397 | 402 | 407 | 412 | 418 | 423 | 429 | 434 |
| 45 | 405 | 410 | 414 | 419 | 424 | 429 | 434 | 439 | 444 | 449 |
| 50 | 423 | 428 | 432 | 436 | 44 I | 445 | 450 | 454 | 459 | 464 |
| 55 | 44 I | 445 | 449 | 453 | 457 | 461 | 465 | 469 | 474 | 478 |
| 60 | $45^{8}$ | 462 | 465 | 469 | 472 | 476 | 480 | 484 | 487 | 491 |
| 65 | 473 | 476 | 480 | 483 | 486 | 489 | 493 | 496 | 500 | 503 |
| 70 | 486 | 489 | 492 | 495 | 498 | 501 | 504 | 507 | 510 | 514 |
| 75 | 497 | 500 | 502 | 505 | 508 | 510 | 513 | 516 | 519 | 522 |
| 80 | 505 | 507 | 510 | 512 | 515 | 517 | 520 | 523 | 525 | 528 |
| 85 | 510 | 512 | 514 | 517 | 519 | 522 | 524 | 527 | 529 | 532 |
| 90 | 511 | 514 | 516 | 518 | 52 r | 523 | 526 | 528 | 53 r | 533 |

[Formula for $\rho_{a}$ given on p. xlv.]

| Azimuth. | LATITUDE. |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $42^{\circ}$ | $43^{\circ}$ | $44^{\circ}$ | $45^{\circ}$ | 46 | 47 | $48^{\circ}$ | $49^{\circ}$ | $50^{\circ}$ | $51^{6}$ |
| $0^{\circ}$ | 6.80373 | 6.80380 | 6.80388 6, | 6,80396 | 6.80 | 6.80 | 16.804 | 6.80426 | 6.80434 | 6.80442 |
| 5 | 374 | 382 | 389 | 397 |  |  | 24 | - 428 | 435 | 443 |
| 10 | 378 | $3^{8} 5$ | 393 | 400 |  |  | 5.4 | 430 | 438 | 445 |
| 15 | 384 | 39 I | 399 | 406 |  |  | - 4 | 435 | 442 | 450 |
| 20 | 392 | 399 | 406 | 413 |  |  | 74 | 441 | 448 | 455 |
| 25 | 402 | 408 | 415 | 422 |  |  | 6 | 2449 | 456 | 463 |
| 30 | 413 | 420 | 426 | 433 |  |  | 6 4 | 2458 | 465 | 471 |
| 35 | 426 | 432 | 438 | 444 |  |  | 6 | 2468 | 474 | 480 |
| 40 | 440 | 446 | 45 I | 457 |  |  | 8 | 4479 | 485 | 490 |
| 45 | 454 | 459. | 464 | 470 |  |  | O 4 | 490 | 495 | 500 |
| 50 | 468 | 473 | 478 | 482 |  |  | 24 | 6 501 | 506 | 510 |
| 55 | 482 | 486 | 490 | 495 |  |  | 35 | 812 | 516 | 520 |
| 60 | 495 | 499 | 502 | 506 |  |  | 45 | 522 | 526 | 530 |
| 65 | 507 | 510 | 514 | 517 |  |  | 45 | 531 | 534 | $53^{8}$ |
| 70 | 517 | 520 | 523 | 526 |  |  | 2 | 5339 | 542 | 545 |
| 75 | 525 | 528 | 530 | 534 |  |  | 95 | 545 | 548 | 551 |
| 80 | 53 I | 534 | 536 | 539 |  |  | 5 | 550 | 553 | 555 |
| 85 | 534 | 537 | 540 | 542 |  |  | 85 | 553 | 555 | 558 |
| 90 | 536 | 538 | 541 | 544 |  |  | 95 | 554 | 556 | 559 |
| Azimuth. | LATITUDE. |  |  |  |  |  |  |  |  |  |
|  | $52^{\circ}$ | $53^{\circ}$ | $54^{\circ}$ | $55^{\circ}$ |  | $56^{\circ}$ | $57^{\circ}$ | $5^{\circ}$ | $59^{\circ}$ | $60^{\circ}$ |
| $0^{\circ}$ | 6.80449 | 6.80457 | 6.80464 | 4 6.80471 |  | 6.80479 | 6.80486 | 6.80493 | 6.80500 | 6.80506 |
| 5 | 450 458 |  | 465 | $5 \quad 472$ |  | 479 | 486 | 493 | 500 | 507 |
| 10 | 453457 |  | 467 | 7 474 |  | $\begin{array}{r} 481 \\ 485 \end{array}$ | 488 | 495 | 502 | 509 |
| 15 | 457 | 464 | 471 |  |  | 492 | 498 | 505 |  |
| 20 | $462 \quad 469$ |  | 476482 | $6 \quad 483$ |  |  | 489495 | 496501 | 502 | 509 | 515 |
| 25 | 469 476 <br> 477 484 |  |  | 2 489 |  | 508 |  |  | 514 | 520 |
| 30 |  |  | 490 |  |  | 495 502 | 508 | 514 | 519 | 525 |
| 35 | 486 |  | 498 | 8 503 |  | 509 | 515 | 520 | 525 | 531537 |
| 40 | 496 501 <br> 505 510 |  | 506 | $5{ }^{512}$ |  | 517525 | 522 | 527 | 532 |  |
| 45 |  |  | 515 | 5520 |  |  | 530 | 534 | 539 | 543 |
| 50 | 515 520 <br> 524 528 <br> 533 537 |  | $\begin{aligned} & 524 \\ & 533 \\ & 541 \end{aligned}$ | $\begin{aligned} & 528 \\ & 537 \\ & 544 \end{aligned}$ |  | $\begin{aligned} & 533 \\ & 541 \\ & 548 \end{aligned}$ | $\begin{aligned} & 537 \\ & 545 \\ & 55^{2} \end{aligned}$ | $\begin{aligned} & 542 \\ & 548 \\ & 555 \end{aligned}$ | $\begin{aligned} & 546 \\ & 552 \\ & 558 \end{aligned}$ | 550556562 |
| 55 |  |  |  |  |  |  |  |  |  |  |
| 60 |  |  |  |  |  |  |  |  |  |  |
| 65 | 541 | 545 | 548 <br> 554 | 8 - 551 |  | 555 | 558 | 561 | 564 | 567 |
| 70 | 548 | 551 |  |    <br> 4 557 560 <br> 62 565  |  |  | 563568 | 566 | 569 | 572 |
| 75 | 554 | 557 | 559 |  |  |  | 570 | 573 | 575 |  |
| 80 |  | 561 | 563566566 | $\begin{aligned} & 566 \\ & 568 \\ & 569 \end{aligned}$ |  | $\begin{aligned} & 568 \\ & 570 \\ & 571 \end{aligned}$ |  | $\begin{aligned} & 571 \\ & 573 \\ & 574 \end{aligned}$ | $\begin{aligned} & 573 \\ & 575 \\ & 576 \end{aligned}$ |  | $\begin{aligned} & 578 \\ & 580 \\ & 580 \end{aligned}$ |
| 85 | 560 | 563 |  |  |  | 578 |  |  |  |  |  |
| 90 | 561 | 564 |  |  |  | 578 |  |  |  |  |  |

Table 13.
LOGARITHMS OF FACTORS $\frac{\rho^{\prime \prime}}{2 \rho_{m} \rho_{n}}$ FOR COMPUTING SPHEROIDAL
EXCESS OF TRIANGLES.

## UNIT = THE ENGLISH FOOT.

[Derivation and use of table explained on p. lviii.]

| $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 0.37498 | $20^{\circ}$ | 0.37429 | $40^{\circ}$ | $0.37255$ | $60^{\circ}$ | $0.37056$ |
| 1 | 498 | 21 | 422 | 41 | 244 | 61 | 047 |
|  | -0.02 |  | -0.12 |  | $-0.17$ | 62 | -0.15 |
| 2 | 497 | 22 | 415 | 42 | ${ }^{234}{ }^{0.17}$ | 62 | $\stackrel{03}{8}_{-0.13}$ |
| 3 | 496 | 23 | 408 | 43 | 224 | 63 | 030 |
|  | -0.02 |  | $-0.12$ |  | $\underline{-214}$ | 64 | -022.13 |
| 4 | $\underline{495}$ | 24 | $\underline{401}$ |  | -0.18 |  | $-0.13$ |
| 5 | 493 | 25 | 393 | 45 | 203 | 65 | 014 |
| 6 | -0.03 | 26 | -0.13 | 46 | -0.17 | 66 | -0.13 |
|  | $\underline{-0.03}$ |  | $\underline{-0.13}$ |  | $-0.17$ |  | -0.13 |
| 7 | 489 | 27 | 377 | 47 | 183 | 67 | 0.36998 |
| 8 | $-0.03$ | 28 | -0.15 | 8 | -0.17 | 68 | -0.12 |
| 8 | $4_{-87}$ | 28 | $\underline{-}$ |  | -0.18 |  | $-0.12$ |
| 9 | 484 | 29 | 360 | 49 | 162 | 69 | 984 |
|  | 0.07 |  | -0.15 |  |  |  |  |
| 10 | 480 -0.07 | 30 | ${ }_{-351}{ }^{\text {3 }}$ | 50 | 152 -0.17 | 70 | ${ }_{977}^{0.10}$ |
| 11 | 476 | 3 I | 342 | 51 | 142 | 71 | 971 |
| 12 | -472 | 32 | 333 | 52 |  | 72 |  |
|  | $-0.07$ |  | $\bigcirc$ |  | -0.17 |  | -0.08 |
| 13 | 468 | 33 |  | 53 |  | 73 | 959 |
| 14 | -0.08 | 34 | ${ }_{314} 0.15$ | 54 | -112 | 74 | -0.10 |
|  | -0.07 |  | -0.17 |  | -0.15 |  | -0.08 |
| 15 | 459 | 35 | 304 | 55 |  | 75 | 948 |
|  | -0.10 |  | -29.15 |  | -0.17 |  | -0.08 |
| 16 | 453. 0.08 | 36 | $\stackrel{295}{-0.17}$ | 56 | $\stackrel{093}{-0.17}$ | 76 | -943.08 |
| 17 | 448 | 37 | 285 | 57 | 083 | 77 | 938 |
| 18 | $-44^{0.10}$ | 38 | -275 ${ }_{2}$ | 58 | -0.14 | 78 | -0.0.07 |
|  | -0.10 |  | -0.17 |  | -0.15 |  | -0.07 |
| 19 | $\underline{436}_{-0.12}$ | 39 | $\begin{aligned} & 265 \\ & -0.17 \end{aligned}$ | 59 | $\begin{aligned} & 065 \\ & -0.15 \end{aligned}$ | 79 | $\xrightarrow{930}$ |
| 20 | ${ }_{-0.12}^{429}$ | 40 | ${ }^{255}$ | 60 | $\begin{aligned} & 0.56 \\ & -0.15 \end{aligned}$ | 80 | 926 |

Smithsonian Tables.

UNIT = THE METRE.
[Derivation and use of table explained on p. lviii.]

| $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. | $\phi$ | log. factor and change per minute. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | I.40695 | $20^{\circ}$ | 1.40626 | $40^{\circ}$ | I. 40452 | $60^{\circ}$ | I. 40253 |
|  | $\underline{695}$ |  | 619 0.12 |  | $-0.18$ |  | $-0.15$ |
| I | 695 <br> 6.02 | 21 | 619 | 4 I | ${ }_{\text {441 }}$ | 61 | 244 -0.15 |
| 2 | 694 | 22 | 612 -0.12 | 42 | ${ }_{\text {43I }}^{\text {- }}$ | 62 | 235 -0.13 |
| 3 | 693 | 23 | 605 | 43 | 421 | 63 | 227 |
|  | $-0.02$ |  | -0.13 |  | -0.17 |  | -0.13 |
| 4 | $\begin{gathered} 692 \\ -0.03 \end{gathered}$ | 24 | 597 -0.12 | 44 | 411 -0.18 | 64 | $\begin{aligned} & 219 \\ & -0.15 \end{aligned}$ |
| 5 | 690 | 25 | 590 | 45 | 400 | 65 | 210 |
|  | $-0.03$ |  | $-0.13$ |  | -0.17 |  | -0.12 |
| 6 | 688 | 26 | 582 | 46 | 390 | 66 | 203 |
|  | -686 0.03 |  | -0.15 |  | $-0.17$ | 67 | $\underline{-0.13}$ |
| 7 | -086 | 27 | 573 | 47 | 380 | 67 | 195 -0.12 |
| 8 | 683 | 28 | 565 | 48 | 369 | 68 | 188 |
|  | -680.05 |  | $\underset{556}{-0.15}$ |  | -0.17 |  | $-0.12$ |
| 9 | - 0.05 | 29 | 556 | 49 | 359 | 69 | $\xrightarrow{\text { 181 }}$ |
| 10 | ${ }_{677}^{-0.07}$ | 30 | ${ }_{548}{ }^{-0.15}$ | 50 | 349 -0.17 | 70 | 174 |
| II | 673 -0.07 | 31 | 539 -0.15 | 51 | 339 -0.17 | 71 | 168 -0.12 |
| 12 | 669 -0.07 | 32 | 530 | 52 | - 329 | 72 | 161 -0.10 |
| 13 | 665 | 33 | 520 | 53 | 319 | 73 | 155 |
|  | -0.08 |  | -0.15 |  | -0.17 |  | -0.08 |
| 14 | 660 -0.08 | 34 | 511 -0.17 | 54 | 309. | 74 | 150 -0.10 |
| 15 | 655 | 35 | 501 | 55 | 299 | 75 | 144 |
| 16 | -0.080 | 36 | -0.17 | 56 | -0.15 | 76 | -0.08 |
| 16 | -0.10 | 36 | $\underline{491}$ | 50 | $\underline{-0.17}$ | 76 | 139 -0.07 |
| 17 | 644 | 37 | 482 | 57 | 280 | 77 | 135 |
|  | -0.08 |  | -0.17 |  | -0.15 |  | - 130 |
| 18 | $\underline{639}$ | 38 | 472 -0.17 | 58 | 271 -0.15 | 78 | 130 -0.07 |
| 19 | $\begin{gathered} 632 \\ -0.10 \end{gathered}$ | 39 | $\begin{gathered} 462 \\ -0.17 \end{gathered}$ | 59 | $\begin{aligned} & 262 \\ & -0.15 \end{aligned}$ | 79 | $\begin{aligned} & 126 \\ & -0.05 \end{aligned}$ |
| 20 | 626 -0.12 | 40 | 452 -0.18 | 60 | $\begin{aligned} & 253 \\ & -0.15 \end{aligned}$ | 80 | 123 |

Smithsonian Tables.

Table 15.
LOGARITHMS OF FACTORS FOR COMPUTINC DIFFERENCES OF LATITUDE, LONGITUDE, AND AZIMUTH IN SECONDARY TRIANCULATION. UNIT = THE ENGLISH FOOT.
[Derivation and use of table explained on p. 1x.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{\text {I }}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ} 0^{\prime}$ | 7.99669 | $7 \cdot 99374$ | $-\infty$ | - | 0.372 | 1000' | 7.99655 | 7.99369 | 9.621 | 9.926 | 0.398 |
| 10 | 669 | 374 | 7.839 | 8.137 | 0.372 | 10 | 655 | - 369 | 9.628 | 9.933 | 0.399 |
| 20 | 669 | 374 | 8.140 | 8.438 | 0.372 | 20 | 654 | 369 | 9.636 | 9.941 | 0.400 |
| 30 | 669 | 374 | $8.3 \times 6$ | 8.614 | 0.372 | 30 | 654 | 369 | 9.643 | 9.948 | 0.401 |
| 40 | 669 | 374 | 8.441 | 8.739 | 0.372 | 40 | 654 | 369 | 9.650 | 9.955 | 0.402 |
| 50 | 669 | 374 | 8.538 | 8.836 | 0.372 | 50 | 653 | 369 | 9.657 | 9.963 | 0.403 |
| 100 | 669 | 374 | 8.617 | 8.915 | 0.372 | 1100 | 653 | 368 | 9.663 | 9.970 | 0.404 |
| 10 | 669 | 374 | 8.684 | 8.982 | 0.372 | 10 | 652 | 368 | 9.670 | 9.977 | 0.404 |
| 20 | 668 | 374 | 8.742 | 9.040 | 0.372 | 20 | 652 | 368 | 9.677 | 9.983 | 0.405 |
| 30 | 668 | 374 | 8.793 | 9.091 | 0.373 | 30 | 651 | 368 | 9.683 | 9.990 | 0.406 |
| 40 | 668 | 374 | 8.839 | 9.137 | 0.373 | 40 | 651 | 368 | 9.690 | 9.997 | 0.407 |
| 50 | 668 | 374 | 8.880 | 9.179 | 0.373 | 50 | 650 | 368 | 9.696 | 0.003 | 0.408 |
| 200 | 668 | 374 | 8.918 | 9.216 | 0.373 | 1200 | 650 | 367 | 9.702 | 0.010 | 0.409 |
| 10 | 668 | 373 | 8.953 | 9.251 | 0.373 | 10 | 649 | 367 | 9.708 | 0.016 | 0.410 |
| 20 | 668 | 373 | 8.985 | 9.283 | 0.373 | 20 | 649 | 367 | 9.714 | 0.023 | 0.412 |
| 30 | 668 | 373 | 9.015 | 9.314 | 0.374 | 30 | 648 | 367 | 9.720 | 0.029 | 0.413 |
| 40 | 668 | 373 | 9.043 | 9.342 | 0.374 | 40 | 648 | 367 | 9.726 | 0.035 | 0.414 |
| 50 | 668 | 373 | 9.069 | 9.368 | 0.374 | 50 | 647 | 367 | 9.732 | 0.04 x | 0.415 |
| 300 | 668 | 373 | 9.094 | 9.393 | 0.374 | 1300 | 646 | 366 | 9.738 | 0.048 | 0.416 |
| 10 | 667 | 373 | 9.118 | 9.417 | 0.375 | 10 | 646 | 366 | 9.744 | 0.054 | 0.417 |
| 20 | 667 | 373 | 9.140 | 9.439 | 0.375 | 20 | 645 | 366 | 9.749 | 0.060 | 0.418 |
| 30 | 667 | 373 | 9.161 | 9.460 | 0.375 | 30 | 645 | 366 | 9.755 | 0.065 | 0.419 |
| 40 | 667 | 373 | 9.182 | 9.481 | 0.376 | 40 | 644 | 366 | 9.761 | 0.07 x | 0.420 |
| 50 | 667 | 373 | 9.201 | 9.500 | 0.376 | 50 | 644 | 365 | 9.766 | 0.077 | 0.422 |
| 400 | 667 | 373 | 9.220 | 9.519 | 0.376 | 1400 | 643 | 365 | 9.771 | 0.083 | 0.423 |
| 10 | 666 | 373 | 9.237 | 9.537 | 0.377 | +10 | 642 | 365 | 9.777 | 0.088 | 0.423 0.424 |
| 20 | 666 | 373 | 9.254 | 9.554 | 0.377 | 20 | 642 | 365 | 9.782 | 0.094 | 0.425 |
| 30 | 666 | 373 | 9.271 | 9.570 | 0.377 | 30 | 641 | 365 | 9.787 | 0.100 | 0.426 |
| 40 | 666 | 373 | 9.287 | 9.586 | 0.378 | 40 | 640 | 364 | 9.792 | 0.105 | 0.428 |
| 50 | 666 | 373 | 9.302 | 9.602 | 0.378 | 50 | 640 | 364 | 9.798 | 0.11 I | 0.429 |
| 500 | 665 | 373 | 9.317 | 9.617 | -0.379 | 1500 | 639 | 364 | 9.803 | $0.1 \times 6$ | 0.430 |
| 10 | 665 | 373 | 9.33 I | 9.63 x | 0.379 | 10 | 639 | 364 | 9.808 | 0.121 | 0.43 I |
| 20 | 665 | 372 | 9.345 | 9.645 | 0.379 | 20 | 638 | 363 | 9.813 | 0.127 | 0.433 |
| 30 | 665 | 372 | $9 \cdot 358$ | 9.659 | 0.380 | 30 | 637 | 363 | 9.818 | 0.132 | 0.434 |
| 40 | 664 | 372 | 9.372 | 9.672 | 0.380 | 40 | 637 | 363 | 9.822 | 0.137 | 0.435 |
| 50 | 664 | 372 | 9.384 | 9.685 | 0.381 | 50 | 636 | 363 | 9.827 | 0.142 | 0.437 |
| 600 | 664 | 372 | 9.397 | 9.697 | $0.3^{81}$ | 1600 | 635 | 363 | 9.832 | 0.147 | 0.438 |
| 10 | 664 | 372 | 9.409 | 9.709 | 0.382 | 10 | 635 | 362 | 9.837 | 0.153 | 0.439 |
| 20 | 663 | 372 | 9.420 | 9.72 I | 0.383 | 20 | 634 | 362 | 9.84 I | 0.158 | 0.447 |
| 30 | 663 | 372 | 9.432 | 9.732 | 0.383 | 30 | 633 | 362 | 9.846 | 0.163 | 0.442 |
| 40 | 663 662 | 372 372 | 9.443 | 9.744 | 0.384 | 40 | 632 | 362 | 9.851 | 0.168 | 0.443 |
| 50 700 | 662 | 372 | 9.453 | 9.755 | 0.384 | 50 | 632 | 361 | 9.855 | 0.173 | 0.445 |
| 700 10 | 662 | 372 371 | 9.464 | 9.765 | 0.385 | 1700 | 631 | $36 x$ | 9.860 | 0.178 | 0.446 |
| 20 | 662 | 371 | 9.474 9.484 | 9.7786 9.786 | 0.386 0.386 | 10 | 630 630 | 361 | 9.864 9.869 | 0.182 | 0.448 |
| 30 | 66 I | 371 | 9.494 | 9.796 | 0.387 | 30 | 629 | 360 | 9.873 | 0.192 | 0.449 0.450 |
| 40 | 661 | 371 | 9.504 | 9.806 | -. 387 | 40 | 628 | 360 | 9.878 | 0.197 | 0.452 |
| 50 | 661 | 37 x | 9.513 | 9.816 | 0.388 | 50 | 627 | 360 | 9.882 | 0.202 | 0.453 |
| 800 | 660 | 37 I | 9.523 | 9.825 | 0.389 | 1800 | 627 | 360 | 9.886 | 0.206 | 0.455 |
| 10 | 660 | 371 | 9.532 | 9.834 | 0.389 | 10 | 626 | 359 | 9.890 | 0.211 | 0.456 |
| 20 | 659 | 371 | $9.54{ }^{1}$ | 9.843 | 0.390 | 20 | 625 | 359 | 9.895 | 0.216 | 0.458 |
| 30 | 659 | 37 I | 9. 549 | 9.852 | 0.391 | 30 | 624 | 359 | 9.899 | 0.220 | 0.459 |
| 40 | 659 658 | 370 | 9.558 | 9.861 | 0.392 | 40 | 624 | 359 | 9.903 | 0.225 | 0.46 x |
| 900 | 658 | 370 | 9. 566 | 9.870 | 0.392 | 50 | 623 | 358 | 9.907 | 0.229 | 0.163 |
| 9 | 657 | 370 370 | 9.575 9.583 | 9.878 9.886 | 0.393 0.394 | 1900 10 | 622 | 358 | 9.911 | 0.234 | 0.464 |
| 20 | 657 | 370 | 9.591 | 9.895 | 0.395 | 20 | 620 | 358 358 | 9.915 | 0.239 | 0.466 |
| 30 | 657 | 370 | 9.598 | 9.903 | 0.396 | 30 | 620 | 357 | 9.923 | 0.248 | 0.469 |
| 40 | 656 656 | 370 | 9.606 | 9.910 | 0.396 | 40 | 619 | 357 | 9.927 | 0.252 | 0.470 |
| 50 | 656 | 369 | 9.614 | 9.918 | 0.397 | 50 | 618 | 357 | 9.931 | 0.256 | 0.472 |
| 1000 | 655 | 369 | 9.621 | 9.926 | 0.398 | 20.00 | 617 | 357 | 9.935 | 0.261 | 0.474 |

Table 15. LOGARITHMS OF FACTORS FOR COMPUTINC DIFFERENCES OF LATITUDE, LONGITUDE, AND AZIMUTH IN SECONDARY TRIANCULATION. UNIT = THE ENGLISH FOOT.
[Derivation and use of table explained on p. lx.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{2}=c_{2}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $20^{\circ} 00^{\prime}$ | 7-99617 | 7.99357 | 9.935 | 0.261 | 0.474 | $30^{\circ} 00^{\prime}$ | 7.99558 | 7.99337 | 0.135 | 0.496 | 0.593 |
| 10 | 616 | 356 | 9.939 | 0.265 | 0.475 | Io | 557 | 337 | 0.138 | 0.500 | 0.595 |
| 20 | 615 | 356 | 9.943 | 0.270 | 0.477 | 20 | 556 | 336 | 0.141 | 0.503 | 0.598 |
| 30 | 615 | 356 | 9.947 | 0.274 | 0.479 | 30 | 555 | 336 | 0.144 | 0.507 | 0.600 |
| 40 | 614 | 355 | 9.951 | 0.278 | 0.480 | 40 | 554 | 335 | 0.146 | 0.51 I | 0.603 |
| 50 | 613 | 355 | 9.955 | 0.282 | 0.482 | 50 | 553 | 335 | 0.149 | 0.514 | 0.605 |
| 2100 | 612 | 355 | 9.958 | 0.287 | 0.484 | 3100 | 552 | 335 | 0.152 | 0.518 | 0.607 |
| го | 6ir | 355 | 9.962 | 0.291 | 0.486 | 10 | 550 | 334 | 0.155 | 0.522 | 0.610 |
| 20 | 610 | 354 | 9.966 | 0.295 | 0.487 | 20 | 549 | 334 | 0.158 | 0.525 | 0.612 |
| 30 | 609 | 354 | 9.970 | 0.299 | 0.489 | 30 | 548 | 333 | 0.161 | 0.529 | 0.615 |
| 40 | 608 | 354 | 9.973 | 0.304 | 0.491 | 40 | 547 | 333 | 0.164 | 0.532 | 0.617 |
| 50 | 608 | 353 | 9.977 | 0.308 | 0.493 | 50 | 546 | 333 | 0.166 | 0.536 | 0.619 |
| 2200 | 607 | 353 | 9.981 | 0.312 | 0.494 | 3200 | 545 | 332 | 0.169 | 0.540 | 0.622 |
| 10 | 606 | 353 | 9.984 | 0.316 | 0.496 | 10 | 544 | 332 | 0.172 | 0.543 | 0.624 |
| 20 | 605 | 353 | 9.988 | 0.320 | 0.498 | 20 | 542 | 332 | 0.175 | 0.547 | 0.627 |
| 30 | 604 | 352 | 9.991 | 0.324 | 0.500 | 30 | 541 | 33 I | 0.177 | 0.550 | 0.629 |
| 40 | 603 | 352 | 9.995 | 0.328 | 0.502 | 40 | 540 | 33 I | 0.180 | 0.554 | 0.632 |
| 50 | 602 | 352 | 9.998 | 0.332 | 0.503 | 50 | 539 | $33^{\circ}$ | 0.183 | 0.558 | 0.634 |
| 2300 | 601 | 351 | 0.002 | 0.336 | 0.505 | 3300 | 538 | 330 | 0.186 | 0.561 | 0.637 |
| 10 | 600 | 351 | 0.005 | 0.340 | 0.507 | 10 | 537 | 330 | 0.188 | 0.565 | 0.639 |
| 20 | 600 | 351 | 0.009 | 0.344 | 0.509 | 20 | 535 | 329 | 0.191 | 0.568 | 0.642 |
| 30 | 599 | 350 | 0.012 | 0.348 | 0.511 | 30 | 534 | 329 | 0.194 | 0.572 | 0.644 |
| 40 | 598 | 350 | 0.016 | 0.352 | 0.513 | 40 | 533 | 328 | 0.197 | 0.575 | 0.647 |
| 50 | 597 | 350 | 0.019 | 0.356 | 0.515 | 50 | 532 | 328 | 0.199 | 0.579 | 0.650 |
| 2400 | 596 | 349 | 0.023 | 0.360 | 0.517 | 3400 | 531 | 328 | 0.202 | 0.583 | 0.652 |
| 10 | 595 | 349 | 0.026 | 0.364 | 0.518 | 10 | 529 528 | 327 | 0.205 | 0. 586 | 0.655 |
| 20 | 594 | 349 | 0.029 | 0.368 | 0.520 | 20 | 528 | 327 | 0.208 | 0.590 | 0.657 |
| 30 | 593 | 348 | 0.033 | 0.372 | 0.522 | 30 | 527 | 326 | 0.210 | 0.593 | 0.660 |
| 40 | 592 | 348 | 0.036 | 0.376 | 0.524 | 40 | 526 | 326 | 0.213 | 0.597 | 0.663 |
| 50 | 591 | 348 | 0.039 | 0.380 | 0.526 | 50 | 525 | 326 | 0.216 | 0.600 | 0.665 |
| 2500 | 590 | 347 | 0.043 | 0.384 | 0.528 | 3500 | 523 | 325 | 0.218 | 0.604 | 0.668 |
| 10 | 588 | 347 | 0.046 | 0.388 | 0. 530 | 10 | 522 | 325 | 0.221 | 0.608 | 0.671 |
| 20 | 588 | 347 | 0.049 | 0.392 | 0-532 | 20 | 521 | 324 | 0.224 | 0.611 | 0.673 |
| 30 | 587 | 346 | 0.052 | 0.396 | 0.534 | 30 | 520 | 324 | 0.226 | 0.615 | 0.676 |
| 40 | 586 | 346 | 0.056 | 0.399 | 0.536 | 40 | 519 | 324 | 0.229 | 0.618 | 0.679 |
| 50 | 585 | 346 | 0.059 | 0.403 | 0.538 | 50 | 517 | 323 | 0.232 | 0.622 | 0.68 I |
| 2600 | 584 | 345 | 0.062 | 0.407 | 0.540 | 3600 | 516 | 323 | 0.234 | 0.625 | 0.684 |
| 10 | 583 | 345 | 0.065 | 0.411 | 0.543 | 10 | 515 | 322 | 0.237 | 0.629 | 0.687 |
| 20 | 582 | 345 | 0.068 | 0.415 | 0.545 | 20 | 514 | 322 | 0.239 | 0.632 | 0.689 |
| 30 | 581 | 344 | 0.072 | 0.418 | 0.547 | 30 | 512 | 322 | 0.242 | 0.636 | 0.692 |
| $40^{\circ}$ | 580 | 344 | 0.075 | 0.422 | 0.549 | 40 | 511 | 32 L | 0.245 | 0.640 | 0.695 |
| 50 | 579 | 344 | 0.078 | 0.426 | 0.551 | 50 | 510 | 32 I | 0.247 | 0.643 | 0.698 |
| 2700 | 578 | 343 | 0.08 I | 0.430 | 0.553 | 3700 | 509 | 320 | 0.250 | 0.647 | 0.700 |
| 10 | 577 | 343 | 0.084 | 0.433 | 0.555 | 10 | 507 | 320 | 0.253 | 0.650 | 0.703 |
| 20 | 576 | 343 | 0.087 | 0.437 | 0.557 | 20 | 506 | 320 | 0.255 | 0.654 | 0.706 |
| 30 | 575 | 342 | 0.090 | 0.441 | 0.559 | 30 | 505 | 319 | 0.258 | 0.657 | 0.709 |
| 40 | 574 | 342 | 0.093 | 0.445 | 0.562 | 40 | 504 | 319 | 0.260 | 0.661 | 0.712 |
| 50 | 573 | 342 | 0.096 | 0.448 | 0.564 | 50 | 503 | 318 | 0.263 | 0.665 | 0.715 |
| 2800 | 571 | 341 | 0.099 | 0.452 | 0.566 | $3^{8} 00$ | 501 | 318 | 0.266 | 0.668 | 0.717 |
| 10 | 570 | 341 | 0.102 | 0.456 | 0. 568 | 10 | 500 | 317 | 0.268 | 0.672 | 0.720 |
| 20 | 569 | 341 | 0. 105 | 0.460 | 0.570 | 20 | 499 | 317 | 0.271 | 0.675 | 0.723 |
| 30 | 568 | 340 | 0.108 | 0.463 | 0.573 | 30 | 498 | 317 | 0.273 | 0.679 | 0.726 |
| 40 | 567 | 340 | 0.1 | 0.467 | 0.575 | 40 | 496 | 316 | 0.276 | 0.683 | 0.729 |
| 50 | 566 | 340 | 0.114 | 0.471 | 0.577 | 50 | 495 | 316 | 0.278 | 0.686 | 0.732 |
| 2900 | 565 | 339 | 0.117 | 0.474 | 0.579 | 39 \% | 494 | 315 | 0.28 I | 0.690 | 0.735 |
| 10 | 564 | 339 | 0.120 | 0.478 | 0.582 |  | 492 | 315 | 0.284 | 0.693 | 0.738 |
| 20 | 563 | 338 | 0.123 | 0.482 | 0.584 | 20 | 49 I | 315 | 0.286 | 0.697 | 0.741 |
| 30 | 562 | 338 | 0.126 | 0.485 | 0.586 | $3{ }^{\circ}$ | $49^{\circ}$ | 314 | 0.289 | 0.701 | 0.744 |
| 40 | 561 | 338 | 0. 129 | 0.489 | 0.588 | 40 | 489 | 314 | 0.291 | 0.704 | 0.747 |
| 50 | 560 | 337 | 0.132 | 0.493 | 0.591 <br> 0.593 | 50 | 487 | 313 313 | 0.294 | 0.708 0.711 | 0.750 0.753 |
| 3000 | 558 | 337 | 0.135 | 0.496 | 0.593 | 4000 | 486 | 313 | 0.296 | 0.711 | 0.753 |

Table 15.
LOGARITHMS OF FACTORS FOR COMPUTINC DIFFERENCES OF LATITUDE, LONGITUDE, AND AZIMUTH IN SECONDARY TRIANGULATION.

UNIT = THE ENGLISH FOOT.
[Derivation and use of table explained on p. Ix.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $40^{\circ} 00^{\prime}$ | 7.99486 | 7.99313 | 0.296 | 0.711 | 0.752 | $50^{\circ} 00^{\prime}$ | 7.99409 | 7.99287 | 0.448 | 0.939 | 0.955 |
| 10 | 485 | 312 | 0.299 | 0.715 | 0.755 | 10 | 408 | 287 | 0.450 | 0.944 | 0.958 |
| 20 | 484 | 312 | 0.301 | 0.719 | 0.759 | 20 | 407 | 287 | 0.453 | 0.948 | 0.962 |
| 30 | 482 | 312 | 0.304 | 0.722 | 0.762 | 30 | 406 | 286 | 0.455 | 0.952 | 0.966 |
| 40 | 481 | 311 | 0.307 | 0.726 | 0.765 | 40 | 404 | 286 | 0.458 | 0.956 | 0.970 |
| 50 | 480 | 3 II | 0.309 | 0.730 | 0.768 | $5^{\circ}$ | 403 | 285 | 0.460 | 0.950 | 0.974 |
| 4100 | 479 | 310 | 0.312 | 0.733 | 0.771 | 5100 | 402 | 285 | 0.463 | 0.964 | 0.978 |
|  | 477 | 310 | 0.314 | 0.737 | 0.774 | 10 | 401 | 284 | 0.466 | 0.968 | 0.982 |
| 20 | 476 | 309 | 0.317 | 0.740 | 0.777 | 20 | 399 | 284 | 0.468 | 0.972 | 0.985 |
| 30 | 475 | 309 | 0.319 | 0.744 | 0.780 | 30 | 398 | 284 | 0.47 I | 0.976 | 0.989 |
| 40 | 473 | 309 | 0.322 | 0.748 | 0.783 | 40 | 397 | 283 | 0.473 | 0.981 | 0.993 |
| 50 | 472 | 308 | 0.324 | 0.75 I | 0.786 | 50 | 396 | 283 | 0.476 | 0.985 | 0.997 |
| 4200 | 471 | 308 | 0.327 | 0.755 | 0.789 | 5200 | 394 | 282 | 0.478 | 0.989 | 1.001 |
| 10 | 470 | 307 | 0.329 | 0.759 | 0.792 | 10 | 393 | 282 | 0.48 I | 0.993 | 1.005 |
| 20 | 468 | 307 | 0.332 | 0.762 | 0.796 | 20 | 392 | 281 | 0.484 | 0.998 | 1.009 |
| 30 | 467 | 306 | 0.334 | 0.766 | 0.799 | 30 | 391 | 281 | 0.486 | 1.002 | 1.0r3 |
| 40 | 466 | 306 | 0.337 | 0.770 | 0.802 | 40 | 389 | 281 | 0.489 | 1.006 | 1.017 |
| 50 | 464 | 306 | 0.339 | 0.774 | 0.805 | 50 | 388 | 280 | 0.491 | 1.010 | 1.021 |
| 4300 | 463 | 305 | 0.342 | 0.777 | 0.808 | 5300 | 387 | 280 | 0.494 | 1.015 | 1.025 |
| 10 | 462 | 305 | 0.344 | 0.781 | 0.812 | 10 | 386 | 279 | 0.497 | 1.019 | 1.030 |
| 20 | 461 | 304 | 0.347 | 0.785 | 0.815 | 20 | 384 | 279 | 0.499 | 1.023 | 1.034 |
| 30 | 459 | 304 | 0.349 | 0.788 | 0.818 | 30 | 383 | 279 | 0.502 | 1.028 | 1.038 |
| 40 | 458 | 303 | 0.352 | 0.792 | 0.821 | 40 | 382 | 278 | 0.505 | 1.032 | 1.042 |
| 50 | 457 | 303 | 0.354 | 0.796 | 0.824 | 50 | 381 | 278 | 0.507 | 1.036 | 1.046 |
| 4400 | 455 | 303 | 0.357 | 0.800 | 0.828 | 5400 | 379 | 277 | 0.510 | 1.041 | 1.050 |
| 10 | 454 | 302 | 0.359 | 0.803 | 0.831 | 10 | 378 | 277 | 0.512 | 1.045 | I. 055 |
| 20 | 453 | 302 | 0.362 | 0.807 | 0.834 | 20 | 377 | 277 | 0.515 | 1.049 | 1.059 |
| 30 | 452 | 301 | 0.364 | 0.811 | 0.838 | $3^{\circ}$ | 376 | 276 | 0.518 | 1.054 | 1.063 |
| 40 | 450 | 301 | 0.367 | 0.815 | 0.84 I | 40 | 375 | 276 | 0.520 | 1.058 | 1.067 |
| 50 | 449 | 300 | 0.370 | 0.818 | 0.844 | 50 | 373 | 275 | 0.523 | 1.063 | 1.072 |
| 4500 | 448 | 300 | 0.372 | 0.822 | 0.848 | 5500 | 372 | 275 | 0.526 | 1.067 | 1.076 |
| 10 | 446 | 300 | 0.375 | 0.826 | 0.851 | 10 | 371 | 275 | 0.528 | 1.072 | 1.080 |
| 20 | 445 | 299 | 0.377 | 0.830 | 0.854 | 20 | 370 | 274 | 0.531 | 1.076 | 1.084 |
| 30 | 444 | 299 | 0.380 | 0.833 | 0.858 | 30 | 369 | 274 | 0.534 | 1.081 | 1.089 |
| 40 | 443 | 298 | 0.382 | 0.837 | 0.861 | 40 | 367 | 273 | 0.537 | 1.085 | 1.093 |
| 50 4600 | 441 | 298 | 0.385 | 0.841 | 0.865 | 50 | 366 | 273 | 0.539 | 1.090 | 1.098 |
| 4600 10 | 440 | 297 | 0.387 | 0.845 | 0.868 | 5600 | 365 | 273 | 0.542 | 1.094 | 1.102 |
| 20 | 439 | 297 | 0.390 | 0.849 | 0.872 | 10 | 364 | 272 | 0.545 | 1.099 | 1.106 |
| 30 | 436 | 296 |  | 0.856 |  | 20 | 363 | 272 | 0.547 | 1.104 | 1.111 |
| 40 | 435 | 296 | 0.397 | 0.860 | 0.882 | 40 | 360 | 271 | 0.550 | I.108 | 1.115 |
| 50 | 434 | 295 | 0.400 | 0.864 | 0.885 | 50 | 359 | 271 271 | 0.553 0.556 | 1.113 | 1.120 1.124 |
| 4700 | 432 | 295 | 0.402 | 0.868 | 0.889 | 57.00 | 358 | 270 | 0.558 | 1.122 | 1.129 |
| 10 | 431 | 294 | 0.405 | 0.872 | 0.892 | 570 | 357 | 270 | 0.558 0.561 | 1.122 | 1.129 1.134 |
| 20 | 430 | 294 | 0.407 | 0.876 | 0.896 | 20 | 356 | 269 | 0.564 | $1.13{ }^{2}$ | 1.138 |
| 30 | 428 | 294 | 0.410 | 0.880 | 0.900 | 30 | 354 | 269 | 0.567 | 1.137 | 1.143 |
| 40 | 427 | 293 | 0.412 | 0.884 | 0.903 | 40 | 353 | 269 | 0.569 | 1.141 | 1.147 |
|  | 426 | 293 | 0.415 | 0.888 | 0.907 | 50 | 352 | 268 | 0.572 | I.I 46 | 1.152 |
| 4800 10 | 425 | 292 | 0.417 | 0.891 | 0.910 | 5800 | 351 | 268 | 0.575 | I. 151 | 1.157 |
| 20 | 422 | 292 | 0.420 0.422 | 0.895 0.899 | 0.914 | 10 | 350 | 267 | 0.578 | 1.156 | 1.162 |
| 30 | 421 | 291 | 0.425 | 0.903 | 0.921 | 30 | 349 | 267 | 0.581 | 1.16I | I. 166 |
| 40 | 420 | 291 | 0.427 | 0.907 | 0.925 | 40 | 347 <br> 346 | 266 | 0.583 | 1.166 | 1.171 |
| 50 | 418 | 290 | 0.430 | 0.911 | 0.929 | 50 | 345 | 266 | 0.589 | 1.175 | 1.176 1.181 |
| 4900 |  |  | 0.432 | 0.915 | 0.932 | 5900 | 344 | 266 | 0.592 | 1.180 | 1.185 |
| 10 20 | 416 414 | 289 | 0.435 | 0.919 | 0.936 | 10 | 343 | 265 | 0. 595 | 1.185 | 1.190 |
| 30 | 414 413 | 289 | 0.438 | 0.923 | 0.940 | 20 | 342 | 265 | 0. 598 | 1.190 | I. 195 |
| 40 | 412 | 288 | -0.443 | 0.927 | 0.943 | 30 | 341 | 264 | 0.600 | 1.195 | 1.200 |
| 50 | 411 | 288 | 0.445 | 0.935 | 0.951 | 50 | 331 338 | 264 | 0.603 | 1.200 | 1.205 |
| 5000 | 409 | 287 | 0.448 | 0.939 | 0.955 | 6000 | 337 | 263 | 0.609 | 1.210 | 1.210 I.215 |

Smithsonian Tables.
[Derivation and use of table explained on p. lx.]

| $\phi$ | $a_{\mathrm{I}}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $60^{\circ} 00^{\prime}$ | 7.99337 | 7.99263 | 0.609 | 1.210 | 1.215 | $70^{\circ} 00^{\prime}$ | 7.99278 | 7.99244 | 0.809 | I. 575 | 1.576 |
| 10 | 336 | 263 | 0.612 | 1.216 | 1.220 | 10 | 277 | 243 | 0.813 | 1.583 | 1.584 |
| 20 | 335 | 263 | 0.615 | 1.221 | 1.225 | 20 | 277 | 243 | 0.817 | I. 590 | 1.591 |
| 30 | 334 | 262 | 0.618 | 1.226 | 1. 230 | 30 | 276 | 243 | 0.821 | 1.598 | 1.599 |
| 40 | 333 | 262 | 0.621 | 1.231 | 1.235 | 40 | 275 | 242 | 0.825 | I. 605 | т. 606 |
| 50 | 332 | 261 | 0.624 | 1.236 | I. 240 | 50 | 274 | 242 | 0.829 | 1.613 | 1.614 |
| 6100 | 331 | 261 | 0.627 | 1.241 | 1.245 | 7100 | 273 | 242 | 0.833 | 1.621 | 1.621 |
| 10 | 329 | 261 | 0.630 | 1.247 | 1.251 | 10 | 273 | 242 | 0.837 | 1.629 | 1.629 |
| 20 | 328 | 260 | 0.633 | 1.252 | 1. 256 | 20 | 272 | 241 | 0.841 | 1.636 | 1.637 |
| 30 | 327 | 260 | 0.636 | 1.257 | 1.261 | 30 | 271 | 241 | 0.845 | 1.644 | r. 645 |
| 40 | 326 | 260 | 0.639 | 1.263 | 1. 266 | 40 | 270 | 241 | 0.849 | 1.652 | 1.653 |
| 50 | 325 | 259 | 0.642 | 1.268 | I. 272 | 50 | 269 | 24 I | 0.854 | 1.660 | 1.661 |
| 6200 | 324 | 259 | 0.645 | 1.273 | 1.277 | 7200 | 269 | 240 | 0.858 | 1. 669 | 1.669 |
| 10 | 323 | 259 | 0.648 | 1.279 | 1.282 | 10 | 268 | 240 | 0.862 | 1.677 | 1.677 |
| 20 | 322 | 258 | 0.651 | I. 284 | 1. 288 | 20 | 267 | 240 | 0.866 | 1.685 | 1.686 |
| 30 | 321 | 258 | 0.654 | 1.290 | 1.293 | 30 | 266 | 240 | 0.871 | 1.694 | 1.694 |
| 40 | 320 | 257 | 0.657 | 1.295 | 1.298 | 40 | 266 | 239 | 0.875 | 1.702 | 1.702 |
| 50 | 319 | 257 | 0.660 | 1.301 | 1.304 | 50 | 265 | 239 | 0.880 | 1.710 | 1.711 |
| 6300 | 318 | 257 | 0.663 | 1.306 | 1. 309 | 7300 | 264 | 239 | 0.884 | 1.719 | 1.720 |
| 10 | 317 | 256 | 0.666 | 1.312 | I. 315 | 10 | 264 | 239 | 0.889 | 1.728 | 1.728 |
| 20 | 316 | 256 | 0.669 | 1.318 | 1.320 | 20 | 263 | 238 | 0.893 | 1.737 | 1.737 |
| 30 | 315 | 256 | 0.672 | 1.323 | 1.326 | 30 | 262 | 238 | 0.898 | 1.745 | 1. 746 |
| 40 | 314 | 255 | 0.676 | 1.329 | 1.332 | 40 | 261 | 238 | 0.903 | 1.754 | I. 755 |
| 50 | 313 | 255 | 0.679 | 1.335 | 1. 337 | 50 | 26 I | 238 | 0.907 | 1.763 | I. 764 |
| 6400 | 312 | 255 | 0.682 | 1.34I | I. 343 | 7400 | 260 | 238 | 0.912 | 1.772 | 1.773 |
| 10 | 311 | 254 | 0.685 | 1.346 | 1. 349 | 10 | 259 | 237 | 0.917 | 1.782 | 1.782 |
| 20 | 310 | 254 | 0.688 | 1.352 | I. 355 | 20 | 259 | 237 | 0.922 | 1.791 | 1.791 |
| 30 | 309 | 254 | 0.692 | 1. 358 | 1.360 | 30 | 258 | 237 | 0.927 | 1.800 | 1.801 |
| 40 | 308 | 253 | 0.695 | 1.363 | I. 366 | 40 | 257 | 237 | 0.931 | 1.810 | r. 810 |
| 50 | 307 | 253 | 0.698 | 1.370 | r. 372 | 50 | 257 | 236 | 0.936 | 1.820 | ז. 820 |
| 6500 | 306 | 253 | 0.701 | 1.376 | I. 378 | 7500 | 256 | 236 | 0.941 | 1.829 | 1. 830 |
| 10 | 305 | 252 | 0.705 | 1. 382 | 1.384 | 10 | 255 | 236 | 0.946 | 1.839 | 1. 839 |
| 20 | 304 | 252 | 0.708 | I. 388 | I. 390 | 20 | 255 | 236 | 0.952 | 1.849 | I. 849 |
| 30 40 | 303 | 252 | 0.711 | 1.394 | I. 396 | 30 | 254 | 236 235 | 0.957 | 1.859 r 869 | 1.859 1. 869 |
| 40 | 302 301 | 251 251 | 0.715 0.718 | 1.400 1.406 | 1.402 1.408 | 40 | 254 253 | 235 235 | 0.962 | 1.869 I .879 | 1.869 r .880 |
| 50 6600 | 301 300 | 251 251 | 0.718 0.721 | 1.406 | 1.408 1.414 | 50 7600 | 253 | 235 | 0.967 | 1.890 | 1. 890 |
| Io | 299 | 250 | 0.725 | I.419 | 1.421 | 10 | 252 | 235 | 0.978 | 1.900 | 1.901 |
| 20 | 298 | 250 | 0.728 | r .425 | 1.427 | 20 | 251 | 235 | 0.984 | 1.917 | 1.911 |
| 30 | 297 | 250 | 0.732 | 1.432 | 1.433 | 30 | 250 | 234 | 0.989 | 1.922 | 1.922 |
| 40 | 296 | 249 | 0.735 | 1.438 | I. 440 | 40 | 250 | 234 | 0.995 | 1.933 | 1.933 |
| 50 | 295 | 249 | 0.739 | 1.444 | 1.446 | 50 | 249 | 234 | 1.000 | 1.944 | 1.944 |
| 6700 | 294 | 249 | 0.742 | 1.45 | I. 452 | 7700 | 249 | 234 | 1.006 | 1.955 | 1.955 |
| 10 | 293 | 249 | 0.746 | 1.457 | 1.459 | 10 | 248 | 234 | 1.012 | 1.966 | 1.966 1.978 |
| 20 | 292 | 248 | 0.749 | I. 464 | 1.465 | 20 | 248 | 233 | 1.018 | 1.978 | 1.978 |
| 30 | 291 | 248 | 0.753 | 1.470 | 1.472 | 30 | 247 | 233 | 1.024 | 1.989 | 1.989 |
| 40 | 290 | 248 | 0.756 | 1.477 | I. 478 | 40 | 247 | 233 | 1.030 1.036 | 2.001 | 2.001 |
| 50 | 289 | 247 | 0.760 | 1.484 | I. 485 | 50 | 246 | 233 | 1.036 | 2.013 | 2.013 |
| 6800 | 289 | 247 | 0.763 | I. 491 | 1.492 | 7800 | 245 | 233 | 1.042 | 2.025 | 2.025 |
| 10 | 288 | 247 | 0.767 | 1.497 | 1.499 | 10 | 245 | 233 | 1.048 | 2.037 | 2.037 2.050 |
| 20 | 287 | 246 | 0.771 | 1.504 | 1.505 | 20 | 244 | 232 | 1.054 | 2.050 | 2.050 2.062 |
| 30 | 286 | 246 | 0.774 | I.51I | 1.512 | 30 | 244 | 232 | 1.061 | 2.062 | $\begin{aligned} & 2.062 \\ & 2.075 \end{aligned}$ |
| 40 | 285 | 246 | 0.778 | 1.518 | 1.519 I. 526 | 40 50 | 243 | 232 232 | 1.074 | 2.088 | 2.088 |
| 50 6900 | 284 | 246 | 0.782 0.786 | 1.525 1.532 | 1.526 <br> 1.533 | 50 7900 | 243 | 232 232 | 1.074 | 2.101 | 2.101 |
| 69 10 10 | 282 | 245 | 0.786 0.789 | I. 539 | 1.533 1.540 |  | 242 | 232 | 1.087 | 2.114 | 2.114 |
| 20 | 282 | 245 | 0.793 | I. 546 | 1. 547 | 20 | 242 | 231 | 1.094 | 2.128 | 2.128 |
| 30 | 281 | 244 | 0.797 | 1.553 | 1.554 | 30 | 241 | 231 | 1.101 | 2.142 | 2.142 |
| 40 | 280 | 244 | 0.801 | 1.56 r | 1.562 | 40 | 241 | 231 | 1.108 | 2.156 | 2.156 |
| 50 | 279 | 244 | 0.805 | 1.568 | 1.569 | 50 | 240 | 231 | 1.116 | 2.170 | 2.170 |
| 7000 | 278 | 244 | 0.809 | I. 575 | 1.576 | 8000 | 240 | 231 | 1.123 | 2.184 | 2.184 |

[Derivation and use of table explained on p. 1x.]

| $\phi$ | $a_{1}$ | $\delta_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $\delta_{\text {I }}=c_{\text {I }}$ | $a_{2}$ | $B_{3}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ} 00^{\prime}$ | 8.51268 | 8.50973 | - ${ }^{\infty}$ | - $\infty$ | I. 404 | $10^{\circ} 00^{\prime}$ | 8.51254 | 8.50968 | 0.653 | 0.958 | 1.430 |
| 10 | 268 | 973 | 8.871 | 9.169 | 1.404 | 10 | 254 | 968 | 0.660 | 0.965 | 1.431 |
| 20 | 268 | 973 | 9.172 | 9.470 | 1.404 | 20 | 253 | 968 | 0.668 | 0.973 | 1.432 |
| 30 | 268 | 973 | 9.348 | 9.646 | 1.404 | 30 | 253 | 968 | 0.675 | 0.980 | 1.433 |
| 40 | 268 | 973 | 9.473 | 9.771 | 1.404 | 40 | 253 | 968 | 0.682 | 0.987 | 1.434 |
| 50 | 268 | 973 | 9.570 | 9.868 | 1.404 | 50 | 252 | 967 | 0.689 | 0.995 | 1.435 |
| 100 | 267 | 973 | 9.649 | 9.947 | I. 404 | 1100 | 252 | 967 | 0.695 | 1.002 | 1.436 |
| 10 | 267 | 973 | 9.716 | 0.014 | 1.404 | 10 | 251 | 967 | 0.702 | 1.009 | 1.436 |
| 20 | 267 | 973 | 9.774 | 0.072 | 1.404 | 20 | 251 | 967 | 0.709 | 1.015 | 1.437 |
| 30 | 267 | 973 | 9.825 | 0.123 | 1.405 | 30 | 250 | 967 | 0.715 | 1.022 | 1.438 |
| 40 | 267 | 973 | 9.871 | 0.169 | 1.405 | 40 | 250 | 967 | 0.722 | 1.029 | 1.439 |
| 50 | 267 | 973 | 9.912 | 0.211 | 1.405 | 50 | 249 | 966 | 0.728 | 1.035 | 1.440 |
| 200 | 267 | 972 | 9.950 | 0.248 | 1.405 | 1200 | 249 | 966 | 0.734 | 1.042 | 1.441 |
| 10 | 267 | 972 | 9.985 | 0.283 | 1.405 | 10 | 248 | 966 | 0.740 | 1.048 | 1.442 |
| 20 | 267 | 972 | 0.017 | 0.315 | 1.405 | 20 | 248 | 966 | 0.746 | 1.055 | 1.444 |
| 30 | 266 | 972 | 0.047 | 0.346 | 1.406 | 30 | 247 | 966 | 0.752 | 1.061 | 1.445 |
| 40 | 266 | 972 | 0.075 | 0.374 | 1.406 | 40 | 246 | 966 | 0.758 | 1.067 | 1.446 |
| 50 | 266 | 972 | 0.101 | 0.400 | 1.406 | 50 | 246 | 965 | 0.764 | 1.073 | 1.447 |
| 300 | 266 | 972 | 0.126 | 0.425 | 1. 406 | 1300 | 245 | 965 | 0.770 | 1.080 | 1.448 |
| 10 | 266 | 972 | 0.150 | 0.449 | I. 407 | 10 | 245 | 965 | 0.776 | 1.086 | 1.449 |
| 20 | 266 | 972 | 0.172 | 0.471 | 1.407 | 20 | 244 | 965 | 0.781 | 1.092 | 1.450 |
| 30 | 266 | 972 | 0.193 | 0.492 | 1.407 | 30 | 244 | 965 | 0.787 | 1.097 | 1.451 |
| 40 | 266 | 972 | 0.214 | 0.513 | 1.408 | 40 | 243 | 964 | 0.792 | 1.103 | 1.452 |
| 50 | 266 | 972 | 0.233 | 0.532 | 1.408 | 50 | 242 | 964 | 0.798 | 1.109 | 1.454 |
| 400 | 265 | 972 | 0.252 | 0.551 | 1. 408 | 1400 | 242 | 964 | 0.803 | 1.115 | 1.455 |
| 10 | 265 | 972 | 0.269 | 0.569 | I. 409 | 10 | 241 | 964 | 0.809 | 1.120 | 1.456 |
| 20 | 265 | 972 | 0.286 | 0.586 | 1.409 | 20 | 241 | 964 | 0.814 | 1.126 | I. 457 |
| 30 | 265 | 972 | 0.303 | 0.602 | I. 409 | 30 | 240 | 963 | 0.819 | 1.132 | I. 458 |
| 40 | 265 | 972 | 0.319 | 0.618 | 1.410 | 40 | 239 | 963 | 0.824 | 1.137 | 1.460 |
| 50 | 264 | 972 | 0.334 | 0.634 | 1.410 | 50 | 239 | 963 | 0.830 | 1.143 | I-46I |
| 500 | 264 | 972 | 0.349 | 0.649 | I.4II | 1500 | 238 | 963 | 0.835 | 1.148 | 1.462 |
| 10 | 264 | 971 | 0.363 | 0.663 | 1.411 | 10 | 237 | 963 | 0.840 | 1.153 | 1.463 |
| 20 | 264 | 971 | 0.377 | 0.677 | 1.411 | 20 | 237 | 962 | 0.845 | I. 159 | 1.465 |
| 30 | 264 | 971 | 0.390 | 0.691 | 1.412 | 30 | 236 | 962 | 0.850 | 1.164 | 1.466 |
| 40 | 263 | 971 | 0.404 | 0.704 | 1.412 | 40 | 235 | 962 | 0.854 | 1.169 | 1.467 |
| 50 | 263 | 971 | 0.416 | 0.717 | 1.413 | 50 | 235 | 962 | 0.859 | 1.174 | 1.469 |
| 600 | 263 | 971 | 0.428 | 0.729 | 1.413 | 1600 | 234 | 961 | 0.864 | 1.179 | 1.470 |
| 10 | 263 | 971 | 0.440 | 0.741 | 1.414 | 10 | 233 | 961 | 0.869 | 1.185 | 1.471 |
| 20 | 262 | 971 | 0.452 | 0.753 | 1.415 | 20 | 233 | 961 | 0.873 | 1.190 | 1.473 |
| $3{ }^{\circ}$ | 262 | 971 | 0.464 | 0.764 | 1.415 | 30 | 232 | 961 | 0.878 | 1.195 | 1.474 |
| 40 | 262 | 971 | 0.475 | 0.776 | 1.416 | 40 | 231 | 961 | 0.883 | 1.200 | 1.475 |
| 50 700 | 261 | 971 | 0.485 | 0.787 | 1.416 | 50 | 231 | 960 | 0.887 | 1.205 | I. 477 |
| 700 | 261 | 970 | 0.496 | 0.797 | 1.417 | 1700 | 230 | 960 | 0.892 | 1.210 | 1.478 |
| 10 | 261 | 970 | 0.506 | 0.808 | 1.417 | 10 | 229 | 960 | 0.896 | 1.214 | 1.480 |
| 20 | 260 | 970 | 0.516 | 0.818 | I. 418 | 20 | 228 | 960 | 0.901 | 1.219 | 1.48I |
| 30 | 260 | 970 | 0.526 | 0.828 | 1.419 | 30 | 228 | 959 | 0.905 | 1.224 | I. $4^{88} 2$ |
| 40 | 260 | 970 | 0.536 | 0.838 | 1.419 | 40 | 227 | 959 | 0.910 | 1.229 | I.484 |
| 50 800 | 259 | 970 | 0.545 | 0.848 | 1.420 | 50 | 226 | 959 | 0.914 | 1.234 | I. 485 |
| 800 | 259 | 970 | 0.555 | 0.857 | I.421 | 1800 | 225 | 959 | 0.918 | 1.238 | 1.487 |
| 10 | 259 | 970 | 0.564 | 0.866 | 1.421 | 10 | 225 | 958 | 0.922 | 1.243 | 1.489 |
| 30 | 258 | 970 | 0.573 | 0.875 | 1.422 | 20 | 224 | 958 | 0.927 | 1.248 | 1.490 |
| 30 40 |  | 969 | 0.58 I 0.590 | 0.884 | 1.423 | 30 | 223 | 958 | 0.931 | 1.252 | r.49r |
| 40 | 258 257 | 969 969 | 0.590 | 0.893 | 1.424 | 40 | 223 | 958 | 0.935 | I. 257 | I. 493 |
| 900 | 257 | 969 | 0.598 0.607 | 0.902 | 1.424 | 50 | 222 | 957 | 0.939 | 1.261 | 1.495 |
| 10 | 256 | 969 | 0.615 | 0.918 | 1.425 1.426 | 1900 10 | 221 220 | 957 | 0.943 | 1.266 | 1.496 1.498 |
| 20 | 256 | 969 | 0.623 | 0.927 | I. 427 | 20 | 219 | 957 | 0.951 | 1.275 | 1.499 |
| 30 | 256 | 969 | 0.630 | 0.935 | 1.428 | 30 | 218 | 956 | 0.955 | 1.279 | 1.501 |
| 40 50 | 255 | 969 | 0.638 | 0.942 | 1.428 | 40 | 218 | 956 | 0.959 | 1.284 | 1.502 |
| 50 10 00 | 255 254 | 968 | 0.646 | 0.950 | 1.429 | 50 | 217 | 956 | 0.963 | 1. 288 | I. 504 |
| 1000 | 254 | 968 | 0.653 | 0.958 | 1.430 | 2000 | 216 | 955 | 0.967 | 1.293 | 1. 506 |

Table 16. LOGARITHMS OF FACTORS FOR COMPUTING DIFFERENCES OF LATITUDE, LONGITUDE, AND AZIMUTH IN SECONDARY TRIANGULATION. UNIT = THE METRE.
[Derivation and use of table explained on p. lx.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 ${ }^{\circ} 00^{\prime}$ | 8.51216 | 8.50955 | 0.967 | 1.293 | 1. 506 | $30^{\circ} 00^{\prime}$ | 8.51 57 | 8.50936 | I. 167 | 1. 528 | 1.625 |
| 10 | 215 | 955 | 0.971 | 1.297 | I. 507 | 10 | 156 | 936 | 1.170 | I. 532 | 1.627 |
| 20 | 214 | 955 | 0.975 | 1.301 | 1. 509 | 20 | 155 | 935 | 1.173 | I. 535 | 1.630 |
| 30 | 214 | 955 | 0.979 | 1. 306 | I.511 | 30 | 154 | 935 | 1.176 | I. 539 | 1.632 |
| 40 | 213 | 954 | 0.983 | 1.310 | 1.512 | 40 | 153 | 934 | 1.178 | I. 543 | 1. 635 |
| 50 | 212 | 954 | 0.987 | 1.314 | 1.514 | 50 | 152 | 934 | 1.181 | I. 546 | 1.637 |
| 2100 | 211 | 954 | 0.990 | 1.319 | 1.516 | 3100 | 151 | 934 | 1.184 | I. $555^{\circ}$ | 1.639 |
| 10 | 210 | 953 | 0.994 | 1.323 | 1.518 | 10 | 149 | 933 | 1.187 | I. 554 | I. 642 |
| 20 | 209 | 953 | 0.998 | 1.327 | 1.519 | 20 | 148 | 933 | 1.190 | 1.557 | 1.644 |
| 30 | 208 | 953 | 1.002 | 1.331 | 1.52I | 30 | 147 | 933 | 1.193 | 1.561 | 1. 646 |
| 40 | 207 | 953 | 1.005 | 1.336 | 1. 523 | 40 | 146 | 932 | 1.195 | 1.564 | 1. 649 |
| 50 | 207 | 952 | 1.009 | I. 340 | 1.524 | 50 | 145 | 932 | 1.198 | 1. 568 | 1.651 |
| 2200 | 206 | 952 | 1.013 | I. 344 | 1. 526 | 3200 | 144 | 931 | 1.201 | 1. 572 | 1.654 |
| 10 | 205 | 952 | 1.01 | I. 348 | 1. 528 | 10 | 143 | 931 | 1.204 | I. 575 | 1. 656 |
| 20 | 204 | 951 | 1.020 | I. 352 | 1.530 | 20 | 141 | 931 | 1.207 | I. 579 | 1. 659 |
| 30 | 203 | 951 | 1.023 | I. 356 | 1. 532 | 30 | 140 | 930 | I. 209 | I. 582 | 1.66I |
| 40 | 202 | 951 | 1.027 | 1.360 | I. 534 | 40 | $\pm 39$ | 930 | 1.212 | I. 586 | I. 664 |
| 50 | 201 | 951 | 1.030 | 1. 364 | 1.535 | 50 | 138 | 929 | 1.215 | 1.590 | 1.666 |
| 2300 | 200 | 950 | 1.034 | 1. 368 | I. 537 | 3300 | 137 | 929 | 1.218 | 1.593 | 1. 669 |
| 10 | 199 | 950 | 1.037 | 1.372 | I. 539 | 10 | 136 | 929 | 1.220 | 1. 597 | 1. 671 |
| 20 | 198 | 950 | 1.041 | 1.376 | 1.541 | 20 | 134 | 928 | 1.223 | 1.600 | 1. 674 |
| 30 | 197 | 949 | 1.044 | 1.380 | I. 543 | 30 | 133 | 928 | 1.226 | I. 604 | 1. 676 |
| 40 | 197 | 949 | 1.048 | I. 384 | 1. 545 | 40 | 132 | 927 | 1.229 | 1.607 | 1.679 |
| 50 | 196 | 949 | 1.051 | 1.388 | 1.547 | 50 | 131 | 927 | 1.231 | I.6II | 1.682 |
| 2400 | 195 | 948 | 1.055 | 1.392 | I. 549 | 3400 | 130 | 927 | I. 234 | 1.615 | I. 684 |
| 10 | 194 | 948 | 1.058 | I.396 | $1.55^{\circ}$ | 10 | 128 | 926 | I. 237 | 1.618 | 1.687 |
| 20 | 193 | 948 | 1.061 | 1.400 | 1.552 | 20 | 127 | 926 | 1.239 | 1.622 | 1.689 |
| 30 | 192 | 947 | 1.065 | 1.404 | I. 554 | 30 | 126 | 925 | 1.242 | 1.625 | 1692 |
| 40 | 191 | 947 | 1.068 | 1.408 | 1.556 | 40 | 125 | 925 | 1.245 | 1.629 | 1. 695 |
| 50 | 190 | 947 | 1.071 | 1.412 | 1.558 | 50 | 124 | 925 | 1.248 | 1.632 | I. 697 |
| 2500 | 189 188 | 946 | 1.075 | 1.416 | 1. 560 | 3500 | 122 | 924 | 1.250 | 1. 636 | 1.700 |
| 10 | 188 | 946 | 1.078 | 1.420 | 1. 562 | 10 | 121 | 924 | 1.253 | I. 639 | 1.702 |
| 20 | 187 | 946 | 1.081 | 1.424 | I. 564 | 20 | 120 | 923 | 1.256 | 1.643 | 1.705 |
| 30 | 186 | 945 | 1.084 | 1.427 | 1.566 | 30 | 119 | 923 | 1.258 | 1. 647 | 1.708 |
| 40 | 185 | 945 | 1.088 | 1.431 | I. 568 | 40 | 118 | 923 | 1.261 | 1.650 | 1.711 |
| 50 | 184 | 945 | 1.091 | 1.435 | 1.570 | 50 | 116 | 922 | 1.264 | 1.654 | 1.713 |
| 2600 | 183 | 944 | 1.094 | 1.439 | 1. 572 | 3600 | 115 | 922 | 1.266 | 1.657 | 1.716 |
| 10 | 182 | 944 | 1.097 | 1.443 | 1. 575 | 10 | 114 | 921 | 1.269 | 1.66I | 1.719 |
| 20 | 181 | 944 | 1.100 | 1.447 | 1. 577 | 20 | 113 | 921 | 1.271 | 1.664 | 1.721 |
| 30 | 180 | 943 | 1.104 | I. 450 | 1. 579 | 30 | 11 | 921 | 1.274 | I. 668 | 1.724 |
| 40 | 179 | 943 | 1.107 | 1.454 | 1.581 | 40 | 110 | 920 | 1.277 | 1.672 | 1.727 |
| 50 | 178 | 943 | I.IIO | 1.458 | 1.583 | 50 | 109 | 920 | 1.279 | 1.675 | 1.730 |
| 2700 | 177 | 942 | 1.113 | 1.462 | 1. 585 | 3700 | 108 | 919 | 1.282 | 1.679 | 1.732 |
| 10 | 176 | 942 | 1.116 | 1. 465 | I. 587 | 10 | 105 | 919 | 1. 288 | 1.682 | 1.735 <br> 1738 |
| 20 | 175 | 942 | 1.119 | 1.469 | 1.589 | 20 | 105 | 919 | 1.287 | I. 686 | 1.738 |
| 30 | 174 | 941 | I. 122 | 1.473 | 1.591 | 30 | 104 | 918 | 1.290 | 1.689 | I.741 |
| 40 | 172 | 941 | 1.125 | 1.477 | 1. 594 | 40 | 103 | 918 | 1.292 1.295 | 1.693 I. 697 | 1.744 1.747 |
| 50 | 171 | 941 | 1.128 | 1.480 | 1. 596 | 50 | 102 | 917 | 1.295 1.298 | 1.697 | 1.747 |
| 2800 | 170 | 940 | 1.13I | 1.484 | 1.598 | 3800 10 | 100 | 917 916 |  | 1.700 1.704 | 1.749 1.752 |
| 10 | 169 168 | 940 940 | I.I 34 I.1 37 I. | 1.488 <br> 1.492 | 1.600 I.602 1. | 10 | 099 098 | 919 | 1.300 1.303 | 1.704 1.707 | 1.752 1.755 |
| 30 | 167 | 939 | $\underline{1.140}$ | r. 495 | 1. 605 | 30 | 097 | 916 | 1.305 | 1.711 | 1.758 |
| 40 | 166 | 939 | I.I43 | I. 499 | 1.607 | 40 | 095 | 915 | 1.308 | 1.715 | 1.761 |
| 50 | 165 | 938 | I.I46 | 1.503 | 1.609 | 50 | 094 | 915 | 1.310 | 1.718 | 1.764 |
| 2900 | 164 | 938 | I.I49 | 1.506 | 1.611 | 3900 | 093 | 914 | 1.313 | 1.722 | 1.767 |
| 10 | 163 | 938 | I.152 | 1.510 | 1.614 |  | 092 | 914 | 1.316 | 1.725 | I.770 |
| 20 | 162 | 937 | I.155 | 1.514 | 1.616 | 20 | 090 | 914 | 1.318 | 1.729 | 1.773 |
| 30 | 161 | 937 | 1.158 | 1.517 | 1.618 |  | 089 | 913 | 1.321 | 1.733 | 1.776 I. 779 |
| 40 50 | 160 158 | 937 936 | 1.161 1.164 | 1.521 1.525 | 1.620 <br> 1.623 |  | 088 | 913 912 | 1.323 1.326 | 1.733 1.740 | 1.779 I .78 I |
| 50 3000 | 158 157 | 936 | 1.164 I.167 | 1.525 1.528 | 1.623 1.625 | + 40 | 085 | 912 | 1. 328 | 1.743 | 1. 784 |

[Derivation and use of table explained on p. 1x.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{\text {I }}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $40^{\circ} 00^{\prime}$ | 8.51085 | 8.50912 | 1.328 | 1.743 | 1.784 | $50^{\circ} 00^{\prime}$ | 8.51008 | 8.50886 | 1.480 | 1.971 | I. 987 |
| 10 | 084 | 9 II | I.331 | 1.747 | 1.787 | 10 | 007 | 886 | 1.482 | 1.975 | 1. 990 |
| 20 | 083 | 911 | 1.333 | 1.751 | 1.790 | 20 | 006 | 885 | 1.485 | 1.980 | 1.994 |
| 30 | 081 | 911 | 1.336 | 1.754 | 1.793 | 30 | 005 | 885 | 1.487 | 1.984 | 1.998 |
| 40 | 080 | 910 | 1.338 | 1.758 | 1.797 | 40 | 003 | 885 | 1.490 | 1.988 | 2.002 |
| 50 | 079 | 910 | 1.341 | 1.762 | 1.800 | 50 | 002 | 884 | 1.492 | 1.992 | 2.006 |
| 4100 | 078 | 909 | 1.344 | 1.765 | 1.803 | 5100 | 001 | 884 | 1.495 | 1.996 | 2.010 |
|  | 076 | 909 | I. 346 | I. 769 | 1.806 | 10 | 000 | 883 | 1.498 | 2.000 | 2.014 |
| 20 | 075 | 908 | 1.349 | 1.772 | 1.809 | 20 | 8.50998 | 883 | 1.500 | 2.004 | 2.017 |
| 30 | 074 | 908 | 1.351 | 1.776 | 1.812 | 30 | 997 | 882 | 1.503 | 2.008 | 2.02I |
| 40 | 072 | 908 | 1.354 | 1.780 | 1.815 | 40 | 996 | 882 | 1.505 | 2.013 | 2.025 |
| 50 | 071 | 907 | 1.356 | 1.783 | I.818 | 50 | 994 | 882 | 1.508 | 2.017 | 2.029 |
| 4200 | 070 | 907 | 1.359 | 1.787 | 1.821 | 5200 | 993 | 881 | 1.510 | 2.021 | 2.033 |
| 10 | .069 | 906 | 1.361 | 1.791 | 1.824 | 10 | 992 | 881 | I. 513 | 2.025 | 2.037 |
| 20 | 067 | 906 | 1. 364 | 1.794 | 1.828 | 20 | 991 | 880 | 1.516 | 2.030 | 2.041 |
| 30 | 066 | 905 | 1.366 | 1.798 | 1.831 | 30 | 990 | 880 | 1.518 | 2.034 | 2.045 |
| 40 | 065 | 905 | I. 369 | 1.802 | 1.834 | 40 | 988 | 880 | 1.521 | 2.038 | 2.049 |
| 50 | 063 | 905 | 1.371 | 1.805 | 1.837 | 50 | 987 | 879 | 1.523 | 2.042 | 2.053 |
| 4300 | 062 | 904 | 1.374 | 1.809 | 1.840 | 5300 | 986 | 879 | 1. 526 | 2.047 | 2.057 |
|  | 061 | 904 | 1.376 | 1.813 | 1.843 | 10 | 985 | 878 | 1.529 | 2.05 I | 2.062 |
| 20 | 060 | 903 | I. 379 | 1.817 | 1.847 | 20 | 983 | 878 | 1.531 | 2.055 | 2.066 |
| 30 | 058 | 903 | 1.381 | 1.820 | 1.850 | 30 | 982 | 877 | 1.534 | 2.060 | 2.070 |
| 40 | 057 | 902 | 1.384 | 1.824 | 1.853 | 40 | 981 | 877. | I. 537 | 2.064 | 2.074 |
| 50 | 056 | 902 | 1.386 | 1.828 | 1.856 | 50 | 980 | 877 | I. 539 | 2.068 | 2.078 |
| 4400 | 054 | 902 | I. 389 | 1.832 | 1.860 | 5400 | 978 | 876 | 1.542 | 2.073 | 2.082 |
| 10 | 053 | 901 | 1.391 | 1.835 | 1.863 | 10 | 977 | 876 | 1.544 | 2.077 | 2.086 |
| 20 | 052 | 901 | I. 394 | 1.839 | 1.866 | 20 | 976 | 875 | 1. 547 | 2.081 | 2.091 |
| 30 | 051 | 900 | 1.396 | I. 843 | 1.870 | 30 | 975 | 875 | 1.550 | 2.086 | 2.095 |
| 40 | 049 | 900 | I. 399 | 1.847 | 1.873 | 40 | 973 | 875 | I. 552 | 2.090 | 2.099 |
| 50 | 048 | 899 | 1.401 | 1.850 | 1.876 | 50 | 972 | 874 | 1.555 | 2.095 | 2.104 |
| 4500 | 047 | 899 | 1.404 | 1.854 | 1.880 | 5500 | 971 | 874 | 1.558 | 2.099 | 2.108 |
| 10 | 045 | 899 | 1.407 | 1.858 | 1. 883 | 10 | 970 | 873 | 1. 560 | 2.104 | 2.112 |
| 20 | 044 | 898 | 1.409 | 1.862 | I. 886 | 20 | 969 | 873 | I. 563 | 2.108 | 2.116 |
| 30 | 043 | 898 | I. 412 | 1.865 | 1.890 | 30 | 967 | 873 | 1.566 | 2.113 | 2.121 |
| 40 | 042 | 897 | I. 414 | 1.869 | 1. 893 | 40 | 966 | 872 | 1.568 | 2.117 | 2.125 |
| $5{ }^{50}$ | 040 | 897 | 1.417 | 1.873 | 1.897 | 50 | 965 | 872 | 1.57 I | 2.122 | 2.130 |
| 4600 10 | 039 038 | 896 896 | 1.419 | 1.877 | 1.900 | 5600 | 964 | 871 | 1.574 | 2.126 | 2.134 |
| то | -038 | 896 896 | 1.422 1.424 | 1.881 1.885 | 1.903 1.907 | 10 | 963 | 871 | 1. 577 | 2.131 | 2.138 |
| 20 | 036 | 896 | 1.424 | 1.885 | 1.907 | 20 | 961 | 871 | 1.579 | 2.136 | 2.143 |
| 30 40 | 035 | 895 | 1.427 | I. 888 | 1.910 | 30 | 960 | 870 | 1.582 | 2.140 | 2.147 |
| 40 50 | 034 | 895 | 1.429 | 1.892 | 1.914 | 40 | 959 | 870 | 1.585 | 2.145 | 2.152 |
| 50 4700 | 033 | 894 | 1.432 | 1.896 | 1.917 | 50 | 958 | 869 | 1.588 | 2.150 | 2.156 |
| 4700 10 | 031 030 | 894 893 | 1.434 I. 437 | 1.900 1.904 | 1.921 1.924 | 57 <br> 10 <br>  <br> 10 | 957 | 869 | 1. 590 | 2.154 | 2.161 |
| 10 | $\bigcirc$ | 893 893 | 1.437 $\mathbf{1} .439$ | 1.904 I. 908 | 1.924 1.928 1.932 | 10 | 956 | 869 868 | 1.593 I. 596 | 2.159 2.164 | 2.166 |
| 30 | 027 | 893 | 1.442 | 1.912 | 1.932 | 30 | 954 | 868 | 1.590 | 2.164 2.169 | 2.170 |
| 40 | 026 | 892 | 1.444 | 1.916 | 1.935 | 40 | 952 | 867 | 1.599 1.601 | 2.169 2.173 | 2.175 2.179 |
| 50 | 025 | 892 | 1.447 | 1.920 | 1.939 | 50 | 951 | 867 | 1.604 | 2.178 | 2.184 |
| 4800 | 024 | 891 | 1.449 | 1.923 | L. 942 | 5800 | 950 | 867 | 1.607 | 2.183 | 2.189 |
| 20 | 022 | 891 800 | 1.452 | 1.927 | 1. 946 | 10 | 949 | 866 | 1.610 | 2.183 | 2.193 |
| 30 | 020 | 890 | 1.454 | I.93I | I. 950 | 20 | 947 | 866 | 1.613 | 2.193 | 2.198 |
| 40 | 019 | 890 | 1.457 1.459 | 1.935 1.939 | 1.953 1.957 | 30 40 | 946 | 866 | 1.615 | 2.197 | 2.203 |
| 50 | 017 | 889 | 1.462 | 1.943 | 1.96ı | 50 | 945 944 | 865 | 1.618 | 2.202 | 2.208 2.213 |
| 4900 | 016 | 889 | I. 464 | 1.947 | 1.964 | 5900 | 943 | 864 | 1.624 | 2.212 | 2.213 2.217 |
| 10 | 015 | 888 | 1. 467 | 1.951 | 1.968 | 10 | 942 | 864 | I. 627 | 2.217 | 2.2122 2.222 |
| 30 | 013 | 888 | 1.469 | 1.955 | 1.972 | 20 | 94 I | 864 | 1.630 | 2.222 | 2.227 |
| 30 40 | 012 | 888 887 | 1.472 <br> I. 475 | 1.959 1.963 | 1.975 <br> $\mathbf{1} .975$ <br> 1.989 | 30 | 939 | 863 | 1.632 | 2.227 | 2.232 |
| 50 | OIO | 887 | 1.475 1.477 | 1.963 1.967 | 1.979 1.983 | 40 50 | 938 937 | 863 | 1.635 | 2.232 | 2.237 |
| 5000 | 008 | 886 | 1.480 | 1.971 | 1.987 | 60 | 937 | 863 | 1.638 | 2.237 | 2.242 |
|  |  |  |  |  |  | 60 | 93 | 862 | 1.641 | 2.242 | 2.247 |

LOGARITHMS OF FACTORS FOR COMPUTING DIFFERENCES OF LATITUDE, LONGITUDE, AND AZIMUTH IN SECONDARY TRIANGULATION. UNIT = THE METRE.
[Derivation and use of table explained on p. lx.]

| $\phi$ | $a_{1}$ | $b_{1}=c_{\text {I }}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ | $\phi$ | $a_{1}$ | $b_{1}=c_{1}$ | $a_{2}$ | $b_{2}$ | $c_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $60^{\circ} 00^{\prime}$ | 8.50936 | 8.50862 | 1.641 | 2.242 | 2.247 | $70^{\circ} 00^{\prime}$ | 8.50877 | 5.50842 | 1.84I | 2.607 | 2.608 |
| 10 | 935 | 862 | 1. 644 | 2.247 | 2.252 | 7 | 876 | 842 | 1.845 | 2.615 | 2.616 |
| 20 | 934 | 86 I | 1.647 | 2.253 | 2.257 | 20 | 875 | 842 | 1. 849 | 2.622 | 2.623 |
| 30 | 933 | 861 | 1.650 | 2.258 | 2.262 | 30 | 875 | 842 | 1.853 | 2.630 | 2.631 |
| 40 | 932 | 861 | 1.653 | 2.263 | 2.267 | 40 | 874 | 841 | 1. 857 | 2.637 | 2.638 |
| 50 | 931 | 860 | 1.656 | 2.268 | 2.272 | 50 | 873 | 841 | 1.86 I | 2.645 | 2.646 |
| 6100 | 929 | 860 | 1.659 | 2.273 | 2.277 | 7100 | 872 | 841 | 1. 865 | 2.653 | 2.653 |
| 10 | 928 | 860 | 1.662 | 2.279 | 2.283 | 10 | 871 | 841 | I. 869 | 2.661 | 2.661 |
| 20 | 927 | 859 | 1.665 | 2.284 | 2.288 | 20 | 87 I | 840 | 1.873 | 2.668 | 2.669 |
| 30 | 926 | 859 | 1.668 | 2.289 | 2.293 | 30 | 870 | 840 | 1.877 | 2.676 | 2.677 |
| 40 | 925 | 858 | 1.671 | 2.295 | 2.298 | 40 | 869 | 840 | ¢. 881 | 2.684 | 2.685 |
| 50 | 924 | 858 | 1. 674 | 2.300 | 2.303 | 50 | 868 | 840 | I. 886 | 2.692 | 2.693 |
| 6200 | 923 | 858 | 1. 677 | 2.305 | 2.309 | 7200 | 868 | 839 | 1. 890 | 2.701 | 2.701 |
| 10 | 922 | 857 | 1.680 | 2.311 | 2.314 | 10 | 867 | 839 | ז. 894 | 2.709 | 2.709 |
| 20 | 92 I | 857 | 1.683 | 2.316 | 2.320 | 20 | 866 | 839 | ז. 898 | 2.717 | 2.718 |
| 30 | 920 | 857 | 1.686 | 2.322 | 2.325 | 30 | 865 | 839 | 1.903 | 2.725 | 2.726 |
| 40 | 919 | 856 | 1.689 | 2.327 | 2.330 | 40 | 865 | 838 | 1.907 | 2.734 | 2.734 |
| 50 | 918 | 856 | 1.692 | 2.333 | 2.336 | 50 | 864 | 838 | 1912 | 2.742 | 2.742 |
| 6300 | 917 | 856 | 1.695 | 2.338 | 2.341 | 7300 | 863 | 838 | 1.916 | 2.751 | 2.751 |
| 10 | 916 | 855 | 1. 698 | 2.344 | 2.347 | 10 | 862 | 838 | 1.921 | 2.760 | 2.760 |
| 20 | 915 | 855 | 1.701 | 2.350 | 2.352 | 20 | 862 | 837 | 1.925 | 2.769 | 2.769 |
| 30 | 913 | 855 | 1.704 | 2.355 | 2.358 | 30 | 861 | 837 | 1.930 | 2.777 | 2.778 |
| 40 | 912 | 854 | 1.708 | 2.36 I | 2.364 | 40 | 860 | 837 | 1.935 | 2.786 | 2.787 |
| 50 | 911 | 854 | 1.711 | 2.367 | 2.369 | 50 | 860 | 837 | I. 939 | 2.795 | 2.796 |
| 6400 | 910 | 854 | 1.714 | 2.373 | 2.375 | 7400 | 859 | 836 | 1.944 | 2.804 | 2.805 |
| 1 | 909 | 853 | 1.717 | 2.378 | 2.381 | 10 | 858 | 836 | 1.949 | 2.814 | 2.814 2.823 |
| 20 | 908 | 853 | 1.720 | 2.384 | 2.387 | 20 | 858 | 836 | I.954 | 2.823 | 2.823 |
| 30 | 907 | 853 | 1.724 | 2.390 | 2.392 | 30 | 857 856 | 836 836 | 1.958 I. 963 | 2.832 | 2.833 2.842 |
| 40 | 906 | 852 852 | 1.727 | 2.396 | 2.398 2.404 | 40 50 | 856 856 | 836 835 | 1.963 | 2.842 | 2.842 |
| 50 | 905 | 852 | 1.730 | 2.402 2.408 | 2.404 | 7500 | 855 | 835 | 1.973 | 2.86 I | 2.861 |
| 10 | 903 | 85 I | 1.737 | 2.414 | 2.416 | 10 | 854 | 835 | 1.978 | 2.87 I | 2.871 |
| 20 | 902 | 851 | I. 740 | 2.420 | 2.422 | 20 | 854 | 835 | 1.984 | 2.881 | 2.881 |
| 30 | 901 | 851 | 1.743 | 2.426 | 2.428 | 30 | 853 | 834 | 1.989 | 2.891 | 2.891 |
| 40 | 900 | 850 | 1.747 | 2.432 | 2.434 | 40 | 852 | 834 | 1.994 | 2.901 | 2.901 |
| 50 | 900 | 850 | 1.750 | 2.438 | 2.440 | 50 | 852 | 834 | 1.999 | 2.911 | 2.912 |
| 6600 | 899 | 850 | I. 753 | 2.445 | 2.446 | 7600 | 851 | 834 | 2.005 | 2.922 | 2.922 |
| 10 | 898 | 849 | 1. 757 | 2.451 | 2.453 | 10 | 851 | 834 | 2.010 | 2.932 | 2.933 |
| 20 | 897 | 849 | 1.760 | 2.457 | 2.459 | 20 | 850 | 833 | 2.015 | 2.943 | 2.943 |
| 30 | 896 | 849 | I. 764 | 2.464 | 2.465 | 30 | 849 | 833 | 2.021 | 2.954 | 2.954 |
| 40 50 | 895 894 | 848 848 | 1.767 1.771 | 2.470 2.476 | 2.472 2.478 | 40 50 | 849 848 | 833 833 | 2.027 2.032 | 2.965 2.976 | 2.965 |
| 50 6700 | 894 | 848 848 | 1.771 1.774 | 2.476 2.483 | 2.478 2.484 | 7700 | 8488 | 833 | 2.038 | 2.987 | 2.987 |
| 67 | 892 | 847 | I. 778 | 2.489 | 2.491 | 10 | 847 | 832 | 2.044 | 2.998 | 2.998 |
| 20 | 891 | 847 | 1.781 | 2.496 | 2.497 | 20 | 847 | 832 | 2.050 | 3.010 | 3.010 |
| 30 | 890 | 847 | I. 785 | 2.502 | 2.504 | 30 | 846 | 832 | 2.056 | 3.021 | 3.021 |
| 40 | 889 | 847 | 1.788 | 2.509 | 2.510 | 40 | 845 | 832 | 2.062 | 3.033 | 3.033 |
| 50 | 888 | 846 | 1.792 | 2.516 | 2.517 | 50 | 845 | 832 | 2.068 | 3.045 | 3.045 |
| 6800 | 887 | 846 | 1.795 | 2.522 | 2.524 | 7800 | 844 | 832 | 2.074 | 3.057 | 3.057 |
| 10 | 887 | 846 | 1.799 | 2.529 | 2.531 | 10 | 844 | 831 | 2.080 | 3.069 | 3.069 3.082 |
| 20 | 886 | 845 | 1.803 | 2.536 | 2.537 | 20 | 843 | 831 | 2.086 | 3.082 | 3.082 |
| 30 | 885 | 845 | 1.806 | 2.543 | 2.544 | 30 | 843 | 831 | 2.093 | 3.094 | 3.094 |
| 40 | 884 | 845 | 1.810 | 2.550 | 2.551 | 40 | 842 842 | 831 831 | 2.099 2.106 | 3.107 3.120 | 3.107 3.120 |
| 50 6900 | 883 | 844 844 | 1.814 1.818 | 2.557 | 2.558 2.565 | 50 79 | 842 841 | 831 | 2.113 | 3.133 | 3.133 |
| 10 | 881 | 844 | 1.821 | 2.57 I | 2.572 | 10 | 841 | 830 | 2.119 | 3.146 | 3.146 |
| 20 | 880 | 844 | 1.825 | 2.578 | 2.579 | 20 | 840 | 830 | 2.126 | 3.160 | 3.160 |
| 30 | 880 | 843 | 1.829 | 2.585 | 2.586 | 30 | 840 | 830 | 2.133 | 3.174 | 3.174 |
| 40 | 879 | 843 | I. 833 | 2.593 | 2.594 | 40 | 839 | 830 830 | 2.140 2.148 | 3.188 3.202 | 3.188 3.202 |
| 50 | 878 | 843 | 1.837 | 2.600 | 2.601 | 50 | 839 | 830 830 | 2.148 2.155 | 3.202 3.216 | 3.202 3.216 |
| 7000 | 877 | 842 | 1.84I | 2.607 | 2.608 | 8000 | 839 | 830 | 2.155 | 3.216 |  |

Table 17.
LENGTHS OF TERRESTRIAL ARCS OF MERIDIAN.
[Derivation of table explained on p. xlvi.]


Smithsonian Tableb.

Table 17.
LENGTHS OF TERRESTRIAL ARCS OF MERIDIAN.
[Derivation of table explained on p. xlvi.]

| Latitude Interval. | $\begin{aligned} & \text { Latitude. } \\ & 25^{\circ} \end{aligned}$ | $\begin{aligned} & \text { Latitude. } \\ & 26^{\circ} \end{aligned}$ | $\underset{27^{\circ}}{\text { Latitude. }}$ | $\begin{aligned} & \text { Latitude. } \\ & 28^{\circ} \end{aligned}$ | Latitude. |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feet. | Feet. | Feet. | Feet. | Feet. |
| $10^{\prime \prime}$ | 1009.49 | 1009.63 | 1009.77 | 1009.92 | 1010.07 |
| 20 | 2088.97 | 2019.25 | 2019. 54 | 2019.83 | 2020.13 |
| 30 | 3028.46 | 3028.88 | 3029.31 | 3029.75 | 3030.20 |
| $4{ }^{\circ}$ | 4037.95 | 4038.51 | 4039.08 | 4039.67 | 4040.27 |
| 60 | 50.47 .44 6056.92 | 5048.13 6057.76 | 5048.85 6058.62 | 5049.58 6059.50 | 50.50.33 6060.40 |
| $10^{\prime}$ | 60569.2 | 60577.6 | 60586.2 | 60595.0 | 60604.0 |
| 20 | 121538.5 | 121155.2 | 121172.3 | 121190.0 | 121208.0 |
| 30 | 181707.7 | 181732.7 | 181758.5 | 181785.0 | 181812.0 |
| 40 | 242276.9 | 2423 ro. 3 | 242344.7 | 242379.9 | 242416.0 |
| 50 60 | 302846.1 | 302887.9 | 302930.9 | 302974.9 | 303019.9 |
|  | 363415.4 | 363465.5 | 363517.1 | 363569.9 | 363623.9 |
|  | $30^{\circ}$ | $31^{\circ}$ | $3^{\circ}$ | $33^{\circ}$ | $34^{\circ}$ |
| $\mathrm{ra}^{\prime \prime}$ | 1010. 22 | roro. 38 | rovo. 54 | 1010.70 | roro. 86 |
| 20 | 2020.44 | 2020.75 | 2021.07 | 2021.40 | 2021.73 |
| 30 | 3030.66 | 3031.13 | 303 r .61 | 3032.10 | 3032.59 |
| 40 | 4040.88 | 4041.51 | 4042.15 | 4042.80 | 4043.46 |
| 50 60 | 5051.10 6061.32 | 5051.89 6062.26 | 5052.68 6063.22 | 5053.50 6064.20 | 5054.32 6065.19 |
|  | 6061.32 | 6062.26 | 6063.22 | 6064.20 | 6065.19 |
| $\mathrm{ra}^{\prime}$ | 60613.2 | 60622.6 | 60632.2 | 60642.0 | 6065 r .9 |
| 20 | 122226.4 | 121245-3 | 121264.4 | 121283.9 | 221303.8 |
| 30 | 181839.7 | 181867.9 | 181896.6 | 181925.9 | r8 r955.7 |
| 40 | 242452.9 | 242490.5 | 242528.8 | 242567.9 | 242607.6 |
| 5060 | 303066.1 363679.3 | 303113.2 363735.8 | 303161.1 363793.3 | 303209.9 36385.8 | 303259.4 |
|  |  |  |  |  |  |
|  | $35^{\circ}$ | $36^{\circ}$ | $37^{\circ}$ | $3^{8}$ | $39^{\circ}$ |
| ${ }^{10} 1$ | motr. 03 | roxt. 20 | roir. 37 | 1011.55 | rorr. 72 |
| 20 | 2022.06 | 2022.40 | 2022.75 | 2023.09 | 2023.44 |
| 30 | 3033.10 | 3033.68 | 3034.12 | 3034.64 | 3035.17 |
| 40 | 4044.13 | ${ }^{4044.81}$ | 4045.50 | 4046.19 | 4046.89 |
| 50 60 | 5055.16 6066.19 | 5056.01 6067.21 | 5056.87 6068.24 | 5057.74 6069.29 | 5058.61 6070.34 |
| $1{ }^{\prime}$ | 6066 r .9 | 60672.1 | 60682.4 | 60692.9 | 60703.4 |
| 20 | 121323.9 | 121344-3 | 121364.9 | 121385.7 | 121406.7 |
| 30 | 18ז985.8 | 182016.4 | 182047.3 | 182078.6 | $182 \mathrm{rio.1}$ |
| 40 | 242647.8 | 242688.5 | 242729.? | 242771.4 | 2428×3.4 |
| 5060 | 303309.7 | 303360.6 | $3034 \times 2.2$ | 303464.3 | 303516.8 |
|  | 36397 1. 7 | 364032.8 | 364094.6 | 364157.1 | 364220.2 |
|  | $40^{\circ}$ | $41^{\circ}$ | $42^{\circ}$ | $43^{\circ}$ | $44^{\circ}$ |
| $1{ }^{\prime \prime}$ | 101.9.90 | 1012.08 | ro12.25 | 1012.43 | rox2.61 |
| 20 | 2023.80 | 2024.15 | 2024.51 | 2024.87 | 2025.23 |
| 30 | 3035.70 | 3036.23 | 3036.77 | 3037.30 | 3037.84 |
| 40 | 4047.60 | 4048.31 | 4049.02 | 4049.74 | 4050.46 |
| 50 60 | 5059.50 6071.39 | 5060.38 6072.46 | 5061.28 6073.53 | 5062.17 6074.61 | 5063.07 6075.69 |
|  |  |  |  |  |  |
| $1{ }^{\prime}$ | 60713.9 | 60724.6 | 60735-3 | 60746.1 | 60756.9 |
| 20 | 121427.9 | 121449.2 | 121470.6 | 121492.2 | 121513.7 |
| 30 | 18214 r .8 | 182173.8 | 182206.0 | 182238.2 | 182270.6 |
| 40 | 242855.8 | 242898.4 | 24294 I .3 | ${ }^{242984.3}$ | 243027.4 |
| 5060 | 303569.7 | 303623.0 | 303676.6 | 303730.4 | 303784.3 |
|  | ${ }^{664283.7}$ | ${ }^{3} 54347.6$ | 3644 II.9 | 364476.5 | 364541.2 |
|  | $45^{\circ}$ | $4^{\circ}$ | $47^{\circ}$ | $48^{\circ}$ | $49^{\circ}$ |
| $10^{\prime \prime}$ | 1012.79 | 1012.97 | 1013.15 | $10 \times 3.33$ | 1013.51 |
| 20 | 2025.59 | 2025.95 | 2026.31 | 2026.67 | 2027.02 |
| 30 | 3038.38 | 3038.92 | 3039.46 | 3040.00 | 3040.54 |
| 40 | 405 I .18 | 405 5 .90 | 402.62 | 4053.34 | 4054.55 |
| ${ }^{50}$ | 5063.97 6076.77 | 5064.87 6077.85 | 5065.77 6078.93 | 5068.67 6080.00 | 5067.56 6081.08 |
|  |  |  | 6078.93 |  |  |
| $10^{\prime}$ | 60767.7 | 60778.5 | 60789.3 | 60800.0 | 60810.8 |
| 20 | 121535.3 | 121556.9 | 121578.5 | 121600.1 | 121621.5 |
| 30 | 182303.0 | 182335-4 | ${ }^{182367.8}$ | 182400.1 | 182432.3 |
| 40 | 243070.6 | 243 r13.9 | 243 57.0 | 243200.1 | 243243.0 |
| 50 | 303838.3 | 303892.4 364670.8 | 303946.3 364735.5 | 304000.1 364800.2 | 304053.8 364864.5 |
| 60 | 364606.0 | 364670.8 | ${ }^{664735.5}$ | 364800.2 | 364864.5 |

Table 17.
LENGTHS OF TERRESTRIAL ARCS OF MERIDIAN.
[Derivation of table explained on p . x xvi.]

| Latitude Interval. | Latitude. $50^{\circ}$ | $\begin{aligned} & \text { Latitude. } \\ & 5 I^{\circ} \end{aligned}$ | $\begin{aligned} & \text { Latitude. } \\ & 5^{2} \end{aligned}$ | $\begin{aligned} & \text { Latitude. } \\ & 53^{\circ} \end{aligned}$ | $\begin{aligned} & \text { Latitude. } \\ & 54^{\circ} \end{aligned}$ | $\begin{aligned} & \text { Latitude. } \\ & 55^{\circ} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feet. | Feet. | Feet. | Feet. | Feet. | Feet. |
| $\mathrm{ro}^{\prime \prime}$ | 1013.69 | 1013.87 <br> 2027.74 | 1014.04 2028.09 | 1014.22 2028.44 | 1014.39 <br> 2028.78 | 1014.56 2029.12 |
| 20 | 2027.38 | 2027.74 304160 | 2028.09 3042.13 | 2028.44 3042.65 | 2028.78 3043.17 | 2029.12 3043.68 |
| $4{ }^{\circ}$ | 4054.76 | + 4055.47 | 4056.17 | 4056.87 | 4057.56 | 4058.24 |
| 50 | 5068.46 | 5069.34 | 5070.22 | 507 r .09 | 5071.96 | 5072.80 |
| 60 | 6082.15 | 6083.21 | 6084.26 | 6085.31 | 6086.35 | 6087.37 |
| $1{ }^{\prime}$ | 60821.5 | 60832.1 | 60842.6 | 60853.1 | 60863.5 | 60873.7 |
| 20 | 121642.9 | 121664.2 | 121685.2 | 121706.2 | 121726.9 | 121747.3 |
| 30 | 182464.4 | 182496.2 | 182527.7 | 182559.2 | 182590.4 | 182621.0 |
| 40 | 243285.8 | 243328.3 | 243370.3 | 243412.3 | 243453.8 | 243494.6 |
| 5060 | 304107.3 364928.8 | 304160.4 | 304212.9 | 304265.4 365118.5 | 304317.3 365180.8 | 304368.3 |
|  |  |  |  | $59^{\circ}$ | $60^{\circ}$ | $61^{\circ}$ |
| $\mathrm{ra}^{\prime \prime}$ | 1014.73 | 1014.90 | 1085.06 | 1015.22 | 1015.38 | 1015.53 |
| 20 | 2029.46 | 2029.79 | 2030.12 | 2030.44 | 2030.76 | 2031.07 |
| 30 | 3044.19 | $3044.69^{-}$ | 3045.18 | 3045.66 | 3046.14 | 3046.60 |
| 40 | 4058.92 | 4059.58 | 4060.24 | 4060.88 | 4061.52 | 4062.14 |
| 50 | 5073.65 | 5074.48 | 5075.30 | 5076.10 | 5076.90 | 5077.67 |
| 60 | 6088.38 | 6089.38 | 6090.36 | 609 r .33 | 6092.27 | 6093.20 |
| $10^{\prime}$ | 60883.8 | 60893.8 | 60903. 6 | 60913.3 | 60922.7 | 60932.0 |
| 20 | 121767.6 | 121787.5 | 121807.2 | 128826.5 | 121845.5 | 121864.1 |
| 30 | 182651.4 | 182681.3 | 182710.8 | 182739.8 | 182768.2 | 182796. |
| $4{ }^{\circ}$ | 243535.2 | 243575.0 | 243614.4 | 243653.0 | 243691.0 | 243728.2 |
| 5060 | 304419.0 | 304468.8 | 304518.0 | 304566.3 | 304613.7 | 304660.2 |
|  | 365302.8 | 365362.6 | 36542 I . 6 | 365479.6 | 365536.4 | 365592.2 |
|  | $62^{\circ}$ | $63^{\circ}$ | $64^{\circ}$ | $65^{\circ}$ | $66^{\circ}$ | $67^{\circ}$ |
| $10^{\prime \prime}$ | ${ }^{1015} 5.69$ | 1015.83 | 1015.98 | 1016.12 | 1016.26 | 1016.39 |
| 20 | 2031.37 | 2031.67 | 2031.96 | 2032.24 | 2032.51 | 2032.78 |
| 30 | 3047.06 | 3047.50 | 3047.94 | 3048.36 | 3048.77 | 3049.16 |
| $4{ }^{\circ}$ | 4062.74 | 4063.34 | 4063.92 | 4064.48 | 4065.02 | 4065.55 |
| 50 | 5078.43 | ${ }_{6}^{5079.17}$ | 5079.90 | 5080.60 | 508 I .28 | 5081.94 |
| 60 | 6094.12 | 6095.00 | 6095.87 | 6096.71 | 6097.54 | 6098.33 |
| ro' | 60941.2 | 60950.0 | 60958.7 | 60967.1 | 60975.4 | 60983.3 |
| 20 | 121882,3 | 121900.1 | 121917.5 | 121934.3 | 1219950.7 | 121966.6 |
| 30 | 188823.5 | 182850.1 | 1888876.2 | 182901.4 | 182926.1 | 182949.8 |
| 40 | 243764.6 | 243800.2 | 243835.0 | 243868.6 | 243901.4 | 243933.1 |
| 5060 | 304705.8 365647.0 | $\begin{aligned} & 304750.2 \\ & 365700.2 \end{aligned}$ | $\begin{aligned} & 304793.7 \\ & 365752.4 \end{aligned}$ | 304835.7 365802.8 | 304876.8 365852.2 | 304916.4 365899.7 |
|  | $68^{\circ}$ | $69^{\circ}$ | $70^{\circ}$ | $71^{\circ}$ | $72^{\circ}$ | $73^{\circ}$ |
| $10^{\prime \prime}$ | 1016.52 | 1016.64 | 1016.76 | 1016.87 | 1016.98 | 1017.09 |
| 20 | 2033.03 | 2033.28 | 2033.52 | 2033.75 | 2033.96 | 2034.17 |
| 30 | 3049.55 | 3049.92 4066.56 | 3050.28 | 3050.62 | 3050.95 | 3051.26 |
| 40 | 4066.07 | 4066.56 | 400704 5083.80 | 4067.49 5084.36 | 4067.93 | 4068.34 |
| ${ }_{60}$ | 5082.58 6099.10 | 5083.20 6199.84 | 5083.80 6100.55 | 5084.36 6101.24 | 5084.91 610189 | 5085.43 6102.52 |
| $1{ }^{\prime}$ | 6099r.o | $6 \mathrm{r998.4}$ | 61005.5 | 61012.4 | 61018.9 | 61025.2 |
| 20 | 121982.0 | 121996.8 | 122011.1 | 122524.8 | 122037.8 | 122050.3 |
| 30 | 182973.1 | 182995.2 | 183016.6 | 183037.1 | 183056.8 | 183075.5 |
| 40 | 243964.1 | 243993.6 | 244022.2 | 244049.5 | 244075.7 | 244100.6 |
| 60 | 304955.1 | 304992.0 | 305027.7 | 30506 x .9 | 305094.6 | 305125.8 |
|  | 365946.1 | 365990.4 | 366033.2 | $366074 \cdot 3$ | 366113.5 | 366151.0 |
|  | $74^{\circ}$ | $75^{\circ}$ | $76^{\circ}$ | $77^{\circ}$ | $78^{\circ}$ | $79^{\circ}$ |
| $10^{\prime \prime}$ | 1017.18 | 1017.28 | 1017.37 | 1017.45 | 1017.53 | 1017.60 |
| 20 | 2034.37 | 2034.56 | 2034.73 | 2034.90 | 2035.05 | 2035.19 |
| 30 40 | 3051.56 4068.74 | 3051.84 4069.12 | 3052.10 4069.46 | 3052.35 4069.80 | 3052.58 | 3052.79 |
| 50 | 5085.92 | 5086.40 | ${ }_{5086.83}$ | 4069.80 5087.24 | 4070.10 5037.63 | 4070.38 5087.98 |
| 60 | 6 ro3.11 | 6103.67 | 6104.20 | 6104.69 | 6 6105.16 | 50105.58 |
| ro' | $6 \mathrm{rO31.1}$ | 61036.7 | 61042.0 | 61046.9 | 6105 r .6 | 61055.8 |
| 20 | 122062.2 | 122073.5 | 122083.9 | 122093.9 | 122103.1 | 122111.5 |
| 30 | 183093.3 | 183110.2 | 183125.9 | 183140.8 | 183154.7 | 183167.3 |
| 40 | 244124.4 | 244147.0 | 244167.8 | 244187.8 | 244206.2 | 244223.0 |
| $6{ }_{60}$ | 305155.5 366186.6 | 305183.7 366230.4 | 305209.8 | 305234.7 | 305257,8 | 305278.8 |
| 60 | 366180.6 | 366220.4 | 366251.8 | 366281.6 | 366309.4 | 366334.6 |

Smithsonian Tables.
[Derivation of table explained on p. xlix.]


Table 18.
LENGTHS OF TERRESTRIAL ARCS OF PARALLEL.
[Derivation of table explained on p. xlix.]

| Longitude Interval. | Latitude. $25^{\circ}$ | Latitude. $26^{\circ}$ | Latitude. $27^{\circ}$ | $\begin{gathered} \text { Latitude. } \\ 28^{\circ} \end{gathered}$ | Latitude. $29^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feet. | Feet. | Feet. | Feet. | Feet. |
| $10^{\prime \prime}$ 20 | 920.03 1840.05 | 912.44 1824.88 | 904.58 1809.16 | 896.44 1792.88 | $\begin{array}{r} 888.03 \\ \mathbf{7 7 6 . 0 6} \end{array}$ |
| 20 30 | 1840.05 2760.08 | 1824.88 2737.33 | 1809.16 2713.74 | 1792.88 2689.32 | $\begin{aligned} & 1776.06 \\ & 2664.09 \end{aligned}$ |
| 30 40 | 2760.08 3680.11 | 2737.33 3649.77 | 27184 368.32 | 3585.76 | 3552.12 |
| 50 | 4600.14 | 4562.21 | 4522.89 | 4482.20 | 4440.15 |
| 60 | 5520.17 | 5474.65 | $5427 \cdot 47$ | 5378.64 | 5328.18 |
| $10^{\prime}$ | 55202.7 | 54746.5 | 54274.7 | 53786.4 | 53281.8 |
| 20 | 110403.3 | 109493.0 | 108549.5 | 107572.9 | 106563.5 |
| 30 | 165605.0 | 164239.5 | 162824.2 | 161359.3 | 159845.3 |
| 40 | 220806.6 | 218986.1 | 217099.0 | 215145.7 | $213 \times 27.1$ |
|  | 276008.3 | 273732.6 | 271373.7 | 268932.2 | 266408.8 |
| 60 | 331209.9 | 328479.1 | 325648.4 | 322718.6 | 319690.6 |
|  | $30^{\circ}$ | $31^{\circ}$ | $32^{\circ}$ | $33^{\circ}$ | $34^{\circ}$ |
| $10^{\prime \prime}$ | 879.35 | 870.40 | 861.18 | $85 \mathrm{s.71}$ | 841.97 |
| 20 | 1758.70 | 1740.80 | $\mathbf{1 7 2 2 . 3 7}$ | 1703.41 | 1683.94 |
| 30 | 2638.04 | 2611.20 | 2583.55 | 2555.12 | 2525.91 |
| 40 | 3517.39 | 3481.59 | 3444.74 | 3406.83 | 3367.88 |
| 50 | 4396.74 | 4351.99 | 4305.92 | 4258.53 | 4209.85 |
| 60 | 5276.09 | 5222.39 | 5167.10 | 5110.24 | 5051.82 |
| $10^{\prime}$ | 52760.9 | 52223.9 | 51671.0 | 51102.4 | 50518.2 |
| 20 | 105521.8 | 104447.8 | 103342.1 | 102204.8 | 101036.4 |
| 30 | 158282.6 | 156671.8 | 155013.1 | 153307.3 | 151554.6 |
| 40 | 2 r1043.5 | 208895.7 | 206684.2 | 204409.7 | 202072.8 |
| 50 | 263804.4 | 261519.6 | 258.355.2 | 255512.1 | 25259 1.0 |
|  | 3 16565.3 | 313343.5 | 310026.3 | 306614.5 | 303109.2 |
|  | $35^{\circ}$ | $36^{\circ}$ | $37^{\circ}$ | $3^{80}$ | $39^{\circ}$ |
| 1011 | 831.98 | 822.73 | 8 xr .23 | 800.48 |  |
| 20 | 1663.95 | 1643.46 | 1622.46 | $\times 600.97$ | $\begin{array}{r}1578.98 \\ \hline\end{array}$ |
| 30 | 2495.93 | 2465.19 | 2433.69 | 2401.45 | 2368.48 |
| 40 | 3327.9 Pr | 3286.91 | 3244.92 | 3201.93 | 3157.97 |
| 50 | 4159.88 | 4108.64 | 4056.15 | 4002.42 | 3947.46 |
| 60 | 4991.86 | 4930.37 | 4867.38 | 4802.90 | 4736.95 |
| $10^{\prime}$ | 49918.6 | 49303.7 | 48673.8 | 48029.0 | 47369.5 |
| 20 | 99837.2 | 98607.4 | 97347.6 | 96058.0 | 94739. 7 |
| 30 | 149755.8 | 147931.2 | 146021.4 | 144087.0 | 142108.6 |
| 40 | 199674.3 | 197214.9 | 194695.2 | 192116.0 | 189478.2 |
| 5060 | 249592.9 | 246518.6 | 243369.0 | 240145.0 | 236847.7 |
|  | 279511.5 | 295822.3 | 292042.8 | 288174.0 | 284217.2 |
|  | $40^{\circ}$ | $41^{\circ}$ | $42^{\circ}$ | $43^{\circ}$ | $44^{\circ}$ |
| $10^{\prime \prime}$ | $77^{8.26}$ | 766.79 | 755.08 | 743.15 | 730.98 |
| 20 | 1556.52 | 1533.58 | 15 r 0.17 | 1486.29 | 1461.96 |
| 30 | 2334.78 | 2300.37 | 2265.25 | 2229.44 | 2192.95 |
| 40 | 3113.04 | 3067.16 | 3020.33 | 2972.59 | 2923.93 |
| 50 | 3891.30 | 3833.94 | 3775.42 | 3715.73 | 3654.91 |
| 60 | 4669.56 | 4600.73 | 4530.50 | 4458.88 | 4385.89 |
| $10^{\prime}$ | 46695.6 | $46007 \cdot 3$ | 45305.0 | 44588.8 | $43^{88} 5^{8.9}$ |
| 20 | 93391.2 | 92014.7 | $906 \mathrm{ro.0}$ | 89177.6 | 87717.9 |
| 30 | 140086.7 | 138022.0 | 1359 ¢5.0 | 133766.4 | ${ }^{131576.8}$ |
| 40 | 186782.3 | 184029.3 | 181220.0 | 178355.2 | 1754.35 .8 |
| 50 60 | $233477 \cdot 9$ 280173.5 | 230036.7 276044.0 | 226525.0 271830.1 | 222944.0 267532.8 | $\begin{aligned} & 219294 \cdot 7 \\ & 263 \times 53.6 \end{aligned}$ |
|  | $45^{\circ}$ | $46^{\circ}$ | $47^{\circ}$ | $48^{\circ}$ | $49^{\circ}$ |
| $10^{\prime \prime}$ |  |  |  |  | 666.87 |
| 20 | 1437.19 | 1411.97 | 1386.32 | $1360.24$ | 1333.75 |
| 30 | 2155.78 | 2117.96 | 2079.48 | 2040.36 | 2000.62 |
| 40 | 2874 -38 | 2823.94 | 2772.64 | 2720.49 | 2667.50 |
| 50 | 3592.97 | 3529.93 | 3465.80 | 3400.61 | 3334.37 |
| 60 | 4311.56 | 4235 -91 | 4158.96 | 4080.73 | 4001.25 |
| $1{ }^{\prime}$ | 43115.6 | 42359.1 | 41589.6 | 40807.3 | 40012.5 |
| 20 | 8623 x .3 | 84718.2 | 83179.2 | 81614.6 | 80024.9 |
| 30 | 129346.9 | 127077.3 | 124768.7 | 122421.9 | 120037.4 |
| 40 | 172462.5 | 169436.5 | 166358.3 | 163229.2 | 160049.9 |
| 50 | 215578.2 | 212795.6 | 207947.9 | 204036.4 | 200062.3 |
| 60 | 258693.8 | 254154.7 | 249537.5 | $244843 \cdot 7$ | 240074.8 |

Smithsonian Tables.
[Derivation of table explained on p. xlix.]

| Longitude Interval. | Latitude. $50^{\circ}$ | Latitude. $5^{\circ}$ | Latitude. $52^{\circ}$ | Latitude. $53^{\circ}$ | Latitude. $54^{\circ}$ | Latitude. $55^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Feat. | Freet. | Feet. | Feet. | Feet. | Feet. |
| $10^{1 /}$ | 653.42 | 639.77 | 625.92 | 61 r .88 | 597.65 | 583.23 |
| 20 | 1306.85 | 1279.54 | 1251.84 | 1223.76 | 1195.30 | 1166.47 |
| 30 | 1960.27 | 1919.31 | 1877.76 | 1835.63 | 1792.94 | 1749.70 |
| 40 | 2613.69 | 2559.08 | 2503.68 | 2447.51 | 2390.59 | 2332.93 |
| 50 60 | 3267.12 3930.54 | 3198.85 | 3129.60 | 3059.39 | 2988.28 | 2916.16 |
| 60 | 3930.54 | 3838.62 | $3755 \cdot 52$ | 3671.27 | 3585.89 | 3499 -40 |
| 10 20 | 39205.4 | 38386.2 | 37555.2 | 36712.7 | 35858.9 | 34994.0 |
| 20 | 78480.8 | 76772.4 | 75110.4 | 73425.4 | 71717.8 | 69988.0 |
| 30 | 117616.1 | 115158.6 | 112665.6 | 110138.0 | 107576.6 | 10498 I .9 |
| 40 50 | 156821.5 | 153544.8 | 150220.8 | 146850.7 | $143435 \cdot 5$ | $139975 \cdot 9$ |
| 50 60 | 196026.9 235232.3 | 191931.0 230317.2 | 187776.0 225331.2 | 183563.4 220276.1 | 179294.4 $2 \times 5153.3$ | $\begin{array}{r} 174969.9 \\ 209963-9 \end{array}$ |
|  | $56^{\circ}$ | $57^{\circ}$ | $58^{\circ}$ | $59^{\circ}$ | $60^{\circ}$ | $61^{\circ}$ |
| $\begin{aligned} & 10^{\prime \prime} \\ & 20 \\ & 30 \\ & 40 \\ & 50 \\ & 60 \end{aligned}$ | 568.64 | 553.87 | 538.93 | 523.82 | 508.55 | 493.13 |
|  | 1137.28 | 1107.74 | 1077.86 | 1047.65 | 1017.11 | 986.26 |
|  | 1705.92 | 166 t .6r | 1616.79 | 1571.47 | 1525.66 | 1479.38 |
|  | 2274.56 | 2215.48 | 2155.72 | 2095.29 | 2034.22 | 1972.52 |
|  | 2843.20 | 2769.35 | 2694.64 | 2619.12 | 2542.77 | 2465.64 |
|  | 3412.83 | 3323.22 | $3233 \cdot 57$ | 3142.94 | 3051.33 | 2958.77 |
| 1020 | 34118.3 68236.7 | 33232.2 | 32335.7 64671.5 |  |  | 29587.7 |
|  | 68236.7 | 66464.4 | 64671.5 | 62858.8 | 61026.6 | 59175-5 |
| 30 | 102355.0 | 99696.6 | 97007.2 | 94288.1 | 91539.9 | 88763.2 |
| 4050 | 136473 -4 | 132928.8 | 120343.0 | 125717.5 | 122053.2 | 118351.0 |
|  | 170591.7 | 166161.0 | 161678.7 | 157146.9 | 152566.5 |  |
| 60 | 204710.0 | 199393.2 | 194014.4 | 188576.3 | 183079.8 | $177526.4$ |
|  | $62^{\circ}$ | $63^{\circ}$ | $64^{\circ}$ | $65^{\circ}$ | $66^{\circ}$ | $67^{\circ}$ |
| 1020 | 477.55 | 461.83 | 445.96 | 429.95 | ${ }_{4} 19.82$ | 397.55 |
|  | 955.10 | 923.65 | 891.98 | 859.91 | 827.63 | 795.10 |
| 30 | 1432.66 | 1385.48 | 1337.88 | 1289.86 | 1241.44 | 1192.64 |
| 4050 | 1910.21 | $1847 \cdot 3$ I | 1783.84 | 1719.81 | 1655.26 | 1590.19 |
|  | 2387.76 | 2309.14 | 2229.80 | 2149.76 | 2069.08 | 1987.74 |
| 60 | 2865.31 | 2770.96 | 2675.75 | 2579.72 | 2482.89 | 2385.29 |
| 1020 | 28653.1 | 27709.6 | 26757.5 | 25797.2 | 24828.9 | 23852.9 |
|  | 57306.2 | 55419.2 | 53515.1 | 51594.4 | 49657.8 | 47705.8 |
| 3040 | 85959.4 | 83128.9 | 80272.6 | 77391.5 | 74486.7 | 71558.6 |
|  | 114612.5 | 110838.5 | 107030.2 | 103288.7 | 99315.6 | 95411.5 |
| 5060 | 143265.6 | 138548.1 | 133787.7 | 128985.9 | 124144.5 | 119264.4 |
|  | 171918.7 | 166257-7 | 160545.2 | 154783.1 | 148973.4 | 143117.3 |
|  | $68^{\circ}$ | $69^{\circ}$ | $70^{\circ}$ | $75^{0}$ | $72^{\circ}$ | $73^{\circ}$ |
| $1^{1 /}$ <br> 20 <br> 30 <br> 40 <br> so <br> 60 <br> $10^{\prime}$ <br> 20 <br> 30 <br> 40 <br> 50 <br> 60 | 381.16 762.32 | 364.65 729.30 | 348.03 696.06 | 331.30 662.60 | 314.47 628.94 | 297.54 595.08 |
|  | 1143.47 | 1093.95 | 1044.09 | 993.90 | 943.41 | 892.62 |
|  | 1524.63 | 1458.60 | 1392.12 | 1325.20 | 1257.88 | 1190.16 |
|  | 1905.79 | 1823.25 | 1740.14 | 1656.50 | 1572.34 | $148770$ |
|  | 2286.95 | 2187.90 | 2088.17 | 1987.81 | 1886.8ı | 1785.23 |
|  | 22869.5 | 21879.0 | 20881.7 | 19878.1 | 18868. 1 | 17852.3 |
|  | 45739.0 | 43758.0 | 41763.5 | 39756.1 | 37736.3 | 35704.7 |
|  | 68608.4 | 65637.0 | 62645.2 | 59634.2 | 56604.4 | 53557.0 |
|  | 91477.9 | 87516.0 | 83527.0 | 79512.2 | 75472.6 | 71409.4 |
|  | 114347.4 137216.9 | 109395.0 | 104408.7 125290.4 | 99390.3 r19368.4 | 94340.7 113208.8 | 89261.7 107114.0 |
|  | 137216.9 | 131274.0 | 125290.4 | $\underline{19268.4}$ | 113208.8 | 107114.0 |
|  | $74^{\circ}$ | $75^{\circ}$ | $76^{\circ}$ | $77^{\circ}$ | $78^{\circ}$ | $79^{\circ}$ |
| $\mathrm{ra}^{\prime \prime}$ | 280.52 | 26.3 .41 | 246.22 | 228.96 | 211.62 |  |
| 20 | 561.04 | 526.82 | 492.44 |  | 423.24 63485 | 388.43 582.64 |
| 30 | 841.56 | 790.23 | 738.66 | 686.86 | 634.85 | 582.64 |
| 40 | 1122.08 | 1053.64 | 984.88 | 915.82 | 846.47 | 776.86 |
| 60 | 1402.60 1683.15 | 1317.06 1580,47 | 1231.10 1477.33 | 1144.78 1373.73 | 1058.09 | 971.08 $1165-29$ |
|  | 1683.15 | 1580,47 | $1477 \cdot 33$ | 1373.73 | 1269.71 | 1165-29 |
| $\mathrm{ra}^{\prime}$ | 1683 I .1 | 15804.7 | 14773.3 | 13737.3 | 12697.1 | 11652.9 |
| 20 | 33662.3 | 31609.3 | 29546.5 | 27474.6 | 25394.2 | 23305.8 |
| 3040 | $50493 \cdot 4$ | 47414.0 | 44319.8 | 41211.9 | 38091.2 | 34958.7 |
|  | 67324.6 | 63218.6 | 59093.0 | 54949.2 68686.5 | 50788.3 63485.4 | 46611.6 58264.5 |
| 5060 | 84155.7 100986.8 | 79023.3 94828.0 | 73866.3 88639.6 | 68686.5 82423.8 | 63485.4 76182.5 | 58264.5 69917.4 |
|  | 100986.8 | 94828.0 | 88639.6 | 82423.8 | 76182.5 | 69917.4 |

table 19.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\frac{25}{250 \pi}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{15}$ ' longitude. |  | $3{ }^{\prime}$ longitude. |  | 45' longitude. |  | $1{ }^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | * | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $0^{\circ} 00^{\prime}$ |  | 4.383 | . 000 | 8.766 | . 000 | 13.148 | . 000 | 17.531 | . 000 |
| 15 | $4 \cdot 353$ | $4 \cdot 383$ | . 000 | 8.766 | . 000 | 13.148 | . 000 | 17.531 | . 001 |
| 30 | 8.706 | 4.383 | . 000 | 8.765 | . 000 | 13.148 | . 001 | 17.530 | . 001 |
| 45 | 13.059 | $4 \cdot 382$ | . 000 | 8.765 | . 001 | 13.147 | . 001 | 17.530 | . 002 |
| 100 | 17.412 | $4 \cdot 382$ | . 000 | 8.764 | . 001 | 13.146 | . 001 | 17.528 | . 003 |
| 15 | 4.353 | 4.382 | . 000 | 8.764 | . 001 | 13.145 | . 002 | 17.527 | . 003 |
| 30 | 8.706 | $4 \cdot 381$ | . 000 | 8.763 | . 001 | 13.144 | . 002 | 17.525 | . 004 |
| 45 | 13.059 | $4 \cdot 381$ | . 000 | 8.762 | . 001 | 13.142 | . 003 | 17.523 | . 005 |
| 200 | 17.412 | 4.380 | . 000 | 8.760 | . 001 | 13.141 | . 003 | 17.521 | . 005 |
| 15 | 4.353 | $4 \cdot 379$ | . 000 | 8.759 | . 001 | 13.138 | . 003 | 17.518 | . 006 |
| 30 | 8.706 | 4.379 | . 000 | 8.757 | . 001 | 13.536 | . 004 | 17.514 | . 007 |
| 45 | 13.059 | $4 \cdot 378$ | . 000 | 8.755 | . 002 | 13.133 | . 004 | 17.511 | . 007 |
| 300 | 17.413 | $4 \cdot 377$ | . 001 | 8.753 | . 002 | 13.130 | . 004 | 17.507 | . 008 |
| 15 | 4.353 | 4.376 | . 001 | 8.751 | . 002 | 13.127 | . 005 | 17.503 | . 008 |
| 30 | 8.706 | 4.375 | . 001 | 8.749 | . 002 | 13.124 | . 005 | 17.498 | . 009 |
| 45 | 13.060 | 4.373 | . 001 | 8.747 | . 002 | 13.120 | . 006 | 17.494 | . 009 |
| 400 | 17.413 | 4.372 | . 001 | 8.744 | . 003 | 13.116 | . 006 | 17.488 | . 010 |
| 15 | 4.353 | 4.371 | . 001 | 8.742 | . 003 | 13.112 | . 006 | 17.483 | . OII |
| 30 | 8.707 | $4 \cdot 369$ | . 001 | 8.739 | . 003 | ${ }^{1} 3.108$ | . 007 | 17.478 | . 012 |
| 45 | 13.060 | $4 \cdot 368$ | . 001 | 8.736 | . 003 | 13.104 | . 007 | 17.472 | . 013 |
| 500 | 17.413 | 4.366 | . 001 | 8.732 | . 003 | 13.099 | . 007 | 17.465 | . 013 |
| 15 | 4.353 | 4.364 | . 001 | 8.729 | . 003 | 13.094 | . 008 | 17.458 | . 014 |
| 30 | 8.707 | $4 \cdot 363$ | . 001 | 8.725 | . 004 | 13.088 | . 008 | 17.451 | . 014 |
| 45 | 13.060 | $4 \cdot 361$ | . 001 | 8.722 | . 004 | 13.082 | . 008 | 17.443 | . 015 |
| 600 | 17.414 | $4 \cdot 359$ | . 001 | 8.718 | . 004 | 13.076 | . 009 | 17.435 | . 016 |
| 15 30 | 4.354 8.707 | 4.357 | . 001 | 8.714 8.710 | . 004 | 13.071 | . 009 | 17.428 | .017 |
| 30 45 | 8.707 13.061 | 4.355 4.353 | . 001 | 8.710 8.705 | . 004 | 13.064 | . 010 | 17.419 | . 017 |
| 45 | 13.061 | 4.353 | . 001 | 8.705 | . 004 | 13.058 | . 010 | 17.410 | . 018 |
| 700 | 17.414 | 4.350 | . 001 | 8.701 | . 005 | 13.051 | . 010 | 17.4r: | . 019 |
| 15 |  | 4.348 | . 001 | 8.696 | . 005 | ${ }_{1} 3.044$ | . 011 |  |  |
| 30 | 8.707 | 4.346 | . 001 | 8.691 | . 005 | 13.036 | . 011 | 17.382 | . 020 |
| 45 | 13.061 | 4.343 | . 001 | 8.686 | . 005 | 13.029 | . 011 | 17.372 | . 020 |
| 800 | 17.415 | 4.340 | . 001 | 8.681 | . 005 | 13.021 | . 012 | 17.362 | . 021 |
| 15 |  | 4.338 | . 001 |  | . 005 | 13.013 | . 012 | 17.351 | . 022 |
| 30 | 8.708 | 4.335 | . 001 | 8.670 | . 006 | 13.005 | . 013 | 17.340 | . 022 |
| 45 | 13.062 | 4.332 | . 002 | 8.664 | . 006 | 12.996 | . 013 | 17.328 | . 023 |
| 900 | 17.416 | 4.329 | . 002 | 8.658 | . 006 | 12.987 | . 013 | 17.316 | . 024 |
| 15 | 4.354 8.708 | 4.326 | . 002 | 8.652 | . 006 | 12,979 | ,014 | 17,305 | . 024 |
| 30 | 8.708 13.062 | 4.323 4.320 | . 0022 | 8.646 8.640 | . 006 | 12.969 | . 014 | $17.292$ | . 025 |
| 45 | 13.062 | 4.320 | . 002 | 8.640 | . 006 | 12,960 | .014 | 17.280 | . 026 |
| 1000 | 17.417 | 4.317 | . 002 | 8.633 | . 006 | 12.950 | . 015 | 17,266 | . 026. |

CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $2 \sigma \delta_{00}$.
[Derivation of table explained on pp. liii - lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{5} 5^{\prime}$ longitude. |  | $30^{\prime}$ loggitude. |  | $45^{\prime}$ longitude. |  | $x^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $10^{\circ} 00^{\prime}$ | . . . . . . | $4 \cdot 317$ | . 002 | 8.633 | . 006 | 12.950 | . 015 | 17.266 | . 026 |
| 15 | 4.354 | 4.313 | . 002 | 8.626 | . 007 | 12.940 | . 015 | 17.253 | . 027 |
| 30 | 8.709 | 4.310 | . 002 | 8.620 | . 007 | 12.930 | . 015 | 17.240 | . 027 |
| 45 | 13.063 | $4 \cdot 306$ | . 002 | 8.6 r 3 | . 007 | 12.919 | .016 | 17.226 | . 028 |
| 1100 | 17.418 | $4 \cdot 303$ | . 002 | 8.606 | . 007 | 12.908 | . 016 | 17.211 | . 029 |
| 15 | $4 \cdot 355$ | 4.299 | . 002 | 8. 598 | . 007 | 12.897 | . 016 | 17.196 | . 029 |
| 30 | 8.709 | 4.295 | . 002 | 8.591 | . 007 | 12.886 | . 017 | 17.182 | . 030 |
| 45 | 13.064 | 4.292 | . 002 | 8.583 | . 008 | 12.875 | . 017 | 17.166 | . 031 |
| 1200 | 17.419 | 4.288 | . 002 | 8.575 | . 008 | 12.863 | .017 | 17.150 | .031 |
| 15 | 4.355 | 4.284 | . 002 | 8.567 | . 008 | 12.851 | . 018 | 17.134 | . 032 |
| 30 | 8.710 | 4.280 | . 002 | 8.559 | . 008 | 12.839 | . 018 | 17.118 | . 032 |
| 45 | 13.065 | 4.275 | . 002 | 8.55 I | . 008 | 12.826 | . 019 | 17.102 | . 033 |
| 1300 | 17.420 | 4.271 | . 002 | 8.542 | . 008 | 12.813 | . 019 | 17.084 | . 034 |
| 15 | 4.355 | 4.267 | . 002 | 8.534 | . 009 | 12.800 | . 019 | 17.067 | . 034 |
| 30 | 8.711 | 4.262 | . 002 | 8.525 | . 009 | 12.787 | . 020 | 17.050 | . 035 |
| 45 | 13.066 | 4.258 | . 002 | 8.516 | . 009 | 12.774 | . 020 | 17.032 | . 035 |
| 1400 | 17.421 | 4.253 | . 002 | 8.507 | . 009 | 12.760 | . 020 | 17.013 | . 036 |
| 15 | 4.356 | 4.249 | . 002 | 8.498 | . 009 | 12.746 | . 021 | 16.995 | . 036 |
| 30 | 8.711 | 4.244 | . 002 | 8.488 | . 009 | 12.732 | . 021 | 16.976 | . 037 |
| 45 | 13.067 | 4.239 | . 002 | 8.479 | . 009 | 12.718 | . 021 | 16.957 | . 038 |
| 1500 | 17.423 | 4234 | . 002 | 8.469 | . 010 | 12.703 | . 022 | 16.938 | .038 |
| 15 | 4.356 | 4.229 | . 002 | 8.459 | . 010 | 12.688 | . 022 | 16.918 | .039 |
| 30 | 8.712 | 4.224 | . 002 | 8.449 | . 010 | 12.673 | . 022 | 16.898 | . 039 |
| 45 | 13.068 | 4.219 | . 002 | 8.439 | . 010 | 12.658 | . 022 | 16.877 | . 040 |
| 1600 | 17.424 | 4.214 | . 003 | 8.428 | . 010 | 12.642 | . 023 | 16.856 | . 041 |
| 15 | 4.356 | 4.209 | . 003 | 8.417 | . 010 | 12.626 | . 023 | 16.835 | . 041 |
| 30 | 8.713 | 4.204 | . 003 | 8.407 | . 010 | 12.610 | . 023 | 16.814 | . 042 |
| 45 | 13.069 | $4 \cdot 198$ | . 003 | 8.396 | . OII | 12.594 | . 024 | 16.792 | . 042 |
| 1700 | 17.426 | 4.192 | . 003 | 8.385 | .orr | 12.577 | . 024 | 16.770 | . 043 |
| 15 | 4.357 | 4.187 | . 003 | 8.374 | . OII | 12.561 | . 024 | 16.748 | . 043 |
| 30 | 8.714 | 4.181 | . 003 | 8.362 | . 011 | 12.544 | . 025 | 16.725 | . 044 |
| 45 | 13.071 | 4.175 | . 003 | 8.35 I | . 011 | 12.526 | . 025 | 16.702 | . 044 |
| 1800 | 17.427 | 4.170 | . 003 | 8.339 | . OLI | 12.509 | . 025 | 16.679 | . 045 |
| 15 | 4.357 | 4.164 | . 003 | 8.327 | . 011 | 12.49 I | . 026 | 16.655 | . 045 |
| 30 | 8.715 | 4.158 | . 003 | 8.316 | . 012 | 12.473 | . 026 | I6.631 | . 046 |
| 45 | 13.072 | 4.152 | . 003 | 8.303 | . 012 | 12.455 | . 026 | 16.606 | . 046 |
| 1900 | 17.429 | 4.145 | . 003 | 8.291 | . 012 | 12.436 | . 026 | 16.582 | . 047 |
| 15 | 4.358 | 4.139 | . 003 | 8.278 | . 012 | 12.418 | . 027 | 16.557 | . 048 |
| 30 | 8.716 | 4.133 | . 003 | 8.266 | . 012 | 12.399 | . 027 | 16.532 | . 048 |
| 45 | 13.073 | 4.127 | . 003 | 8.253 | . 012 | 12.380 | . 027 | 16.506 | . 048 |
| 2000 | 17.431 | 4.120 | . 003 | 8.240 | . 012 | 12.360 | . 028 | 16.480 | . 049 |

Table 19.

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR- |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $15^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | 45 loagitude. |  | $x^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inchas. | Inches. |
| 20 ${ }^{\circ} 0^{\prime}$ |  | 4.120 | . 003 | 8.240 | . 012 | 12.360 | . 028 | 16.480 | . 049 |
| 15 | 4.358 | 4.114 | . 003 | 8.227 | . 012 | 12.340 | . 028 | 16.454 | . 050 |
| 30 | 8.717 | 4.107 | . 003 | 8.214 | . 113 | 12.321 | . 028 | 16.428 | . 050 |
| 45 | 13.075 | 4.100 | . 003 | 8.200 | .013 | 12.301 | . 029 | 16.401 | . 051 |
| 2100 | 17.433 | 4.094 | . 003 | 8.187 | .013 | 12.280 | . 029 | 16.374 | .051 |
| 15 | 4.359 | 4.087 | . 003 | 8.173 | .OI3 | 12.260 | . 029 | 16.346 | .052 |
| 30 | 8.718 | 4.080 | . 003 | 8.159 | . 013 | 12.239 | . 029 | 16.318 | . 052 |
| 45 | 13.076 | 4.073 | . 003 | 8.145 | . 013 | 12.218 | . 030 | 16.291 | . 053 |
| 2200 | 17.435 | 4.066 | . 003 | 8.13I | . 013 | 12.197 | . 030 | 16.262 | . 053 |
| 15 | 4.359 | 4.058 | . 003 | 8.117 | .013 | 12.175 | . 030 | 16.234 | . 054 |
| 30 | 8.719 | 4.051 | . 003 | 8.102 | . 014 | 12.154 | .030 | 16.205 | . 054 |
| 45 | 13.078 | 4.044 | . 003 | 8.088 | . 014 | 12.132 | .031 | 16.176 | . 055 |
| 2300 | 17.437 | 4.036 | . 003 | 8.073 | . 014 | 12.109 | . 031 | 16.146 | . 055 |
| 15 | 4.360 | 4.029 | . 003 | 8.058 | . 014 | 12.087 | .031 | 16.116 | . 055 |
| 30 | 8.720 | 4.021 | . 003 | 8.043 | . 014 | 12.064 | . 031 | 16.086 | . 056 |
| 45 | 13.080 | 4.014 | . 004 | 8.028 | . 014 | 12.041 | .032 | 16.055 | . 056 |
| 2400 | 17.439 | 4.006 | . 004 | 8.012 | . 014 | 12.018 | . 032 | 16.024 | .057 |
| 15 | 4.360 | 3.998 | . 004 | 7.997 | . 014 | 1 l .995 | . 032 | 15.993 | . 057 |
| 30 | 8.721 | 3.990 | . 004 | 7.981 | . 014 | 11.971 | . 032 | 15.962 | . 058 |
| 45 | 13.081 | 3.982 | . 004 | 7.965 | .015 | 11.948 | . 033 | 15.930 | . 058 |
| 2500 | 17.442 | 3.974 | . 004 | 7.949 | . 015 | 11.923 | . 033 | 15.898 | .059 |
| 15 | 4.361 | 3.966 | . 004 | 7.933 | .015 | 11.899 | . 033 | ${ }^{1} 5.865$ | . 059 |
| 30 | 8.722 | 3.958 | . 004 | 7.916 | .015 | 11.874 | . 033 | 15.832 | . 059 |
| 45 | 13.083 | 3.950 | . 004 | 7.900 | .015 | 11.850 | . 034 | 15.800 | .060 |
| 2600 | 17.444 | 3.942 | . 004 | 7.883 | . 015 | 11.825 | . 034 | 15.767 | . 060 |
| 15 30 | 4.362 8.723 | 3.933 3.925 | . 004 | 7.866 7.849 | . 015 | 11.800 | . 034 | 15.733 | . 061 |
| 30 | 8.723 13.085 | 3.925 | . 004 | 7.849 | . 015 | 11.774 | . 034 | 15.699 | .06r |
| 45 | ${ }^{13} 3.085$ | 3.916 | . 004 | 7.833 | .015 | 11.749 | . 035 | 15.665 | .06I |
| 2700 | 17.446 | 3.908 | . 004 | 7.816 | . 015 | 11.723 | . 035 | 15.631 | . 062 |
| 15 | 4.362 | 3.899 | . 004 | 7.798 | . 016 | 11.697 | . 035 | 15.596 |  |
| 30 | 8.724 | 3.890 | . 004 | 7.780 | . 016 | 11.671 | . 035 | 15.561 | .063 |
| 45 | 13.087 | 3.88 I | . 004 | 7.763 | .016 | 11.644 | . 036 | 15.526 | .063 |
| 2800 | 17.449 | 3.873 | . 004 | 7.745 | . 016 | 11.618 | . 036 | 15.490 | . 064 |
| 15 30 | 4.363 8.726 | 3.863 3.854 | . 004 | 7.727 7.709 | . 016 | 11.591 | . 036 | 15.454 |  |
| 45 | 13.726 13.088 | 3.854 3.845 | . 004 | 7.709 7.691 | . .016 | 11.563 11.536 | .036 .036 | 15.418 15.382 | . 064 |
| 2900 | 17.451 | 3.836 | . 004 | 7.673 | . 016 | 11.509 | . 036 | 15.345 | . 065 |
| 15 30 | 4.363 8.727 | 3.827 3.817 | . 004 | 7.654 | . 016 | 11.48 I | . 037 | ${ }^{1} 5 \cdot 308$ | .065 |
| 45 | 8.727 13.091 | 3.817 3.808 | .004 .004 | 7.635 7.616 | . .1016 | 11.453 11.425 | .037 .037 | 15.270 15.233 | . 066 |
| 3000 | 17.454 | 3.799 | . 004 | $7 \cdot 598$ | .017 | 11.396 | . 037 | 15.195 | . 066 |

[^23][Derivation of table explained on pp. liii-lvi.]

| 능 |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{5} 5^{\prime}$ longitude. |  | 30' longitude. |  | $45^{\prime}$ longitude. |  | $\mathrm{I}^{\circ} \mathrm{l}$ longitude. |  |
|  |  | x | $y$ | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $30^{\circ} 00^{\prime}$ |  | 3.799 | . 004 | 7.598 | . 017 | II. 396 | . 037 | 15.195 | . 066 |
| 15 | 4.364 | 3.789 | . 004 | 7.578 | . 017 | 11.367 | . 037 | 15.156 | . 067 |
| 30 | 8.728 | 3.779 | . 004 | 7.559 | .017 | $\underline{1} .338$ | . 038 | 15.118 | . 067 |
| 45 | 13.092 | 3.770 | . 004 | 7.540 | .017 | 11.309 | .038 | 15.079 | . 067 |
| 3100 | 17.457 | 3.760 | . 004 | $7 \cdot 520$ | . 017 | II. 280 | . 038 | 15.040 | . 068 |
| 15 | 4.365 | 3.750 | . 004 | 7.500 | .017 | 11.250 | . 038 | 15.001 | . 068 |
| 30 | 8.730 | 3.740 | . 004 | 7.480 | . 017 | 11.221 | . 038 | 14.961 | . 068 |
| 45 | 13.095 | 3.730 | . 004 | $7 \cdot 460$ | .017 | 11.191 | . 038 | 14.92 I | . 068 |
| 3200 | 17.460 | 3.720 | . 004 | 7.441 | .017 | 11.161 | . 039 | 14.881 | . 069 |
| 15 | $4 \cdot 366$ | 3.710 | . 004 | 7.420 | .017 | 11.130 | . 039 | 14.840 | . 069 |
| 30 | 8.73 I | $3.700^{\prime}$ | . 004 | 7.400 | .017 | 11.100 | . 039 | 14.799 | . 069 |
| 45 | 13.097 | 3.690 | . 004 | 7.379 | . 017 | 11.069 | . 039 | 14.758 | . 070 |
| 3300 | 17.462 | 3.679 | . 004 | 7.359 | . 017 | 11.038 | . 039 | 14.718 | . 070 |
| 15 | 4.366 | 3.669 | . 004 | 7.338 | . 018 | 11.007 | . 039 | 14.676 | . 070 |
| 30 | 8.733 | 3.658 | . 004 | $7 \cdot 317$ | . 018 | 10.975 | . 040 | 14.633 | . 070 |
| 45 | 13.099 | 3.648 | . 004 | 7.296 | . 018 | 10.943 | . 040 | 14.591 | . 071 |
| 3400 | 17.465 | 3.637 | . 004 | 7.275 | . 018 | 10.912 | . 040 | 14.549 | . 071 |
| 15 | $4 \cdot 367$ | 3.626 | . 004 | 7.253 | . 018 | 10.879 | . 040 | 14.506 | . 071 |
| $3{ }^{\circ}$ | 8.734 | 3.616 | . 004 | 7.231 | . 018 | 10.847 | . 040 | 14.463 | . 071 |
| 45 | 13.101 | 3.605 | . 004 | 7.210 | .c18 | 10.815 | . 040 | 14.420 | . 072 |
| 3500 | 17.468 | 3-594 | . 004 | 7.188 | . 018 | 10.782 | . 040 | 14.376 | . 072 |
| 15 | 4.368 | 3.583 | . 004 | 7.166 | .018 | 10.749 | .041 | 14.332 | . 072 |
| 30 | 8.735 | 3.572 | . 004 | 7.144 | . 018 | 10.716 | .041 | 14.288 | . 072 |
| 45 | 13.103 | $3 \cdot 561$ | . 005 | 7.122 | . 018 | 10.683 | . 041 | 14.244 | . 073 |
| 3600 | 17.471 | 3.550 | . 005 | 7.100 | .oı8 | 10.650 | . 041 | 14.200 | . 073 |
| 15 | 4.368 | $3 \cdot 539$ | . 005 | 7.077 | . 018 | 10.616 | . 041 | 14.154 | . 073 |
| 30 | 8.736 | $3 \cdot 527$ | . 005 | 7.054 | . 018 | 10.582 | . 041 | 14.109 | . 073 |
| 45 | 13.105 | 3.516 | . 005 | 7.032 | . 018 | 10.547 | .04I | 14.053 | . 073 |
| 3700 | 17.473 | $3 \cdot 504$ | . 005 | 7.009 | . 018 | 10.513 | . 041 | 14.018 | . 074 |
|  | $4 \cdot 369$ | 3.493 | . 005 | 6.986 | . 018 | 10.479 | . 041 | 13.972 | . 074 |
| $3{ }^{\circ}$ | 8.738 | 3.481 | . 005 | 6.963 | .or8 | 10.444 | . 042 | 13.925 | . 074 |
| 45 | 13.108 | 3.470 | . 005 | 6.939 | .aI8 | 10.409 | . 042 | 13.879 | . 074 |
| 3800 | 17.477 | $3 \cdot 45^{8}$ | . 005 | 6.916 | . 019 | 10.374 | . 042 | 13.832 | . 074 |
| 15 | 4.370 | 3.446 | . 005 | 6.892 | . 019 | 10.339 | . 042 | 13.785 | . 074 |
| 30 | 8.740 | 3.434 | . 005 | 6.869 | . 019 | 10.303 | . 042 | 13.737 | . 075 |
| 45 | 13.110 | 3.422 | . 005 | 6.845 | .019 | 10.267 | . 042 | 13.690 | . 075 |
| 3900 | 17.480 | 3.41I | . 005 | 6.821 | . 019 | 10.232 | . 042 | 13.642 | . 075 |
| 15 |  | 3.398 | . 005 | 6.797 | . 019 | 10.195 | . 042 | 13.594 | . 075 |
| 30 | 8.74 I | $3 \cdot 386$ | . 005 | 6.773 | .or9 | 10.159 | . 042 | 13.545 | . 075 |
| 45 | 13.112 | 3.374 | . 005 | 6.748 | .or9 | 10.123 | . 042 | 13.497 | . 075 |
| 4000 | 17.483 | $3 \cdot 362$ | . 005 | 6.724 | . 019 | 10.086 | . 042 | 13.448 | . 075 |

Smitisonian Tables.
[Derivation of table explained on p. Hiii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }_{15}{ }^{\text {' }}$ longitude. |  | 30 longitude. |  | 45' longitude. |  | $\mathrm{x}^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $40^{\circ} 00^{\prime}$ |  | 3.362 | . 005 | 6.724 | . 019 | 10.086 | . 042 | 13.448 | . 075 |
| 15 | 4.371 | $3 \cdot 350$ | . 005 | 6.699 | . 019 | 10.049 | . 042 | 13.399 | . 075 |
| 30 | 8.743 | 3.337 | . 005 | 6.675 | . 019 | 10.012 | . 043 | 13.349 | . 076 |
| 45 | 13.114 | 3.325 | . 005 | 6.650 | . 019 | 9.975 | . 043 | 13.300 | . 076 |
| 4100 | 17.486 | $3 \cdot 312$ | . 005 | 6.625 | . 019 | 9.937 | . 043 | 13.250 | . 076 |
| 15 | $4 \cdot 372$ | $3 \cdot 300$ | . 005 | 6.600 | . 019 | 9.900 | . 043 | 13.200 | . 076 |
| 30 | 8.744 | 3.287 | . 005 | 6.575 | . 019 | 9.862 | . 043 | 13.149 | . 076 |
| 45 | 13.117 | 3.275 | . 005 | 6.549 | . 019 | 9.824 | . 043 | 13.098 | . 076 |
| 4200 | 17.489 | 3.262 | . 005 | 6.524 | . 019 | 9.786 | . 043 | 13.048 | . 076 |
| 15 | 4.373 8.746 | 3.249 | . 005 | 6.498 | . 019 | 9.747 | . 043 | 12.996 | . 076 |
| 30 | $\begin{array}{r}8.746 \\ \\ \hline 3\end{array}$ | 3.236 | . 005 | 6.472 | . 019 | 9.709 | . 043 | 12.945 | . 076 |
| 45 | 13.119 | 3.223 | . 005 | 6.447 | . 019 | 9.670 | . 043 | 12.893 | . 076 |
| 4300 | 17.492 | 3.210 | . 005 | 6.421 | . 019 | 9.631 | . 043 | 12.842 | . 076 |
| 15 | 4.374 | 3.197 | . 005 | 6.394 | . 019 | 9. 592 | . 043 | 12.789 | . 076 |
| 30 | 8.747 | 3.184 | . 005 | 6.368 | . 019 | 9.552 | . 043 | 12.736 | . 076 |
| 45 | 13.121 | 3.170 | . 005 | 6.342 | . 019 | 9.513 | . 043 | 12.684 | . 076 |
| 4400 | 17.495 | 3.158 | . 005 | 6.316 | . 019 | 9.473 | . 043 | 12.631 | . 077 |
| 15 | . 4.375 | 3.144 | . 005 | 6.289 | . 019 | 9.433 | . 043 | 12.578 | . 077 |
| 30 | 8.749 | 3.131 | .005 | 6.262 | . 019 | $9 \cdot 393$ | . 043 | 12.524 | . 077 |
| 45 | 13.124 | 3.118 | . 005 | 6.235 | . 019 | 9.353 | . 043 | 12.471 | . 077 |
| 4500 | 17.498 | 3.104 | . 005 | 6.209 | . 019 | 9.313 | . 043 | 12.417 | . 077 |
| 15 | 4.375 8.757 | 3.091 | . 005 | 6.181 | . 019 | 9.272 | . 043 | 12.363 | . 077 |
| 30 | 8.751 | 3.077 | . 005 | 6.154 | . 019 | 9.231 | . 043 | 12.308 | . 077 |
| 45 | 13.126 | 3.063 | . 005 | 6.127 | . 019 | 9.190 | . 043 | 12.254 | . 077 |
| 4600 | 17.501 | 3.050 | . 005 | 6.100 | . 019 | 9.150 | . 043 | 12.200 | . 077 |
| 15 | 4.376 8.752 | 3.036 | . 005 | 6.072 | . 019 | 9. 108 | . 043 | 12.144 | . 077 |
| 30 | $\begin{array}{r}8.752 \\ \hline\end{array}$ | 3.022 | . 005 | 6.044 | . 019 | 9.067 | . 043 | 12.089 | . 077 |
| 45 | 13.128 | 3.008 | . 005 | 6.017 | . 019 | 9.025 | . 043 | 12.033 | . 077 |
| 4700 | 17.504 | 2.994 | . 005 | 5.989 | . 019 | 8.983 | . 043 | 11.978 | . 076 |
| 15 | 4.377 | 2.980 | . 005 | 5.961 | .019 | 8.941 | . 043 | 11.922 | . 076 |
| 30 | 8.754 | 2.966 | . 005 | 5.933 | . 019 | 8.899 | . 043 | 11.865 | . 076 |
| 45 | 13.131 | 2.952 | . 005 | 5.904 | .019 | 8.857 | . 043 | 11.809 | . 076 |
| 4800 | 17.508 | 2.938 | . 005 | 5.876 | . 019 | 8.814 | . 043 | 11.752 | . 076 |
| 15 |  | 2.924 | . 005 | 5.848 | . 019 | 8.771 | . 043 |  |  |
| 30 | 8.755 | 2.909 | . 005 | 5.819 | . 019 | 8.728 8.686 | . 043 | 11.638 | . 076 |
| 45 | 13.133 | 2.895 | . 005 | 5.790 | .019 | 8.686 | . 043 | 11.581 | . 076 |
| 4900 | 17.511 | 2.88I | . 005 | 5.762 | . 019 | 8.643 | . 043 | 11.524 | . 076 |
| 15 |  | 2.866 | . 005 |  | . 019 |  | . 043 |  |  |
| $30$ | $\begin{array}{r}8.757 \\ \hline\end{array}$ | 2.852 2.837 | . 005 | 5.704 | .019 | 8.555 | . 043 | 11.407 | . 076 |
| 45 | 13.135 | 2.837 | . 005 | 5.675 | . 019 | 8.512 | . 042 | 11.349 | . 076 |
| 5000 | 17.514 | 2.823 | . 005 | 5.646 | . 019 | 8.468 | . 042 | 11.291 | . 076 |

[Derivation of table explained on p. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR- |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15' longitude. |  | $30^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $\mathrm{r}^{\circ} \mathrm{longitude}$. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $50^{\circ} 00^{\prime}$ |  | 2.823 | . 005 | 5.646 | . 019 | 8.468 | . 042 | 1 I .291 | . 076 |
| 15 | $4 \cdot 379$ | 2.808 | . 005 | 5.616 | . 019 | 8.424 | . 042 | 1 I .232 | . 075 |
| 30 | 8.758 | 2.793 | . 005 | $5 \cdot 587$ | . 19 | 8.380 | . 042 | 1 I .174 | . 075 |
| 45 | 13.137 | 2.779 | . 005 | 5.557 | .019 | 8.336 | . 042 | 11.114 | . 075 |
| 5100 | 17.517 | 2.764 | . 005 | 5.528 | . 019 | 8.291 | . 042 | 11.055 | . 075 |
| 15 | 4.380 | 2.749 | . 005 | 5.498 | . 019 | 8.247 | . 042 | 10.996 | . 075 |
| 30 | 8.760 | 2.734 | . 005 | $5 \cdot 468$ | . 019 | 8.202 | . 042 | 10.936 | . 075 |
| 45 | 13.140 | 2.719 | . 005 | $5 \cdot 438$ | .019 | 8.157 | . 042 | 10.876 | . 075 |
| 5200 | 17.520 | 2.704 | . 005 | 5.408 | .019 | 8.112 | . 042 | 10.816 | . 074 |
| 15 | 4.38 I | 2.689 | . 005 | $5 \cdot 378$ | . 019 | 8.067 | . 042 | 10.756 | . 074 |
| 30 | 8.761 | 2.674 | . 005 | $5 \cdot 347$ | . 019 | 8.021 | . 041 | 10.695 | . 074 |
| 45 | 13.142 | 2.659 | . 005 | $5 \cdot 317$ | . 018 | 7.976 | . 041 | 10.634 | . 074 |
| 5300 | 17.523 | 2.643 | . 005 | 5.287 | . 018 | 7.930 | . 041 | 10.573 | . 074 |
| 15 | 4.38 I | 2.628 | . 005 | 5.256 | . 018 | 7.884 | . 041 | 10.512 | . 074 |
| 30 | 8.763 | 2.613 | . 005 | 5.225 | . 018 | 7.838 | . 041 | 10.451 | . 073 |
| 45 | 13.144 | 2.597 | . 005 | 5.195 | . 018 | 7.792 | . 041 | 10.389 | . 073 |
| 5400 | 17.526 | 2.582 | . 005 | 5.164 | . 018 | 7.745 | . 041 | 10.327 | . 073 |
| 15 | 4.382 | 2.566 | . 005 | 5.133 | . OI 8 | 7.699 | . 041 | 10.266 | . 073 |
| 30 | $\begin{array}{r}8.764 \\ \hline\end{array}$ | 2.551 | . 005 | 5.102 | . 18 | 7.652 | . 041 | 10.203 | . 073 |
| 45 | 13.147 | 2.535 | . 005 | 5.070 | . 018 | 7.606 | . 041 | 10.145 | . 072 |
| 5500 | 17.529 | 2.520 | . 005 | 5.039 | . 018 | 7.559 | . 041 | 10.078 | . 072 |
| 15 | 4.383 | 2.504 | . 004 | 5.008 | . 018 | 7.512 | . 040 | 10.016 | . 072 |
| 30 | $\begin{array}{r}8.766 \\ \\ \hline\end{array}$ | 2.488 2.472 | . 0004 | 4.976 | . O . 88 | 7.465 7.417 | . 040 | 9.953 9.890 | . 072 |
| 45. | 13.149 | 2.472 | . 004 | 4.945 | . 018 | 7.417 | . 040 | 9.890 | . 071 |
| 5600 | 17.532 | 2.456 | . 004 | 4.913 | . 018 | $7 \cdot 370$ | . 040 | 9.826 | . 071 |
| 15 | 4.384 | 2.441 | . 004 | 4.88 I | . 018 | 7.322 | . 040 | 9.763 | . 071 |
| 30 | 8.767 | 2.425 | . 004 | 4.849 | . 018 | 7.274 | . 040 | 9.699 | . 071 |
| 45 | 13.151 | 2.409 | . 004 | 4.817 | . 018 | 7.226 | . 040 | 9.635 | . 070 |
| 5700 | 17.535 | 2.393 | . 004 | 4.785 | . 018 | 7.178 | . 039 | 9.571 | . 070 |
| 15 |  | 2.377 | . 004 | 4.753 | .017 | 7.130 | . 039 | 9.507 | . 070 |
| 30 | $\begin{array}{r}8.769 \\ \hline\end{array}$ | 2.361 2.344 | . 004 | 4.721 4.680 | . 017 | 7.082 7.033 | . 039 | 9.442 9.378 | .070 .069 |
| 145 | 13.153 | 2.344 | . 004 | 4.689 | . 017 | 7.033 | . 039 | 9.378 | . 069 |
| 5800 | 17.537 | 2.328 | . 004 | 4.656 | . 017 | 6.985 | . 039 | 9.313 | . 069 |
| 15 | 4.385 | 2.312 | . 004 | 4.624 | . 017 | 6.936 | . 039 | 9.248 | . 069 |
| 30 | 8.770 | 2.296 | . 004 | $4 \cdot 591$ | .017 | 6.887 | . $33^{8}$ | 9.183 | . 068 |
| 45 | 13.155 | 2.279 | . 004 | 4.559 | . 017 | 6.838 | . 038 | 9.117 | . 068 |
| 5900 | 17.540 | 2.263 | . 004 | 4.526 | . 017 | 6.789 | . 038 | 9.052 | . 068 |
| 15 |  | 2.246 | . 004 |  | .017 |  |  | 8.986 |  |
| 30 | $\begin{array}{r}4.3872 \\ \hline\end{array}$ | 2.230 | . 004 | 4.460 4.427 | . 017 | 6.690 6.641 | . 038 | 8.920 8.854 | . 067 |
| 45 | 13.157 | 2.214 | . 004 | 4.427 | . 017 | 6.641 | . 038 | 8.854 | . 067 |
| 6000 | 17.543 | 2.197 | . 004 | 4.394 | . 017 | 6.591 | . 037 | 8.788 | . 067 |

Table 19.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\overline{2 \delta 0000 . ~}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{5} 5^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | 45' longitude. |  | $\mathrm{s}^{0}$ longitude. |  |
|  |  | $\mathbf{x}$ | y | x | y | $x$ | y | x | y |
|  | Iuches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Incines. | Inches. |
| $60^{\circ} 00^{\prime}$ |  | 2.197 | . 004 | 4.394 | . 017 | 6.591 | . 037 | 8.788 | . 067 |
| 15 | $4 \cdot 386$ | 2.180 | . 004 | 4.361 | . 017 | 6.541 | . 037 | 8.722 | . 066 |
| 30 | 8.773 | 2.164 | . 004 | $4 \cdot 327$ | . 016 | 6.491 | . 037 | 8.655 | . 066 |
| 45 | 13.159 | 2.147 | . 004 | 4.294 | . 016 | 6.441 | . 037 | 8.588 | . 066 |
| 6 x 00 | 17.546 | 2.130 | . 004 | 4.261 | . 016 | 6.391 | . 037 | 8.521 | . 065 |
| 15 | 4.387 | 2.114 | . 004 | 4.227 | . 016 | 6.340 | . 036 | 8.454 | . 065 |
| 30 | 8.774 | 2.097 | . 004 | 4.194 | . 016 | 6.290 | . 036 | 8.387 | . 064 |
| 45 | 13.161 | 2.080 | . 004 | 4.160 | . 016 | 6.240 | . 036 | 8.320 | . 064 |
| 6200 | 17.548 | 2.063 | . 004 | 4.126 | . 016 | 6.189 | . 036 | 8.252 | . 064 |
| 15 | 4.388 | 2.046 | . 004 | 4.092 | . 016 | 6.138 | . 036 | 8.184 | . 063 |
| 30 | 8.776 | 2.029 | . 004 | 4.058 | . 016 | 6.088 | . 035 | 8.117 | .063 |
| 45 | 13.163 | 2.012 | . 004 | 4.024 | . 016 | 6.036 | . 035 | 8.048 | . 063 |
| 6300 | 17.551 | 1.995 | . 004 | 3.990 | . 015 | 5.985 | . 035 | 7.980 | . 062 |
| 15 | 4.388 | 1. 978 | . 004 | 3.956 | .OI 5 | 5.934 | . 035 | 7.912 | . 062 |
| 30 | 8.777 | 1.961 | . 004 | 3.922 | . 015 | 5.883 | . 034 | 7.844 | .061 |
| 45 | 13.165 | 1.944 | . 004 | 3.887 | . 015 | 5.83 I | . 034 | 7.775 | . 061 |
| 6400 | 17.554 | 1. 926 | . 004 | 3.853 | . 015 | 5.780 | . 034 | 7.706 | . 060 |
| 15 | 4.389 | 1.909 | . 004 | 3.819 | . 015 | 5.728 | . 034 | 7.637 | . 060 |
| 30 | 8.778 | 1.892 | . 004 | 3.784 | . 015 | 5.676 | . 034 | 7.568 | . 060 |
| 45 | 13.167 | 1.875 | . 004 | 3.749 | . 015 | 5.624 | . 033 | 7.499 | . 059 |
| 6500 | 17.556 | 1.857 | . 004 | 3.715 | . 015 | 5.572 | . 033 | 7.430 | . 059 |
| 15 |  | 1.840 | . 004 | 3.680 | .015 | $5 \cdot 520$ | . 033 | 7.360 | . 059 |
| 30 | 8.779 | 1.823 | . 004 | 3.645 | . 014 | $5 \cdot 468$ | . 033 | 7.290 | . 058 |
| 45 | 13.169 | I. 805 | . 004 | 3.610 | . 014 | 5.45 | .032 | 7.220 | . 058 |
| 6600 | 17.559 | 1.788 | . 004 | 3.575 | . 014 | 5.363 | . 032 | 7.151 | . 057 |
| 15 | 4.390 | 1.770 | . 004 | 3.540 | . 014 | $5 \cdot 310$ | . 032 | 7.080 | . 057 |
| 30 | 8.780 | 1.753 | . 004 | 3.505 | . 014 | 5.258 | . 032 | 7.010 | . 056 |
| 45 | 13.171 | 1.735 | . 003 | 3.470 | . 014 | 5.205 | .031 | 6.940 | . 056 |
| 6700 | 17.561 | 1.717 | . 003 | 3.435 | . 014 | $5 \cdot 152$ | .03I | 6.870 | . 055 |
| 15 | 4.391 8.782 | 1.700 1.682 | . 003 | 3.400 3.364 | . 014 | 5.099 | .031 | 6.799 6.728 | . 055 |
| 30 45 | 8.782 13.172 | 1.682 1.664 | .003 .003 | 3.364 3.329 | . 01014 | 5.046 4.993 | . 31 | 6.728 6.658 | . 054 |
| 6800 | 17.563 | 1.647 | . 003 | 3.293 | . 013 | 4.940 | . 030 | 6.586 | . 053 |
| 15 |  | 1. 629 | . 003 | 3.258 | .013 | 4.886 | . 030 | 6.515 |  |
| 30 | $\begin{array}{r}8.783 \\ \hline\end{array}$ | 1.6II | . 003 | 3.222 | . 013 | 4.833 | . 029 | 6.444 | . 052 |
| 45 | 13.174 | 1.593 | . 003 | 3.186 | . 013 | 4.780 | . 029 | 6.373 | . 052 |
| 6900 | 17.565 | 1.575 | . 003 | 3.151 | . 013 | 4.726 | . 029 | 6.301 | . 051 |
| 15 |  | 1.557 | .003 | 3.115 | .013 | 4.672 |  |  |  |
| 30 | $\begin{array}{r}8.784 \\ \hline\end{array}$ | 1. 540 | . 003 | 3.079 | . 013 | 4.618 | . 028 | 6.158 | . 051 |
| 45 | 13.176 | 1.522 | . 003 | 3.043 | .012 | $4 \cdot 564$ | . 028 | 6.086 | . 050 |
| 7000 | 17.568 | 1.504 | . 003 | 3.007 | . 012 | 4.510 | . 028 | 6.014 | . 049 |

CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\frac{101}{250000 .}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $15^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | 45' longitude. |  | $5^{\circ}$ longitude. |  |
|  |  | $x$ | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $70^{\circ} 00^{\prime}$ |  | 1.504 | . 003 | 3.007 | . 012 | 4.510 | . 028 | 6.014 | . 049 |
| 15 | 4.392 | 1.486 | . 003 | 2.971 | . 012 | 4.456 | . 028 | 5.942 | . 049 |
| 30 | 8.785 | 1.467 | . 003 | 2.935 | . 01 | 4.402 | . 027 | 5.870 | . 048 |
| 45 | 13.177 | I. 449 | . 003 | 2.899 | . 012 | $4 \cdot 348$ | . 027 | 5.797 | . 048 |
| 7100 | 17.570 | 1.431 | . 003 | 2.862 | . 012 | 4.294 | . 027 | 5.725 | . 047 |
| 15 | 4.393 | 1.413 | . 003 | 2.826 | . 012 | 4.239 | . 026 | 5.652 | . 047 |
| 30 | 8 | 1.395 | . 003 | 2.790 | . 011 | 4.185 | . 026 | 5.580 | . 046 |
| 45 | 13.179 | 1.377 | . 003 | 2.753 | . OI | 4.130 | . 026 | $5 \cdot 507$ | . 046 |
| 7200 | 17.572 | 1. $35^{8}$ | . 003 | 2.717 | . 011 | 4.075 | . 025 | $5 \cdot 434$ | . 045 |
| 15 | 4.393 | 1.340 | . 003 | 2.681 | . OI 1 | 4.021 | . 025 | $5 \cdot 36 \mathrm{I}$ | . 045 |
| 30 | 8.787 | I. 322 | .003 | 2.644 | . OLI | 3.966 | . 025 | 5.288 | . 044 |
| 45 | 13.180 | 1.304 | . 003 | 2.607 | . OI 1 | 3.911 | . 024 | 5.215 | . 044 |
| 7300 | 17.573 | 1.285 | . 003 | 2.57 I | . 011 | 3.856 | . 024 | 5.142 | . 043 |
| 15 | 4.394 | 1.267 | . 003 | 2.534 | . OI 1 | 3.801 | . 024 | 5.068 | .043 |
| 30 | 8.788 | 1. 249 | . 003 | 2.497 | .oio | 3.746 | . 024 | 4.994 | . 042 |
| 45 | 13.181 | 1.230 | . 003 | 2.461 | . 010 | 3.691 | . 023 | 4.921 | . 041 |
| 7400 | 17.575 | 1.212 | . 003 | 2.424 | . 010 | 3.636 | . 023 | 4.848 | . 041 |
| 15 | 4.394 | 1.193 | . 003 | 2.387 | . 010 | $3 \cdot 580$ | . 023 | 4.774 | . 040 |
| 30 | 8.788 | 1.175 | . 002 | 2.350 | . 010 | 3.525 | . 022 | 4.700 | . 040 |
| 45 | 13.183 | I.I 56 | . 002 | 2.313 | . 010 | 3.470 | . 022 | 4.626 | . 039 |
| 7500 | 17.577 | 1.138 | . 002 | 2.276 | . 010 | 3.414 | . 022 | 4.552 | . 038 |
| 15 | 4.395 | 1.119 | . 002 | 2.239 | . 009 | $3 \cdot 358$ | . 021 | $4 \cdot 478$ | .038 |
| 30 | 8.789 | 1.101 | . 002 | 2.202 | . 009 | $3 \cdot 303$ | . 021 | 4.404 | . 037 |
| 45 | 13.184 | 1.082 | . 002 | 2.165 | . 009 | 3.247 | . 021 | 4.329 | . 037 |
| 7600 | 17.579 | 1.064 | . 002 | 2.127 | . 009 | 3.191 | . 020 | 4.255 | . 036 |
| 15 | 4.395 | 1.045 | . 002 | 2.090 | . 009 | 3.135 | . 020 | 4.180 | . 036 |
| 30 | 8.790 | 1.026 | . 002 | 2.053 | . 009 | 3.079 | . 020 | 4.106 | . 035 |
| 45 | 13.185 | 1.008 | . 002 | 2.016 | . 009 | 3.023 | . 019 | 4.031 | . 034 |
| 7700 | 17.580 | 0.989 | . 002 | 1.978 | . 008 | 2.967 | . 019 | 3.956 | . 034 |
| 15 | 4.395 | 0.970 | . 002 | - 1.941 | . 008 | 2.911 | . 019 | 3.882 | . 033 |
| 30 | 8.79 T | 0.952 | . 002 | 1.903 | . 008 | 2.855 | . 018 | 3.807 | . 033 |
| 45 | 13.186 | 0.933 | . 002 | 1.866 | . 008 | 2.799 | . 018 | 3.732 | . 032 |
| 7800 | 17.582 | 0.914 | . 002 | 1.828 | . 008 | 2.743 | . 018 | 3.657 | .031 |
| 15 | $4 \cdot 396$ | 0.895 | . 002 | 1.791 | . 008 | 2.686 | .017 | 3.582 | .031 |
| 30 | 8.791 | 0.877 | . 002 | 1.753 | .008 | 2.630 | .017 | 3.506 | . 030 |
| 45 | 13.187 | 0.858 | . 002 | 1.716 | . 008 | 2.573 | . 017 | 3.43 I | . 030 |
| 7900 | 17.583 | 0.839 | . 002 | ェ. 678 | . 007 | 2.517 | .016 | 3.356 | . 029 |
| 15 | 4.396 | 0.820 | . 002 | 1.640 | . 007 | 2.461 | . 016 | 3.28I | . 028 |
| 30 | 8.792 | 0.801 | . 002 | 1.603 | . 007 | 2.404 | . 016 | 3.205 | . 028 |
| 45 | 13.188 | 0.782 | . 002 | 1.565 | . 007 | 2.348 | . 015 | 3.130 | . 027 |
| 8000 | 17.584 | 0.764 | . 002 | 1.527 | . 007 | 2.29 I | . 015 | 3.054 | . 026 |

[Derivation of table explained on pp. liii- Ivi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5{ }^{\prime}$ | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $3{ }^{\prime}$ |  |  |  |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Iuches. | Inches. |  |  |  |
| $0^{\circ} 00^{\prime}$ |  | 2.922 | 5.844 | 8.765 | 11.687 | 14.609 | 17.531 | 盛気 | $0^{\circ}$ | $1{ }^{0}$ |
| 10 | 5.804 | 2.922 | 5.843 | 8.765 | 11.687 | 14.608 | 17.530 | 운..․․ |  |  |
| 20 | 11.608 | 2.922 | 5.843 | 8.765 | 11.686 | 14.608 | 17.530 |  |  |  |
| 30 | 17.412 | 2.922 | 5.843 | 8.765 | 11.686 | 14.608 | $17.53{ }^{\circ}$ |  | Inches. | Inches. |
| 40 | 23.216 | 2.922 | 5.843 | 8.764 | 11.686 | 14.608 | 17.529 | 5 | 0.000 | 0.000 |
| 50 | 29.020 | 2.921 | 5.843 | 8.764 | 11.686 | 14.607 | 17.528 | 15 | 0.000 .000 | . 0000 |
| 100 |  | 2.921 | 5.843 | 8.764 | 11.685 | 14.606 | 17.528 | 15 | . 000 | . 000 |
| 10 | 5.840 | 2.921 | 5.842 | 8.763 | 11.684 | 14.606 | 17.527 | 20 | . 000 | . 001 |
| 20 | 11.608 | 2.921 | 5.842 | 8.763 | 11.684 | 14.604 | 17.525 | 25 | . 000 | . 001 |
| 30 | 17.412 | 2.921 | 5.841 | 8.762 | 11.683 | 14.604 | 17.524 | 30 | ,00 | . 01 |
| 40 | 23.216 | 2.920 | 5.841 | 8.761 | 11.682 | 14.602 | 17.522 |  |  |  |
| 50 | 29.020 | 2.920 | 5.840 | 8.761 | 11.681 | 14.601 | 17.521 |  |  |  |
| 200 |  | 2.920 | 5.840 | 8.760 | 11.680 | 14.600 | 17.520 |  | $2{ }^{\circ}$ | $3^{\circ}$ |
| 10 | 5.804 | 2.920 | 5.839 | 8.759 | 11.678 | 14.598 | 17.518 |  |  |  |
| 20 | 11.608 | 2.919 | 5.839 | 8.758 | 11.677 | 14.596 | 17.516 | 5 | 0.000 | 0.000 |
| 30 | 17.412 | 2.919 | 5.838 | 8.757 | 11.676 | 14.594 | 17.513 | 10 | . 000 | . 000 |
| 40 | 23.216 | 2.918 | 5.837 | 8.756 8.755 | 11.674 | 14.592 | 17.511 | 15 | . 001 | . 001 |
| 50 | 29.020 | 2.918 | 5.836 | 8.755 | 11.673 | 14.591 | 17.509 | 20 | . 001 | . 002 |
| 300 |  | 2.918 | 5.836 | 8.753 | 11.671 | 14.589 | 17.507 | 25 30 | . 002 | . 003 |
| 10 | 5.804 | 2.917 | 5.835 | 8.752 | 11.669 | 14.586 | 17.504 | 30 | . 003 |  |
| 20 | 11.608 | 2.917 | 5.834 | 8.750 | 11.667 | 14.584 | 17.501 |  |  |  |
| 30 | 17.413 | 2.916 | 5.832 | 8.749 | 11.665 | 14.581 | 17.497 |  |  |  |
| 40 | 23.217 | 2.916 | 5.831 | 8.747 | I 1.663 | 14.578 | 17.494 |  | $4^{\circ}$ | $5^{\circ}$ |
| 50 | 29.021 | 2.915 | 5.830 | 8.746 | 11.661 | 14.576 | 17.491 |  |  |  |
| 400 |  | 2.915 | 5.829 | 8.744 | II. 659 | 14.574 | 17.488 | 5 | 0.000 | 0.000 |
| 10 | 5.804 | 2.914 | 5.828 | 8.742 | 11.656 | 14.570 | 17.484 | 10 | . 001 | . 001 |
| 20 | 11.609 | 2.913 | 5.827 | 8.740 | 11.654 | 14.567 | 17.480 | 15 | . 001 | . 002 |
| 30 | 17.413 | 2.913 | 5.825 | 8.738 | 11.651 | 14.564 | 17.476 | 20 | . 002 | . 003 |
| 40 | 23.217 | 2.912 | 5.824 | 8.736 | 11.648 | 14.560 | 17.473 | 25 | . 004 | . 005 |
| 50 | 29.022 | 2.911 | 5.823 | 8.734 | 11.646 | 14.557 | 17.468 | 30 | . 005 | . 007 |
| 500 |  | 2.911 | 5.822 | 8.732 | 11.643 | 14.554 | 17.465 |  |  |  |
| 10 | 5.804 | 2.910 | 5.820 | $8.73{ }^{\circ}$ | 11.640 | 14.550 | 17.459 |  | $6^{\circ}$ | ${ }^{\circ}$ |
| 20 | 11.609 17.414 | 2.909 2.908 | 5.818 5.817 | 8.727 8.725 | 11.636 | 14.546 | 17.455 |  | 6 | $7{ }^{\circ}$ |
| 40 | 23.218 | 2.908 | 5.815 | 8.722 | $11.63{ }^{\circ}$ | 14.542 12.538 | 17.450 17.445 |  |  |  |
| 50 | 29.022 | 2.907 | 5.813 | 8.720 | 11.627 | 14.534 | 17.445 | 10 | 0.000 .001 | $\begin{array}{r} 0.000 \\ .001 \end{array}$ |
| 600 |  | 2.906 | 5.812 | 8.718 | 11.624 | 14.530 | 17.435 | 15 | . 002 | . 002 |
| 10 | 5.805 | 2.905 | 5.810 | 8.715 | 11.620 | 14.524 | 17.429 | 20 | . 004 | . 004 |
| 20 | 11.609 | 2.904 | 5.808 | 8.712 | 11.616 | 14.520 | 17.424 | 25 | . 006 | . 006 |
| 30 | 17.414 | 2.903 | 5.806 | 8.709 | 11.612 | 14.515 | 17.418 | $3{ }^{\circ}$ | . 008 | . 009 |
| 40 | 23.219 29.024 | 2.902 | 5.804 | 8.706 | 11.608 | 14.510 | 17.413 |  |  |  |
| 50 | 29.024 | 2.901 | 5.802 | 8.703 | 11.604 | 14.506 | 17.407 |  | 80 |  |
| 700 |  | 2.900 | 5.800 | 8.701 | 11.601 | 14.501 | 17.401 |  | 8 |  |
| 10 | 5.805 | 2.899 | 5.798 | 8.697 | 11.596 | 14.496 | $17.395$ |  |  |  |
| 20 | 11.610 17.415 | 2.898 2.897 | 5.796 | 8.694 | 11.592 | $14.49^{\circ}$ | $17 \cdot 387$ | 5 | 0.000 |  |
| 30 | 17.415 | 2.897 | 5.794 | 8.690 | 11.587 | 14.484 | 17.381 | 10 | . 001 |  |
| 40 | 23.220 29.025 | 2.896 | 5.79 I | 8.687 | 11.583 | 14.478 | $17 \cdot 374$ | 15 | . 003 |  |
| 50 | 29.025 | 2.895 | 5.789 | 8.684 | 11.578 | 14.473 | 17.368 | 20 | . 005 |  |
| 800 |  | 2.894 | 5.787 | 8.680 | 11.574 | 14.468 | 17-361 | 25 30 | . 0107 |  |

©mithsonian Tables.
[Derivation of table explained on pp. liii-lvi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ordinates of DEVELOPED parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ | $10^{\prime}$ | $15^{\prime \prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |  |  |  |
| $8^{\circ} 00^{\prime}$ |  | 2.894 | 5.787 | 8.680 | 11.574 | 14.468 | 17.361 | 碞鮞 |  |  |
| 10 | 5.805 | 2.892 | 5.784 | 8.677 | 11.569 | 14.46 r | 17.353 |  |  |  |
| 20 | 11.610 | 2.891 | 5.782 | 8.673 | 11.564 | 14.455 | 17.346 |  |  |  |
| 30 | 17.416 | 2.890 | 5.779 | 8.669 | 11.559 | 14.448 | 17.338 |  | Inches. | Inches |
| 40 | 23.221 | 2.888 | 5.777 | 8.666 | 11.554 | 14.442 | 17.331 | 5 | 0.000 | 0.000 |
| 50 | 29.026 | 2.887 | 5.775 | 8.662 | 11.549 | 14.436 | 17.324 | 10 | . 001 | . 001 |
| 900 |  | 2.886 | 5.772 | 8.658 | II. 544 | 14.430 | 17.3 | 15 | . 003 | . 003 |
| 10 | 5.806 | 2.885 | 5.769 | 8.654 | 11.539 | 14.424 | 17.308 | 20 | . 005 | . 0008 |
| 20 | II.6II | 2.883 | 5.767 | 8.650 | 1 I .533 | 14.416 | 17.300 | 30 | . 010 | . 008 |
| 30 | 17.417 | 2.882 | 5.764 | 8.646 | Ir. 528 | 14.410 | 17.291 |  |  |  |
| 40 | 23.222 | 2.88 r | 5.761 | 8.642 | 11.522 | 14.402 | 17.283 |  |  |  |
| 50 | 29.028 | 2.879 | 5.758 | 8.637 | 11.516 | 14.396 | 17.275 |  |  |  |
| 1000 |  | 2.878 | 5.755 | 8.633 | II.511 | 14.388 | 17.266 |  | $10^{\circ}$ | $\mathrm{II}^{\circ}$ |
| 10 | 5.806 | 2.876 | 5.752 | 8.628 | 11.504 | 14.380 | 17.257 |  |  |  |
| 20 | 11.612 | 2.875 | 5.749 | 8.624 | 11.498 | 14.373 | 17.248 | 5 | 0.000 | 0.000 |
| 30 | 17.417 | 2.873 | 5.746 | 8.619 | 11.492 | 14.366 | 17.239 | 10 | . 001 | . 002 |
| 40 | 23.223 | 2.872 | 5.743 | 8.614 | 11.486 | 14.358 | 17.229 | 15 | . 003 | . 004 |
| 50 | 29.029 | 2.870 | 5.740 | 8.610 | 11.480 | 14.350 | 17.220 | 20 | . 006 | . 006 |
| 1100 |  | 2.869 | 5.737 | 8.606 |  | 14.342 | 17.211 | 25 | .009 | . 010 |
| 10 | 5.806 | 2.867 | 5.734 | 8.60I | 11.468 | 14.334 | 17.201 | 30 | . 13 | . 014 |
| 20 | 11.612 | 2.865 | 5.730 | 8.596 | II 461 | 14.326 | 17.191 |  |  |  |
| 30 | 17.419 | 2.864 | 5.727 | 8.590 | 11.454 | 14.318 | 17.181 |  |  |  |
| 40 | 23.225 | 2.862 | 5.724 | 8.585 | 11.447 | 14.309 | 17.171 |  | $12^{\circ}$ | $13^{\circ}$ |
| 50 | 29.031 | 2.860 | $5 \cdot 720$ | 8.580 | 11.440 | 14.300 | 17.161 |  |  |  |
| 1200 |  | 2.858 | 5.717 | 8.575 | 11.434 | 14.292 | 17.150 | 5 | 0.000 | 0.000 |
| 10 | 5.807 | 2.857 | 5.713 | 8.570 | 11.426 | 14.282 | 17.139 | 10 | . 002 | . 002 |
| 20 | 11.613 | 2.855 | 5.709 | 8.564 | 11.419 | 14.274 | 17.128 | 15 | . 004 | . 004 |
| 30 | 17.420 | 2.853 | 5.706 | 8.559 | 11.412 | 14.264 | 17.117 | 20 | . 007 | . 007 |
| 40 | 23.226 | 2.851 | 5.702 | 8.553 | 1 I .404 | 14.256 | 17.107 | 25 | . 011 | . 012 |
| 50 | 29.033 | 2.849 | 5.698 | 8.548 | 11.397 | 14.246 | 17.095 | 30 | . 016 | .017 |
| 1300 |  | 2.847 | 5.695 | 8.542 | 11.390 | 14.237 | 17.084 |  |  |  |
| 10 | 5.807 1.654 | 2.846 2.844 | 5.691 5.687 | 8.536 8.530 | 11.382 11.374 | 14.228 14.218 | 17.073 |  | $14^{\circ}$ | $15^{\circ}$ |
| 20 | 11.614 17.421 | 2.844 2.842 | 5.687 5.683 | 8.530 | 11.374 11.366 | 14.218 | 17.001 17.049 |  |  |  |
| 40 | 17.4228 | 2.840 | 5.679 | 8.519 | 11.358 | 14.198 | 17.038 |  |  |  |
| 50 | 29.035 | 2.838 | 5.675 | 8.513 | 11.350 | 14.188 | 17.026 | 15 | 0.000 .002 | $\begin{array}{r} 0.001 \\ .002 \end{array}$ |
| 1400 |  | 2.836 | 5.671 | 8.507 | 11.342 | 14.178 | 17.014 | 15 20 | .004 .008 | $\begin{aligned} & .005 \\ & .009 \end{aligned}$ |
| 10 | 5.808 | 2.834 | 5.667 | 8.500 | 11.334 | 14.168 | 17.001 16.988 | 25 | . 012 | . 013 |
| 20 | 11.615 | 2.831 | 5.663 | 8.494 | 11.326 | 14.157 | 16.988 | 30 | . 018 | . 019 |
| 30 | 17.422 | 2.829 | 5.658 | 8.488 | 11.317 | 14.146 | 16.975 |  |  |  |
| 40 | 23.230 | 2.827 | 5.654 | 8.48 I | 11.308 | 14.136 | 16.963 |  |  |  |
| 50 | 29.038 | 2.825 | 5.650 | 8.475 | 11.300 | 14.125 | 16.950 |  | $16^{\circ}$ |  |
| 1500 |  | 2.823 | 5.646 | 8.469 | 11. 292 | 14.114 | 16.937 |  |  |  |
| 10 | 5.808 | 2.821 | 5.641 | 8.462 | 1 I .282 | 14.103 | 16.924 |  |  |  |
| 20 | 11.616 | 2.818 | 5.637 | 8.455 | 11.274 | 14.092 | 16.910 |  | 0.001 |  |
| 30 40 | 17.424 23.232 | 2.816 2.814 | 5.632 5.628 | 8.448 8.44 I | 11.264 11.255 | 14.080 14.069 | 16.897 16.883 | 10 | . 002 |  |
| 40 50 | 23.232 29.040 | 2.814 2.812 | 5.628 5.623 | 8.44 I 8.435 | 11.255 11.246 | 14.069 14.058 | 16.883 16.870 | 15 20 | .005 .009 |  |
| 50 | 29.040 | 2.812 | 5.623 | 8.435 | 11.246 | 14.0 |  | 25 | . 014 |  |
| 1600 |  | 2.809 | 5.619 | 8.428 | 11.237 | 14.046 | 16.856 | 30 | . 020 |  |

Table 20.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $12 \frac{1}{260 \sigma} \cdot$
[Derivation of table explained oa pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $10^{\prime}$ |  | $20^{\prime}$ | 25 | $30^{\prime}$ |  |  |  |
| $32^{\circ} 00^{\prime}$ | Inches. | Inches. | Inches. | Inches. | Inches. | Inches.$12.401$ | Inches. |  | $32^{\circ}$ | $33^{\circ}$ |
|  |  | 2.480 | 4.960 | 7.441 | 9.921 |  | $\begin{aligned} & 14.881 \\ & 14.854 \end{aligned}$ |  |  |  |
|  | 5.821 | 2.476 | 4.951 | 7.427 | 9.903 | 12.379 |  |  |  |  |
| 20 | I1.642 | 2.471 | 4.942 | 7.413 | 9.884 | 12.355 | 14.827 |  | Inches.$0.001$ | Inches. <br> 0.001 |
| 30 | 17.462 | 2.467 | 4.933 | $7 \cdot 400$ | 9.866 | 12.333 | 14.800 |  |  |  |
| 40 | 23.283 | 2.462 | 4.924 | $7 \cdot 386$ | 9.848 | 12.310 | 14.772 |  |  |  |
| 50 | 29.104 | 2.458 | 4.915 | 7.373 | 9.830 | 12.288 | 14.745 | 10 | . 004 | . 0.004 |
| 3300 |  | 2.453 | 4.806 | 7.359 | 9.812 | 12.265 | 14.717 | 15 20 | .009 .015 | . 0099 |
| 10 | 5.822 | 2.448 | 4.896 | 7.345 | 9.793 | 12.241 | 14.689 | 20 25 | . .024 | . .024 |
| 20 | 11.643 | 2.444 | 4.887 | 7.33 I | 9.774 | 12.218 | 14.661 | 30 | . 034 | . 035 |
| 30 | 17.465 | 2.439 | 4.878 | $7 \cdot 316$ | 9.755 | 12.194 | 14.633 |  |  |  |
| 40 | 23.287 | 2.434 | 4.868 | 7.302 | 9.736 | 12.171 | 14.605 |  |  |  |
| 50 | 29.109 | 2.429 | 4.859 | 7.288 | 9.718 | 12.147 | 14.576 |  |  |  |
| 3400 |  | 2.425 | 4.850 | 7.274 | 9.699 | 12.124 | 14.549 |  | $34^{\circ}$ | $35^{\circ}$ |
| 10 | 5.823 | 2.420 | 4.840 | 7.260 | 9.680 | 12.100 | 14.520 |  |  |  |
| 20 | 11.645 | 2.415 | 4.830 | 7.246 | 9.661 | 12.076 | 14.491 | 5 | 0.001 | 0.001 |
| 30 40 | 17.468 | 2.410 | 4.82 I | 7.231 | 9.642 | 12.052 | 14.462 | 10 | . 004 | . 004 |
| 40 | 23.291 | 2.406 | 4.81 II | 7.217 | 9.622 | 12.028 | 14.434 | 15 | . 009 | . 009 |
| 50 | 29.113 | 2.401 | 4.802 | 7.203 | 9.604 | 12.004 | 14.405 | 20 | . 016 | . 016 |
| 3500 |  | 2.396 | 4.792 | 7.188 | 9.584 | 11.980 | 14.376 | 25 30 | .025 | . 025 |
| 10 | 5.824 | 2.391 | 4.782 | 7.174 | 9.565 | 11.956 | 14.347 |  |  |  |
| 20 | 11.647 | 2.386 | 4.773 | 7.159 | 9.545 | 11.932 | 14.318 |  |  |  |
| 30 | 17.471 | 2.381 | 4.763 | 7.144 | 9.526 | 11.907 | 14.288 |  |  |  |
| 40 | 23.294 | 2.377 | 4.753 | 7.130 | 9.506 | 11.883 | 14.259 |  | $36^{\circ}$ | $37^{\circ}$ |
| 50 | 29.118 | 2.372 | 4.743 | 7.115 | 9.486 | 11.858 | 14.230 |  |  |  |
| 3600 | $\cdots$ | 2.367 | 4.733 | 7.099 | 9.466 | 11.833 | 14.200 | 5 | 0.001 | 0.001 |
|  | 5.824 | 2.362 | 4.723 | 7.085 | 9.446 | 11.808 | 14.170 | 10 | . 004 | . 004 |
| 20 | 11.649 | 2.357 | 4.713 | 7.070 | 9.426 | 11.783 | 14.139 | 15 | . 009 | . 009 |
| 30 | 17.473 | 2.351 | 4.703 | 7.055 | 9.406 | 11.757 | 14.109 | 20 | . 016 | . 016 |
| 40 | 23.297 | 2.346 | 4.693 | 7.039 | 9.386 | 11.732 | 14.078 | 25 | . 025 | . 026 |
| 50 | 29.122 | 2.34 I | 4.683 | 7.024 | 9.366 | 11.707 | 14.048 | 30 | . 036 | . 037 |
| 3700 |  | 2.336 | 4.673 | 7.009 | 9.345 | II. 682 | 14.018 |  |  |  |
| 10 | 5.826 | 2.331 | 4.662 | 6.994 | 9.325 | 11.656 | 13.987 |  |  |  |
| 20 | 11.651 | 2.326 | 4.652 | 6.978 | 9.304 | 11.630 | 13.956 |  | $3^{8}$ | $39^{\circ}$ |
| 30 | 17.477 | 2.321 | 4.642 | 6.963 | 9.284 | 11.605 | 13.925 |  |  |  |
| 40 | 23.302 | 2.316 | 4.631 | 6.947 | 9.263 | 11.579 | 13.894 |  | 0.001 | 0.001 |
| 50 | 29.128 | 2.311 | 4.621 | 6.932 | 9.242 | 11.553 | 13.864 | 10 | . 004 | . 004 |
| $3^{80} 0$ |  | 2.305 | 4.6II | 6.916 | 9.222 | 11.527 | 13.832 | 15 | . 009 | . 009 |
| 10 <br> 20 <br>  | 5.827 11.653 | 2.300 2.295 | 4.600 | 6.900 6.884 | 9.200 | 11.501 | 13.801 | 20 | . 017 | . 017 |
| 20 30 | 11.653 17.480 | 2.295 2.290 | 4.590 4.579 | 6.884 6.869 | 9.179 | 11.474 | 13.769 | 30 | . 037 | . 037 |
| 30 40 | 17.400 23.306 | 2.290 2.284 | 4.579 4.568 | 6.869 6.853 | 9.158 | 11.448 | 13.737 |  |  |  |
| 50 | 29.133 | 2.279 | 4.568 4.55 | 6.853 6.837 | 9.137 9.116 | 11.421 | 13.705 13.673 |  |  |  |
|  |  |  |  |  |  |  |  |  | $40^{\circ}$ |  |
| 3900 10 | 5.828 | 2.274 | 4.548 4.537 | 6.821 | 9.095 | 11.369 | 13.642 |  |  |  |
| 20 | 11.635 | 2.263 | 4.526 | 6.789 | 9.052 | 11.315 | 13.15 | 5 | 0.001 |  |
| 30 | 17.483 | 2.258 | 4.515 | 6.773 | 9.030 | 11.288 | 13.545 | 10 | . 004 |  |
| 40 | 23.310 | 2.252 | 4.504 | 6.756 | 9.008 | 11.261 | 13.513 | 15 | . 009 |  |
| 50 | 29.138 | 2.247 | 4.493 | 6.740 | 8.987 | 11.234 | 13.480 | 20 | .017 |  |
| 4000 |  | 2.241 | 4.483 | 6.724 | 8.965 | 11.207 | 13.448 | 25 30 | . 0288 |  |

[Derivation of table explained on pp. liii-lvi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | 25 | $30^{\prime \prime}$ |  |  |  |
| $40^{\circ} 00^{\prime}$ | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |  |  |  |
|  |  | 2.24 I | 4.483 | 6.724 | 8.965 | 11.207 | 13.448 | 葹号 | $40^{\circ}$ | $41^{\circ}$ |
| 10 | 5.829 | 2.236 | 4.472 | 6.707 | 8.943 | 11.179 | 13.415 | 운 |  |  |
| 20 | 11.657 | 2.230 | 4.46 I | 6.69 I | 8.921 | II.I52 | 13.382 |  |  |  |
| 30 | 17.486 | 2.225 | 4.450 | 6.674 | 8.899 | 11.124 | 13.349 |  | Inches. | Inches. |
| 40 | 23.314 | 2.219 | 4.439 | 6.658 | 8.877 | 11.097 | 13.316 | $5^{\prime}$ | 0.001 | 0.001 |
| 50 | '29.143 | 2.214 | 4.428 | 6.641 | 8.855 | 11.069 | 13.283 | 10 | 0.001 .004 | 0.001 .004 |
| 410010 |  | 2.208 | 4.417 | 6.625 | 8.834 | 11.042 | 13.250 | 15 | . 009 | . 009 |
|  | 5.830 | 2.203 | 4.406 | 6.608 | 8.81 I | 11.014 | 13.250 13.217 | 20 | . 017 | . 017 |
| 20 | 11.659 | 2.197 | 4.394 | 6.591 | 8.788 | 10.985 | 13.183 | 25 30 | . .026 |  |
| 30 | 17.489 | 2.192 | 4.383 | 6.575 | 8.766 | 10.958 | 13.149 |  |  |  |
| 40 | 23.319 | 2.186 | 4.372 | 6.558 | 8.744 | 10.929 | 13.115 |  |  |  |
| 50 | 29.149 | 2.180 | 4.360 | 6.54 I | 8.721 | 10.901 | 13.081 |  |  |  |
| 420010 |  | 2.175 | 4.349 | 6.524 | 8.698 | 10.873 | 13.048 |  | $42^{\circ}$ | $43^{\circ}$ |
|  | 5.83 I | 2.169 | 4.338 | 6.507 | 8.676 | 10.844 | 13.013 |  |  |  |
| 20 | 11.661 | 2.163 | $4 \cdot 326$ | 6.490 | 8.653 | 10.816 | 12.979 | 5 | 0.001 | 0.001 |
| 30 | 17.492 | 2.157 | 4.315 | 6.472 | 8.630 | 10.787 | 12.945 | 10 | . 004 | . 004 |
| 40 | 23.323 | 2.152 | 4.303 | 6.455 | 8.607 | 10.759 | 12.910 | 15 | . 010 | . 010 |
| 50 | 29.154 | 2.146 | 4.292 | 6.438 | 8.584 | 10.730 | 12.876 | 20 | . 017 | . 017 |
| 4300 |  | 2.140 | 4.281 | 6.421 | 8.561 | 10.702 | 12.842 | 25 30 | .026 .038 | .027 .038 |
| 10 | 5.832 | 2.135 | 4.269 | 6.403 | 8.538 | 10.672 | 12.807 | 30 | . $3^{8}$ |  |
| 20 | I1. 663 | 2.129 | 4.257 | 6.386 | 8.514 | 10.643 | 12.772 |  |  |  |
| 30 | 17.495 | 2.123 | 4.246 | 6.368 | 8.491 | 10.614 | 12.737 |  |  |  |
| 40 | 23.327 | 2.117 | 4.234 | 6.351 | 8.468 | 10.585 | 12.701 |  | $44^{\circ}$ | $45^{\circ}$ |
| 50 | 29.159 | 2.111 | 4.222 | 6.333 | 8.444 | IO. 556 | 12.667 |  |  |  |
| 4400 |  | 2.105 | 4.210 | 6.316 | 8.42 I | 10.526 | 12.631 | 5 | 0.001 | 0.001 |
|  | 5.833 | 2.099 | 4.199 | 6.298 | 8.397 | 10.496 | 12.596 | 10 | . 004 | . 004 |
| 20 | 1 I .666 | 2.093 | 4.187 | 6.280 | 8.373 | I0.467 | 12.560 | 15 | . 010 | . 010 |
| 30 | 17.498 | 2.087 | $4 \cdot 175$ | 6.262 | 8.350 | 10.437 | 12.524 | 20 | . 017 | . 017 |
| 40 | 23.331 | 2.081 | 4.163 | 6.244 | 8.326 | 10.407 | 12.489 | 25 | . 027 | . 027 |
| 50 | 29.164 | 2.076 | 4.151 | 6.227 | 8.302 | 10.378 | 12.453 | 30 | . 038 | . 038 |
| 450 | $\cdots$ | 2.070 | 4.139 | 6.209 | 8.278 | 10.348 | 12.417 |  |  |  |
| 10 | 5.834 | 2.064 | 4.127 | 6.191 | 8.254 | 10.317 | 12.38 I |  |  |  |
| 20 | 11.668 | 2.057 | 4.115 | 6.172 | 8.230 | 10.288 | 12.345 |  | $46^{\circ}$ | $47^{\circ}$ |
| 30 | 17.501 | 2.051 | 4.103 | 6.154 | 8.206 | 10.257 | 12.308 |  |  |  |
| 40 | 23.335 29.169 | 2.045 2.039 | 4.091 4.079 | 6.136 6.118 | 8.181 8.157 | 10.226 | 12.272 | 5 | 0.001 | 0.001 |
| 50 | 29.169 | 2.039 | 4.079 | 6.118 | 8.157 | 10.197 | 12.236 | 10 | . 004 | . 004 |
| 4600 |  | 2.033 | 4.067 | 6.100 | 8.133 | 10.166 | 12.199 | 15 20 | . .1010 | . 010 |
| 10 | 5.835 | 2.027 | 4.054 | 6.08 I | 8.108 | 10.136 | 12.163 | 20 | . 017 | . 017 |
| 20 | 11.670 17.504 | 2.021 | 4.042 4.030 | 6.063 | 8.084 | 10.104 | 12.125 | 30 | . 038 | . 038 |
| 30 | 17.504 23.339 | 2.015 | 4.030 4.017 | 6.044 | 8.059 8.034 | 10.074 10.043 | 12.089 |  |  |  |
| 50 | 23.339 29.174 | 2.003 | 4.005 | 6.008 | 8.010 | 10.013 | 12.015 |  |  |  |
| 4700 |  | I. 996 | 3.992 | 5.989 | 7.985 | 9.98I | 11.978 |  | $4^{\circ}$ |  |
| 10 | 5.836 | I. 990 | 3.980 | 5.970 | 7.960 | 9.951 | 11.94 I |  |  |  |
| 20 | 11.672 | I. 984 | 3.968 | 5.951 | 7.935 | 9.919 | 11.903 | 5 | 0.001 |  |
| 30 | 17.508 | 1. 978 | 3.955 | 5.933 | 7.910 7885 | 9.888 | 11.866 | 10 | . 004 |  |
| 40 | 23.344 29.180 | 1.971 I. 965 | 3.943 | 5.914 | 7.885 | 9.857 | 11.828 | 15 | . 010 |  |
| 50 | 29.180 | 1.965 | 3.930 | 5.895 | 7.860 | 9.826 | I 1.791 | 20 25 | . 017 |  |
| 4800 |  | 1.959 | 3.917 | 5.876 | 7.835 | 9.794 | II 1.752 | 30 | . 038 |  |

Table 20.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE रहुणन.
[Derivation of table explained on pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $56^{\circ} 00^{\prime}$10 | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |  |  |  |
|  |  | I. 638 | 3.275 | 4.913 | 6.551 | 8.188 | 9.826 | 熍 | $56^{\circ}$ | $57^{\circ}$ |
|  | 5.845 | 1.63I | 3.265 | 4.892 | 6.522 | 8.153 | 9.784 | -1. |  |  |
| 20 | 11.690 | 1.624 | 3.247 | 4.870 | 6.494 | 8.118 | 9.74 I |  |  |  |
| 30 | 17.535 | 1.6i6 | 3.233 | 4.849 | 6.466 | 8.082 | 9.698 |  | Inches. | Inches. |
| 40 | 23.380 | 1.609 | 3.219 | 4.828 | 6.437 | 8.046 | 9.656 | 5 | 0.001 | 0.001 |
| 50 | 29.224 | 1.602 | 3.204 | 4.807 | 6.409 | 8.011 | 9.613 | 10 | . 004 | . 004 |
| 570010 |  | 1. 595 | 3.190 | 4.785 | 6.380 | 7.976 | 9.571 | 15 | . 009 | . 009 |
|  | 5.846 | 1.588 | 3.176 | 4.764 | 6.352 | 7.940 | 9.527 | 20 | . 016 | . 016 |
| 20 | 11.692 | 1.581 | 3.162 | 4.742 | 6.323 | 7.904 | 9.485 | 25 30 | . 036 | . 024 |
| 30 | 17.537 | 1. 574 | 3.147 | 4.721 | 6.294 | 7.868 | 9.442 |  |  |  |
| 40 | 23.383 | L. 566 | 3.133 | 4.699 | 6.266 | 7.832 | 9.398 |  |  |  |
| 50 | 29.229 | 1.559 | 3.119 | 4.678 | 6.237 | 7.796 | 9.356 |  |  |  |
| 5800 |  | 1.552 | 3.104 | 4.656 | 6.208 | 7.760 | 9.313 |  | $5^{\circ}$ | $59^{\circ}$ |
| 10 | 5.847 | 1. 545 | 3.090 | 4.634 | 6.179 | 7.724 | 9.269 |  |  |  |
| 20 | 11.694 | 1.538 | 3.075 | 4.613 | 6.150 | 7.688 | 9.226 | 5 | 0.001 | 0.001 |
| 30 | 17.540 | I. $53{ }^{\circ}$ | 3.061 | $4 \cdot 59 \mathrm{I}$ | 6.122 | 7.652 | 9.182 | Io | . 004 | . 004 |
| 40 | 23.387 | I. 523 | 3.046 | 4.569 | 6.092 | 7.616 | 9.139 | 15 | . 009 | . 008 |
| 50 | 29.234 | 1.516 | 3.032 | 4.547 | 6.063 | 7.579 | 9.095 | 20 | . 015 | . 015 |
| 590010 |  | I. 509 | 3.017 | 4.526 | 6.034 | 7.543 | 9.052 | 25 | .024 .034 | . 024 |
|  | 5.848 | I. 501 | 3.003 | 4.504 | 6.005 | 7.506 | 9.008 | 30 | . 034 | . 034 |
| 20 | 11.695 | I. 494 | 2.988 | 4.482 | 5.976 | 7.470 | 8.963 |  |  |  |
| 30 | 17.543 | 1. 487 | 2.973 | 4.460 | 5.946 | 7.433 | 8.920 |  |  |  |
| 40 | 23.391 | 1.479 | 2.959 | 4.438 | 5-917 | 7.396 | 8.876 |  | $60^{\circ}$ | $6 \mathrm{I}^{\circ}$ |
| 50 | 29.238 | 1.472 | 2.944 | 4.416 | 5.888 | $7 \cdot 360$ | 8.831 |  |  |  |
| 6000 |  | 1. 465 | 2.929 | 4.394 | 5.858 | $7 \cdot 323$ | 8.788 | 5 | 0.001 | 0.001 |
| 10 | 5.849 | I. 457 | 2.914 | $4 \cdot 372$ | 5.829 | 7.286 | 8.743 | Io | . 004 | . 004 |
| 20 | $\underline{11.697}$ | 1.450 | 2.900 | $4 \cdot 349$ | 5.799 | 7.249 | 8.699 | 15 | . 008 | . 008 |
| 30 | 17.546 | I. 442 | 2.885 | 4.327 | 5.770 | 7.212 | 8.654 | 20 | . 015 | . 014 |
| 40 | 23.394 | 1.435 | 2.870 | 4.305 | 5.740 | 7.175 | 8.610 | 25 | . 023 | . 023 |
| 50 | 29.243 | 1.428 | 2.855 | 4.283 | 5.710 | 7.138 | 8.566 | 30 | . 033 | . 033 |
| 6ı 00 |  | 1.420 | 2.840 | 4.261 | 5.681 | 7.101 | 8.521 |  |  |  |
| 10 | 5.850 | I. 413 | 2.825 | 4.238 | 5.651 | 7.064 | 8.476 |  |  |  |
| 20 | 11.699 | 1.405 | 2.810 | 4.216 | 5.621 | 7.026 | 8.43 I |  | $62^{\circ}$ | $63^{\circ}$ |
| 30 | 17.549 | 1. 398 | 2.795 | 4.193 | $5 \cdot 591$ | 6.988 | 8.386 |  |  |  |
| 40 50 | 23.398 29.248 | 1.390 1. 383 | 2.781 2.766 | 4.171 4.148 | 5.561 | 6.952 6.914 | 8.342 8.297 | 5 | 0.001 | 0.001 |
| 50 | 29.248 | 1.383 | 2.766 | 4.148 | $5 \cdot 53 \mathrm{I}$ | 6.914 | 8.297 | 10 | . 004 | . 003 |
| 6200 |  | 1. 375 | 2.751 | 4.126 | $5 \cdot 501$ | 6.877 | 8.252 | 15 20 | . 008 | . 008 |
| 10 | 5.850 | 土. 368 | 2.736 | 4.103 | 5.471 | 6.839 | 8.207 | 25 | . 0142 | . 014 |
| 20 | 11.701 | I. 360 | 2.720 | 4.081 | 5.441 | 6.801 | 8.16I | 30 |  | .031 |
| 30 | 17.551 | I. 353 | 2.705 | 4.058 | 5.410 | 6.763 | 8.116 | 3 | . 32 | . 3 |
| 50 | 23.402 | 1. 345 | 2.690 | 4.035 | $5 \cdot 380$ | 6.726 6.688 | 8.071 8.026 |  |  |  |
|  | 29.252 | r. 338 | 2.675 | 4.013 | $5 \cdot 350$ | 6.688 | 8.026 |  | $64^{\circ}$ |  |
| 6300 |  | 1. 330 | 2.660 | 3.990 | 5.320 | 6.650 | 7.980 |  |  |  |
| 10 | 5.851 | I. 322 | 2.645 | 3.967 | 5.290 | 6.6 I 2 | 7.934 |  |  |  |
| 20 | 1 I .702 | I.315 | 2.630 | 3.944 | 5.259 | 6.574 | 7.889 | 5 | 0.001 |  |
| 30 | 17.554 | 1. 307 | 2.614 | 3.921 | 5.228 | 6.536 | 7.843 | 10 | . 003 |  |
| 40 | 23.405 | I. 300 | 2. 599 | 3.899 | 5.198 | 6.498 | 7.797 | 15 | . 008 |  |
| 50 | 29.256 | 1. 292 | 2.584 | 3.876 | 5.168 | 6.460 | 7.751 | 20 25 | . 013 |  |
| 6400 |  | 1. 284 | 2.569 | 3.853 | 5.1 37 | 6.422 | 7.706 | 30 | . 030 |  |

Smithsonian Tables.

Table 20.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE 12$\}^{h} \delta 0_{0}$
[Derivation of table explained on pp. liii-jvi.]

[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ | $10^{\prime \prime}$ | 15 | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime \prime}$ |  |  |  |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |  |  |  |
| $72^{\circ} 00^{\prime}$ |  | . 906 | 1.8rx | 2.717 | 3.623 | 4.529 | $5 \cdot 434$ |  | $72^{\circ}$ | $73^{\circ}$ |
| 10 | 5.858 | . 898 | 1.795 | 2.693 | 3.590 | 4.488 | 5.386 | 우․․․ |  |  |
| 20 | 11.716 | . 889 | 1.779 | 2.668 | 3.558 | 4.447 | 5.336 |  |  |  |
| 30 | 17.573 | .881 | 1.763 | 2.644 | 3.525 | 4.407 | 5.288 |  | Inches. | Inches. |
| 40 | 23.43 I | .873 | 1.746 | 2.620 | 3.493 | 4.366 | 5.239 | $5{ }^{\prime}$ | 0.001 | 0.001 |
| 50 | 29.289 | . 865 | $1.73{ }^{\circ}$ | 2.595 | 3.460 | $4 \cdot 325$ | 5.190 | 10 | . 003 | . 002 |
| 7300 |  | . 857 | 1.714 | 2.571 | 3.428 | 4.285 | 5.141 | 15 | . 006 | . 005 |
| 10 | 5.858 | . 849 | 1. 697 | 2.546 | 3.428 3.395 | 4.244 | 5.092 | 20 | . 010 | . 010 |
| 20 | 11.717 | .841 | 1. 68 r | 2.522 | 3.362 | 4.203 | 5.044 | 35 | . 016 | . 021 |
| 30 | 17.575 | . 832 | 1. 665 | 2.497 | 3.330 | 4.162 | 4.994 |  |  |  |
| 40 | 23.434 | . 824 | 1.648 | 2.473 | 3.297 | 4.121 | 4.945 |  |  |  |
| 50 | 29.292 | . 816 | r. 632 | 2.448 | 3.264 | 4.081 | 4.897 |  |  |  |
| 7400 |  | . 808 | I. 616 | 2.424 | 3.232 | 4.040 | 4.847 |  |  |  |
| 10 | 5.859 | . 800 | I. 599 | 2.399 | 3.199 | 3.999 | 4.798 |  |  |  |
| 20 | 11.718 | .791 | 1.583 | 2.374 | 3.160 | 3.957 | 4.748 |  | $74^{\circ}$ | $75^{\circ}$ |
| 30 | 17.577 | .783 | 1.566 | 2.350 | 3.133 | 3.916 | 4.699 |  |  |  |
| 40 | 23.436 | . 775 | 1.550 | 2.325 | 3.100 | 3.875 | 4.650 |  |  |  |
| 50 | 29.295 | . 767 | I. 534 | 2.300 | 3.067 | 3.834 | 4.601 | 10 | 0.001 | 0.001 .002 |
| 7500 |  | . 759 | I. 517 | 2.276 | 3.034 | 3.793 | 4.552 | 15 | . 005 | . 005 |
| 10 | 5.860 | .750 | 1. 501 | 2.251 | 3.002 | 3.752 | 4.502 | 20 | . 009 | . 009 |
| 20 | 1 I .719 | . 742 | 1. 484 | 2.226 | 2.968 | 3-7 11 | 4.453 | 25 | . 014 | . 013 |
| 30 | 17.578 | . 734 | I. 468 | 2.201 | 2.935 | 3.669 | 4.403 | 30 | . 020 | . 019 |
| 40 | 23.438 | .726 | I.451 | 2.177 | 2.902 | 3.628 | 4.354 |  |  |  |
| 50 | 29.298 | .717 | I. 435 | 2.152 | 2.870 | $3 \cdot 587$ | 4.304 |  |  |  |
| 7600 |  | . 709 | 1.418 | 2.127 | 2.836 | 3.546 | 4.255 |  |  |  |
| 10 | 5.860 | .701 | 1.402 | 2.102 | 2.803 | 3.504 | 4.205 |  |  |  |
| 20 | 1 I .720 | . 692 | I. 385 | 2.078 | 2.770 | 3.463 | 4.155 |  | $76^{\circ}$ | $77^{\circ}$ |
| 30 | 17.580 23.440 | . 684 | I. 368 | 2.053 | 2.737 | 3.421 | 4.105 |  |  |  |
| 40 | 23.440 29.300 | . 676 | I. 352 I. 335 | 2.028 2.003 | 2.704 2.671 | 3.380 3.339 | 4.056 4.006 |  |  |  |
| 50 | 29.300 | . 668 | 1.335 | 2.003 | 2.671 | $3 \cdot 339$ | 4.006 | 10 | 0.001 | 0.000 .002 |
| 7700 |  | . 659 | 1.319 | 1.978 | 2.638 | 3.297 | 3.956 | 15 | . 005 | . 004 |
| 10 | 5.860 | . 651 | 1.302 | 1.953 | 2.604 | 3.256 | 3.907 | 20 | . 008 | . 007 |
| 20 | 11.721 | . 643 | 1.285 | 1.928 | 2.571 | 3.214 | 3.856 | 25 | . 013 | . 012 |
| 30 40 | 17.582 23.442 | . 634 | 1. 269 | 1.903 | 2.538 | 3.172 | 3.806 | 30 | . 018 | . 017 |
| 40 | 23.442 | . 626 | 1.252 | 1.878 | 2.504 | 3.131 | 3.757 |  |  |  |
| 50 | 29.302 | . 618 | 1.235 | 1.853 | 2.471 | 3.089 | 3.706 |  |  |  |
| 7800 |  | .609 | 1.219 | 1. 828 | 2.438 | 3.047 | 3.656 |  |  |  |
| 10 | 5.861 | . 601 | 1.202 | 1.803 | 2.404 | 3.005 | 3.606 |  |  |  |
| 20 | 11.722 | - 593 | 1.185 | 1.778 | 2.371 | 2.964 | 3.556 |  | $78^{\circ}$ | $79^{\circ}$ |
| 30 | 17.583 | -584 | 1.169 | 1.753 | 2.338 | 2.922 2.880 | 3.506 3.456 |  |  |  |
| 40 | 23.444 | . 576 | I.I 52 | 1.728 | 2.304 2.270 | 2.880 2.838 | 3.456 3.406 |  |  |  |
| 50 | 29.304 | -568 | 1.I35 | 1.703 | 2.270 | 2.838 | $3 \cdot 406$ | 5 | 0.000 | 0.000 |
| 7900 |  | -559 | I.119 | 1. 678 | 2.237 | 2.797 | 3.356 | 10 | . 0002 | . 0002 |
| 10 | 5.861 | -55I | 1.102 | 1. 653 | 2.204 | 2.755 | $3 \cdot 305$ | 20 | . 007 | . 006 |
| 20 | 11.723 | -542 | 1.085 | 1. 628 | 2.170 | 2.713 | 3.255 3.205 | 25 | . 0111 | .010 |
| 30 40 | 17.584 23.445 | . 534 | 1.068 | 1.602 I. 577 | 2.136 2.103 | 2.671 2.629 | 3.205 3.155 | 30 | . 016 | .014 |
| 50 | 29.306 | . 517 | 1. 035 | 1. 552 | 2.070 | 2.587 | 3.104 |  |  |  |
| 8000 |  | $\cdot 509$ | 1.018 | I.527 | 2.036 | 2.545 | 3.054 |  |  |  |

Table 21.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $12{ }^{1} 720$.
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15' longitude. |  | $3^{0}$ dongitude. |  | $4 s^{\prime}$ longitude. |  | $\mathrm{I}^{\circ}$ longitude. |  |
|  |  | x | $y$ | x | $y$ | x | $y$ | x | $y$ |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $0^{\circ} 00^{\prime}$ |  | 8.647 | . 000 | 17.293 | . 000 | 25.940 | . 000 | 34.586 | . 000 |
| 15 | 8.588 | 8.646 | . 000 | 17.293 | . 001 | 25.939 | . 001 | 34.585 | . 001 |
| 30 | 17.176 | 8.646 | . 000 | 17.292 | . 001 | 25.938 | . 001 | 34.584 | . 003 |
| 45 | 25.764 | 8.646 | . 000 | 17.291 | . 001 | 25.937 | . 002 | 34.582 | . 004 |
| 100 | $34 \cdot 35{ }^{2}$ | 8.645 | . 000 | 17.291 | . 001 | 25.936 | . 003 | 34.581 | . 005 |
| 15 | 8.588 | 8.644 | . 000 | 17.289 | . 002 | 25.933 | . 003 | 34.577 | . 007 |
| 30 | 17.176 | 8.643 | . 000 | 17.287 | . 002 | 25.930 | . 004 | 34.573 | . 008 |
| 45 | 25.764 | 8.642 | .001 | 17.285 | . 002 | 25.927 | . 005 | 34.569 | . 009 |
| 200 | 34:352 | 8.64 x | . 001 | 17.283 | . 003 | 25.924 | . 006 | 34.565 | . 011 |
| 15 | 8.588 | 8.640 | . 001 | 17.279 | . 003 | 25.919 | . 007 | 34.559 | . 012 |
| 30 | 17.176 | 8.638 | . 001 | 17.276 | . 003 | 25.914 | . 007 | 34.552 | . 014 |
| 45 | 25.765 | 8.636 | .001 | 17.273 | . 004 | 25.909 | . 008 | 34.546 | . 015 |
| 300 | 34.353 | 8.635 | . 001 | 17.270 | . 004 | 25.904 | . 009 | 34.539 | . 016 |
| 15 | 8.588 | 8.633 | . 001 | 17.265 | . 004 | 25.898 | . 009 | 34.530 | . 018 |
| 30 | 17.177 | 8.630 | . 001 | 17.260 | . 005 | 25.898 | . 010 | 34.52 I | . 019 |
| 45 | 25.765 | 8.628 | . 001 | 17.256 | . 005 | 25.884 | . 011 | $34 \cdot 512$ | . 020 |
| 400 | 34.353 | 8.626 | . 001 | 17.251 | . 005 | 25.877 | . 012 | 34.502 | . 021 |
| 15 | 8.589 | 8.623 | . 001 | 17.245 | . 006 | 25.868 | . 012 | 34.491 | . 023 |
| 30 | 17.177 | 8.620 | . 001 | 17.240 | . 006 | 25.859 | . 013 | 34.479 | . 024 |
| 45 | 25.766 | 8.617 | . 002 | 17.234 | . 006 | 25.850 | . 014 | $34 \cdot 467$ | . 025 |
| 500 | 34.354 | 8.614 | . 002 | 17.228 | . 007 | 25.842 | . 015 | 34.456 | . 026 |
| 15 | 8.589 | 8.610 | . 002 | 17.221 | . 007 | 25.83 r | . 016 | 34.441 |  |
| 30 | 17.177 | 8.607 | . 002 | 17.213 | . 007 | 25.820 | . 016 | 34.427 | . 029 |
| 45 | 25.766 | 8.603 | . 002 | 17.206 | . 008 | 25.809 | . 017 | 34.412 | . 030 |
| 600 | 34.355 | 8.600 | . 002 | 17.199 | . 008 | 25.799 | . 018 | 34.398 | .03I |
| 15 |  | 8.595 | . 002 | 17.191 | . 008 | 25.786 | . 019 | 34.381 | . 033 |
| 30 | 17.178 | 8. 59 I | . 002 | 17.182 | . 008 | 25.773 | . 020 | 34.364 | . 034 |
| 45 | 25.767 | 8.587 | . 002 | 17.174 | . 009 | 25.760 | . 021 | 34.347 | . 035 |
| 700 | 34.356 | 8.583 | . 002 | 17.165 | . 009 | 25.748 | .02I | 34.330 | . 037 |
| 15 | 8.589 | 8. 578 | . 002 | 17.155 | . 009 | 25.733 | . 022 | 34.310 | .038 |
| 30 | 17.179 | 8.573 | . 003 | 17.145 | . 009 | 25.718 | . 022 | 34.291 | . 040 |
| 45 | 25.768 | 8. 568 | . 003 | 17.136 | . 0 | 25.704 | . 023 | 34.272 | .04I |
| 800 | 34.358 | 8.563 | . 003 | 17.126 | . 010 | 25.689 | . 023 | 34.252 | . 042 |
| 15 | 8.590 | 8. 558 | . 003 | 17.115 | . 010 | 25.673 | . 024 |  |  |
| 30 | 17.180 | 8. 552 | . 003 | 17.104 | . OII | 25.656 | . 024 | 34.208 | . 045 |
| 45 | 25.769 | 8.546 | . 003 | 17.093 | .OII | 25.639 | . 025 | 34.186 | . 046 |
| 900 | 34.359 | 8.541 | . 003 | 17.082 | . 012 | 25.622 | . 026 | 34.163 | . 047 |
| $15$ |  |  | . 003 | 17.069 | .012 | 25.604 | . 027 | 34.138 | . 048 |
| $\begin{aligned} & 30 \\ & 45 \end{aligned}$ | $17.180$ $25.771$ | 8.528 8.522 | . 003 | 17.057 17.045 | . 012 | 25.585 | . 027 | 34.114 | . 040 |
| 45 | 25.771 | 8.522 | . 003 | 17.045 | .013 | 25.567 | . 028 | 34.089 | . 051 |
| 1000 | 34.36r | 8.516 | . 003 | 17.032 | . 013 | 25.548 | :029 | 34.064 | . 052 |

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15' longitude. |  | 30' longitude. |  | $45^{\prime}$ longitude. |  | $\mathrm{I}^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Tuches. | Inches. | Inches, | Inches. |
| $10^{\circ} 00^{\prime \prime}$ |  | 8.516 | . 003 | 17.032 | .or3 | 25.548 | . 029 | 34.064 | . 052 |
| 15 | 8.591 | 8.509 | . 003 | 17.019 | . 013 | 25.528 | . 030 | 34.037 | . 054 |
| 30 | 17.181 | 8.502 | . 003 | 17.005 | . 013 | 25.507 | . 031 | 34.010 | . 055 |
| 45 | 25.772 | 8.496 | . 003 | 16.991 | . 014 | 25.487 | . 032 | 33.982 | . 056 |
| 1100 | 34.363 | 8.489 | . 004 | 16.977 | . 014 | 25.466 | . 032 | 33.955 | . 057 |
| 15 | 8.59 r | 8.48 x | . 004 | 16.962 | .014 | 25.444 | . 033 | 33.925 | . 058 |
| 30 | 17.183 | 8.474 | . 004 | 16.947 | . 015 | 25.421 | . 033 | 33.895 | . 059 |
| 45 | 25.774 | 8.466 | . 004 | 16.933 | . 015 | $25 \cdot 399$ | . 034 | 33.865 | .060 |
| 1200 | 34.365 | 8.459 | . 004 | 16.918 | . 015 | $25 \cdot 376$ | . 035 | 33.835 | .061 |
| 15 | 8.592 | 8.451 | . 004 | 16.901 | . 016 | 25.352 | . 035 | 33.803 | .063 |
| 30 | 17.184 | 8.443 | . 004 | I6.885 | . 016 | $25 \cdot 328$ | . 036 | 33.770 | . 064 |
| 45 | 25.776 | 8.434 | . 004 | 16.869 | . 016 | 25.304 | .036 | 33.738 | . 065 |
| 1300 | 34.368 | 8.426 | . 004 | 16.853 | .017 | 25.279 | . 037 | 33.706 | . 066 |
| 15 | 8.592 | 8.418 | . 004 | 16.835 | . 017 | 25.253 | .038 | 33.67 r | . 067 |
| 30 | 17.185 | 8.409 | . 004 | 16.818 | . 017 | 25.227 | . 039 | 33.636 | . 069 |
| 45 | 25.778 | 8.400 | . 004 | 16.800 | . or 8 | 25.201 | . 040 | 33.601 | . 070 |
| 1400 | 34.370 | 8.39 r | . 004 | 16.783 | . 018 | 25.174 | . 040 | 33.566 | . 071 |
| 15 | 8.593 | 8.382 | . 005 | 16.764 | . 018 | 25.146 | . 041 | 33.528 | . 072 |
| 30 | 17.186 | 8.373 | . 005 | 16.745 | . 018 | 25.118 | . 041 | 33.490 | . 073 |
| 45 | 25.780 | 8.363 | . 005 | 16.726 | . 019 | 25.090 | . 042 | 33.453 | . 074 |
| 1500 | 34-373 | 8.354 | . 005 | 16.708 | .or9 | 25.061 | . 042 | 33.415 | . 075 |
| 15 | 8.594 | 8.344 | . 005 | 16.688 | . 019 | 25.031 | . 043 | 33.375 | . 077 |
| 30 | 17.188 | 8.334 | . 005 | 16.668 | . 019 | 25.001 | . 044 | $33 \cdot 335$ | . 078 |
| 45 | 25.782 | 8.324 | . 005 | r6.647 | . 020 | 24.971 | . 045 | 33.295 | . 079 |
| 1600 | 34.376 | 8.314 | . 005 | 16.627 | . 020 | 24.941 | . 045 | 33.255 | .080 |
| 15 | 8.595 | 8.303 | . 005 | 16.606 | . 020 | 24.909 | . 045 | 33.212 | .08r |
| 30 | 17.190 | 8.292 | . 005 | 16.585 | . 020 | 24.877 | . 046 | 33.170 | . 082 |
| 45 | 25.784 | 8.282 | . 005 | 16.564 | . 021 | 24.845 | . 046 | 33.127 | . 083 |
| 1700 | 34.379 | 8.27 r | . 005 | 16.542 | . 021 | 24.813 | . 047 | 33.084 | . 084 |
| 15 | 8.596 | 8.260 | . 005 | 16.520 | . 021 | 24.779 | . 048 | 33.039 | . 085 |
| 30 | 17.191 25.787 | 8.249 8.237 | . 005 | 16.497 | . 021 | 24.746 | . 049 | 32.994 | . 087 |
| 45 | 25.787 | 8.237 | . 006 | 16.475 | . 022 | 24.712 | . 050 | 32.949 | . 088 |
| 1800 | 34.382 | 8.226 | . 006 | 16.452 | . 022 | 24.678 | . 050 | 32.904 | . 089 |
| 15 | 8.596 | 8.214 | . 006 | 16.428 | . 022 | 24.642 | . 051 | 32.856 | . 090 |
| 30 | 17.193 | 8.202 | . 006 | 16.404 | . 023 | 24.607 | . 051 | 32.809 | . 091 |
| 45 | 25.790 | 8.190 | . 006 | 16.38 r | . 023 | 24.57 I | . 052 | 32.761 | . 092 |
| 1900 | 34.386 | 8.178 | . 006 | 16.357 | . 023 | 24.535 | . 052 | 32.714 | . 093 |
| 15 | 8.597 | 8.166 | . 006 | 16.332 | . 023 | 24.498 | . 053 | 32.664 | . 094 |
| 30 | 17.195 | 8.153 | . 006 | 16.3307 | . 024 | 24.460 | . 054 | 32.614 | . 095 |
| 45 | 25.792 | 8.14 4 | . 006 | 16.282 | . 024 | 24.422 | . 055 | 32.563 | . 096 |
| 2000 | 34.390 ${ }^{-}$ | 8.128 | . 006 | 16.257 | . 024 | 24.385 | . 055 | 32.513 | . 097 |

Table 21.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\frac{128 \sigma^{*} 0^{\circ}}{}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $15^{\prime}$ lougitude. |  | $3{ }^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $\mathrm{r}^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | $y$ | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $20^{\circ} 00^{\prime}$ |  | 8.128 | . 006 | 16.257 | . 024 | 24.385 | . 055 | 32.513 | . 097 |
| 15 | 8.598 | 8.115 | . 006 | 16.230 | . 024 | 24.346 | . 056 | 32.461 | . 098 |
| 30 | 17.197 | 8.102 | . 006 | 16.204 | . 025 | 24.306 | .056 | 32.408 | . 099 |
| 45 | 25.795 | 8.089 | . 006 | 16.178 | . 025 | 24.267 | . 057 | 32.356 | . 100 |
| 2100 | $34 \cdot 394$ | 8.076 | . 006 | 16.152 | . 025 | 24.227 | . 057 | 32.303 | .roi |
| 15 | 8. 599 | 8.062 | . 006 | 16.124 | . 025 | 24.186 | . 058 | 32.248 | . 102 |
| 30 | 17.199 | 8.048 | . 006 | 16.097 | . 026 | 24.145 | . 058 | 32.193 | .103 |
| 45 | 25.798 | 8.035 | . 007 | 16.069 | . 026 | 24.104 | . 059 | 32.138 | . 104 |
| 2200 | 34.398 | 8.021 | . 007 | 16.042 | . 026 | 24.062 | . 059 | 32.083 | . 105 |
| 15 | 8.600 | 8.006 | . 007 | 16.013 | . 026 | 24.019 | . 060 | 32.026 | .106 |
| 30 | 17.201 | 7.992 | . 007 | 15.984 | . 027 | 23.976 | . 060 | 31.968 | .107 |
| 45 | 25.801 | 7.978 | . 007 | 15.955 | . 027 | 23.933 | . 061 | 31.911 | . 108 |
| 2300 | 34.402 | 7.963 | . 007 | 15.927 | . 027 | 23.890 | . 061 | 31.853 | .109 |
| 15 | 8.602 | 7.948 | . 007 | 15.897 | . 027 | 23.845 | . 062 | 31.794 | .109 |
| 30 | 17.203 | 7.933 | . 007 | 15.867 | . 028 | 23.800 | . 062 | 31.734 | . 110 |
| 45 | 25.804 | 7.918 | . 007 | 15.837 | . 028 | 23.756 | . 063 | 31.674 | . 111 |
| 2400 | 34.406 | 7.904 | . 007 | 15.807 | . 028 | 23.71 I | . 063 | 31.614 | . 112 |
| 15 | 8.603 | 7.888 | . 007 | 15.776 | . 028 | 23.664 | . 064 | 3 L .552 | .113 |
| 30 | 17.205 | 7.872 | . 007 | 15.745 | . 029 | 23.617 | .064 | 31.489 | .114 |
| 45 | 25.808 | 7.857 | . 007 | 15.713 | . 029 | 23.570 | . 065 | 31.427 | .115 |
| 2500 | 34.410 | 7.841 | . 007 | 15.682 | . 029 | 23.524 | . 065 | 31.365 | . 116 |
| 15 | 8.604 | 7.825 | . 007 | 15.650 | . 029 | 23.475 | . 065 | 31.300 | . 117 |
| 30 | 17.207 | 7.809 | . 007 | 15.617 | . 029 | 23.426 | . 066 | 31.235 | .117 |
| 45 | 25.811 | 7.793 | . 007 | 15.585 | .030 | 23.378 | . 067 | 31.170 | . 118 |
| 2600 | 34.415 | 7.776 | . 007 | 15.553 | . 030 | 23.329 | . 067 | 31.106 | . 119 |
| 15 | 8.605 | 7.760 | . 007 | 15.519 | . 030 | 23.279 | . 067 | 31.039 | . 120 |
| 30 | 17.210 | 7.743 | . 008 | 15.486 | . 030 | 23.229 | . 068 | 30.972 | . 121 |
| 45 | 25.814 | 7.726 | . 008 | 15.452 | . 030 | 23.179 | . 068 | 30.905 | . 121 |
| 2700 | 34.419 | 7.709 | . 008 | 15.419 | .031 | 23.128 | . 069 | 30.838 | . 122 |
| 15 | 8.606 | 7.692 | . 008 | 15.384 | .031 | 23.076 | . 069 | 30.769 | . 123 |
| 30 | 17.212 | 7.675 | . 008 | 15.350 | .03I | 23.024 | . 070 | 30.699 | .124 |
| 45 | 25.818 | 7.657 | . 008 | 15.315 | . 031 | 22.972 | . 070 | 30.630 | . 124 |
| 2800 | 34.424 | 7.640 | . 008 | 15.280 | . 031 | 22.920 | . 070 | 30.560 | . 125 |
| 15 | 8.607 | 7.622 | . 008 | 15.244 | . 031 | 22.866 | . 071 | 30.489 | . 126 |
| 30 | 17.215 | 7.604 | . 008 | 15.208 | . 032 | 22.813 | . 071 | 30.417 | . 127 |
| 45 | 25.822 | 7.586 | . 008 | 15.173 | .032 | 22.759 | . 072 | 30.345 | . 127 |
| 2900 | 34.430 | $7 \cdot 568$ | . 008 | 15.137 | . 032 | 22.705 | . 072 | 30.274 | . 128 |
|  | 8.609 | 7.550 | . 008 | 15.100 | . 032 | 22.650 | . 072 | 30.200 | . 129 |
| $30$ | 17.217 25.826 | $7.531$ | . 008 | 15.063 | . 032 | 22.594 | . 073 | 30.125 | . 130 |
| 45 | 25.826 | 7.513 | . 008 | 15.026 | . 033 | 22.539 | . 073 | 30.051 | . 30 |
| 3000 | 34.435 | 7.494 | . 008 | 14.989 | . 033 | 22.483 | . 074 | 29.978 | $13^{1}$ |

[Derivation of table explaiued on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR- |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }_{15}{ }^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $2^{\circ}$ lougitude. |  |
|  |  | x | y | x | y | x | y | x | $y$ |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inckes. | Inckes. | Inches. |
| $30^{\circ} 00^{\prime}$ |  | 7.494 | . 008 | 14.989 | . 033 | 22.483 | . 074 | 29.978 | . 31 |
| 15 | 8.610 | 7.475 | . 008 | 14.951 | . 033 | 22.426 | . 074 | 29.902 | .131 |
| 30 | 17.220 | $7 \cdot 456$ | . 008 | 14.913 | . 033 | 22.369 | . 074 | 29.825 | .132 |
| 45 | 25.830 | 7.437 | . 008 | 14.874 | . 033 | 22.312 | . 075 | 29.749 | . 133 |
| $3{ }^{1} 00$ | 34.440 | 7.418 | . 008 | 14.836 | . 033 | 22.254 | . 075 | 29.672 | . 133 |
| 15 | 8.611 | 7.398 | . 008 | 14.797 | .033 | 22.195 | . 075 | 29.594 | . 134 |
| 30 | 17.213 | 7.379 | . 008 | 14.758 | . 034 | 22.137 | . 076 | 29.515 | . 135 |
| 45 | 25.834 | 7.359 | . 008 | 14.718 | . 034 | 22.078 | . 076 | 29.437 | . 35 |
| 3200 | 34.446 | $7 \cdot 340$ | . 008 | 14.679 | . 034 | 22.019 | . 076 | 29.358 | . 136 |
| 15 | 8.613 | 7.319 | . 008 | 14.639 | . 034 | 21.958 | . 077 | 29.278 | . 136 |
| 30 | 17.225 | 7.299 | . 009 | 14.598 | . 034 | 21.898 | . 077 | 29.197 | . 137 |
| 45 | 25.838 | 7.279 | . 009 | 14.558 | . 034 | 21.837 | . 077 | 29.116 | . 137 |
| 3300 | 34.451 | 7.259 | . 009 | 14.518 | . 034 | 21.777 | . 078 | 29.036 | . 138 |
| 15 | 8.614 | 7.238 | . 009 | 14.476 | . 035 | 21.714 | . 078 | 28.953 | . 138 |
| 30 | 17.228 | 7.217 | . 009 | 14.435 | . 035 | 21.652 | . 078 | 28.869 | . 139 |
| 45 | 25.842 | 7.197 | . 009 | 14.393 | . 035 | 21.590 | . 078 | 28.786 | . 39 |
| 3400 | 34-456 | 7.176 | . 009 | 14.352 | . 035 | 21.527 | . 079 | 28.703 | . 140 |
| 15 | 8.615 | 7.154 | . 009 | 14.309 | . 035 | 21.464 | . 079 | 28.618 | .14I |
| 30 | 17.23 r | 7.133 | . 009 | I4.266 | . 035 | 21.400 | . 079 | 28.533 | . 141 |
| 45 | 25.846 | 7.112 | . 009 | 14.224 | . 035 | 21.336 | . 080 | 28.448 | . 142 |
| 3500 | 34.462 | 7.091 | . 009 | 14.18I | . 035 | 21.272 | . 080 | 28.362 | . 142 |
| 15 | 8.617 | 7.069 | . 009 | 14.138 | .036 | 21.207 | . 880 | 28.275 | . 142 |
| 30 | 17.234 | 7.047 | . 009 | 14.094 | . 036 | 21.141 | . 080 | 28.188 | . 143 |
| 45 | 25.851 | 7.025 | . 009 | 14.050 | . 036 | 21.076 | . 080 | 28.101 | . 143 |
| 3600 | 34.468 | 7.003 | . 009 | 14.007 | . 036 | 21.010 | . 081 | 28.014 | . 144 |
| 15 | 8.618 | 6.981 | . 009 | 13.962 | . 036 | 20.943 | . 081 | 27.924 | . 144 |
| 30 | 17.237 | 6.959 | . 009 | 13.917 | .036 | 20.876 | .081 | 27.835 | . 144 |
| 45 | 25.855 | 6.936 | . 009 | 13.873 | . 036 | 20.809 | . 081 | 27.745 | . 145 |
| 3700 | 34.474 | 6.914 | . 009 | 13.828 | . 036 | 20.742 | . 082 | 27.655 | . 145 |
| 15 | 8.620 | 6.89 I | . 009 | 13.782 | . 036 | 20.673 | . 082 | 27.564 |  |
| 30 | 17.240 | 6.868 | . 009 | 13.736 | . 036 | 20.604 | . 082 | 27.472 | . 146 |
| 45 | 25.860 | 6.845 | . 009 | 13.690 | . 037 | 20.536 | . 082 | 27.381 | . 146 |
| 3800 | 34.480 | 6.822 | . 009 | 1 3.645 | . 037 | 20.467 | . 082 | 27.289 | . 147 |
| 15 | 8.62 I | 6.799 | . 009 | 13.598 | . 037 | 20.397 | . 083 | 27.196 | . 147 |
| 30 | 17.243 | 6.775 | . 009 | 13.551 | . 037 | 20.326 | . 083 | 27.102 | . 147 |
| 45 | 25.864 | 6.752 | . 009 | 13.504 | . 037 | 20.256 | . 083 | 27.008 | . 147 |
| 3900 | 34.485 | 6.729 | . 009 | 13.457 | . 037 | 20.186 | . 083 | 26.914 | . 148 |
|  |  | 6.705 | . 009 | I 3.409 | . 037 | 20.114 | . 083 | 26.819 | . 148 |
| $30$ | 17.246 | 6.681 | . 009 | I $3 \cdot 36 \mathrm{I}$ | . 037 | 20.042 | . 083 | 26.723 | . 148 |
| 45 | 25.868 | 6.657 | . 009 | 13.314 | . 037 | 19.970 | . 084 | 26.627 | . 148 |
| 4000 | 34.491 | 6.633 | . 009 | 13.266 | . 037 | 19.899 | . 084 | 26.532 | . 149 |

[Derivation of table explained on pp. Hiii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR- |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15 ${ }^{\text {l }}$ longitude. |  | $30^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $\mathrm{r}^{\circ}$ longitude. |  |
|  |  | x | $y$ | x | y | $x$ | $y$ | x | y |
|  | Inches. | Inches. | Inihes. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $40^{\circ} 00^{\prime}$ |  | 6.633 | . 009 | 13.266 | .037 | 19.899 | .084 | 26.532 | . 149 |
| 15 | 8.624 | 6.608 | . 009 | 13.217 | . 037 | 19.825 | . 084 | 26.434 | . 149 |
| 30 | 17.249 | 6.584 | . 009 | 13.168 | . 037 | 19.752 | . 084 | 26.336 | . 149 |
| 45 | 25.873 | 6.560 | . 009 | 13.119 | . 037 | 19.679 | . 084 | 26.238 | .149 |
| 4100 | 34.497 | 6.535 | . 009 | 13.070 | 037 | 19.605 | . 084 | 26.140 | .150 |
| 15 | 8.625 | 6.510 | . 009 | 13.020 | . 037 | 19.530 | . 084 | 26.041 | . 150 |
| 30 | 17.250 | 6.485 | . 009 | 12.970 | . 037 | 19.456 | . 084 | 25.94 I | .150 |
| 45 | 25.875 | 6.460 | . 009 | 12.920 | . 037 | 19.381 | . 084 | 25.841 | .150 |
| 4200 | 34.500 | 6.435 | . 009 | 12.87 I | . 037 | 19.306 | . 085 | 25.74 I | . 150 |
| 15 | 8.627 | 6.410 | . 009 | 12.820 | . 037 | 19.230 | .085 | 25.640 | . 150 |
| 30 | 17.255 | 6.385 | . 009 | 12.769 | . 038 | 19.154 | .085 | 25.538 | . 51 |
| 45 | 25.882 | 6.359 | . 009 | 12.718 | . 038 | 19.077 | . 085 | 25.436 | . 51 |
| 4300 | 34.510 | 6.334 | . 009 | 12.667 | .038 | 19.001 | . 085 | 25.335 | . 51 |
| 15 | 8.629 | 6.308 | . 009 | 12.615 | . 038 | 18.923 | . 085 | 25.231 | . 151 |
| 30 | 17.257 | 6.282 | . 009 | 12.563 | . 338 | 18.845 | . 085 | 25.127 | . 151 |
| 45 | 25.886 | 6.256 | . 009 | 12.512 | . 038 | 18.767 | . 085 | 25.023 | . 51 |
| 4400 | 34.515 | 6.230 | . 009 | 12.460 | . 038 | 18.689 | . 085 | 24.919 | . 151 |
| 15 | 8.630 | 6.203 | . 009 | 12.407 | . 038 | 18.610 | .085 | 24.814 | . 151 |
| 30 | 17.261 | 6.177 | . 009 | 12.354 | . 038 | 18.531 | .085 | 24.708 | . 151 |
| 45 | 25.891 | 6.151 | . 009 | 12.301 | . 038 | 18.452 | . 085 | 24.603 | . 51 |
| 4500 | 34.522 | 6.124 | . 009 | 12.249 | . 038 | 18.373 | . 085 | 24.497 | . 151 |
| 15 | 8.632 | 6.097 | . 009 | 12.195 | . 038 | 18.292 | . 085 | 24.390 | . 151 |
| 30 | 17.264 | 6.071 | . 009 | 12.141 | . 038 | 18.212 | . 085 | 24.283 | . 51 |
| 45 | 25.896 | 6.044 | . 009 | 12.088 | . 038 | 18.131 | . 085 | 24.175 | . 515 |
| 4600 | 34.528 | 6.017 | . 009 | 12.034 | . 038 | 18.051 | .085 | 24.068 | . 151 |
| 15 | 8.633 | 5.990 | . 009 | 11.979 | . 038 | 17.969 | .085 | 23.959 |  |
| 30 | 17.267 | 5.962 | . 009 | 11.925 | . 338 | 17.887 | .085 | 23.849 | . 55 |
| 45 | 25.901 | 5.935 | . 009 | 11.870 | . 038 | 17.805 | . 085 | 23.740 | . 51 |
| 4700 | 34.534 | 5.908 | . 009 | 11.815 | . 038 | 17.723 | . 085 | 23.631 | . 151 |
| 15 | 8.635 | 5.880 | . 009 | 11.760 | .038 | 17.640 | . 085 | 23.520 | . 151 |
| 30 | 17.270 | 5.852 | . 009 | 11.704 | . 038 | 17.556 | . 085 | 23.408 | . 151 |
| 45 | 25.905 | 5.824 | . 009 | 11.648 | . 038 | 17.473 | . 085 | 23.297 | . 51 |
| 4800 | 34.540 | 5.796 | . 009 | 11.593 | .038 | 17.389 | . 085 | 23.186 | . 150 |
| 15 | 8.637 | 5.768 | . 009 | I1.536 | . 038 | 17.305 | .085 | 23.073 | .150 |
| 30 | 17.273 | 5.740 | . 009 | 11.480 | . 038 | 17.220 | .084 | 22.960 | .150 |
| 45 | 25.910 | 5.712 | . 009 | I 1.424 | . 037 | 17.135 | . 084 | 22.847 | . 150 |
| 4900 | 34.546 | 5.684 | . 009 | 11.367 | . 037 | 17.051 | . 084 | 22.734 | . 150 |
|  | 8.638 17.276 |  | . 009 | 11.310 | . 037 | 16.965 | . 084 | 22.620 |  |
| 30 | 17.276 25.914 | 5.626 | . 009 | 11.253 | . 037 | 16.879 | . 084 | 22.505 | . 150 |
| 45 | 25.914 | $5 \cdot 598$ | . 009 | 1 I .195 | . 037 | 16.793 | . 084 | 22.39 I | . 150 |
| 5000 | 34.552 | 5.569 | . 009 | IJ. 138 | . 037 | 16.707 | . 084 | 22.276 | .150 |

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED Parallel for - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $15^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $x^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $50^{\circ} 00^{\prime}$ |  | 5.569 | . 009 | 11.138 | . 037 | 16.707 | . 084 | 22.276 | .150 |
| 15 | 8.640 | 5.540 | . 009 | 11.080 | . 037 | 16.620 | . 084 | 22.160 | . 149 |
| 30 | 17.279 | 5.51 I | . 009 | 11.022 | . 037 | 16.532 | . 084 | 22.043 | . 149 |
| 45 | 25.919 | $5 \cdot 482$ | . 009 | 10.963 | . 037 | 16.445 | . 083 | 21.927 | . 149 |
| 5100 | $34 \cdot 558$ | $5 \cdot 453$ | . 009 | 10.905 | . 037 | 16.358 | . 083 | 21.810 | . 148 |
| 15 | 8.641 | 5.423 | . 009 | 10.846 | . 037 | 16.269 | . 083 | 21.692 | . 148 |
| 30 | 17.282 | 5.394 | . 009 | 10.787 | . 037 | 16.181 | . 083 | 21.574 | . 148 |
| 45 | 25.924 | $5 \cdot 364$ | . 009 | 10.728 | . 037 | 16.092 | . 083 | 21.456 | . 147 |
| 5200 | 34.565 | $5 \cdot 334$ | . 009 | 10.669 | . 037 | 16.004 | . 083 | 21.338 | . 147 |
| 15 | 8.643 | 5.305 | . 009 | 10.609 | . 036 | 15.914 | . 082 | 21.218 | . 146 |
| 30 | 17.285 | 5.275 | . 009 | 10.549 | . 036 | 15.824 | . 082 | 21.099 | . 146 |
| 45 | 25.928 | 5.245 | . 009 | 10.490 | . 036 | 15.734 | . 082 | 20.979 | . 145 |
| 5300 | 34.571 | 5.215 | . 009 | 10.430 | . 036 | 15.645 | . 082 | 20.860 | . 145 |
| 15 | 8.644 | 5.185 | . 009 | 10.369 | . 036 | I 5.554 | . 082 | 20.738 | . 145 |
| 30 | 17.288 | 5.154 | . 009 | 10.309 | . 036 | 15.463 | .08I | 20.617 | . 144 |
| 45 | 25.932 | 5.124 | . 009 | 10.248 | . 036 | 15.372 | .08I | 20.496 | . 144 |
| 5400 | 34.576 | 5.094 | . 009 | 10.187 | . 036 | '15.281 | .08I | 20.374 | . 144 |
| 15 | 8.646 | 5.063 | . 009 | 10.126 | . 036 | 15.189 | . 081 | 20.252 | . 143 |
| 30 | 17.291 | 5.032 | . 009 | 10.064 | . 036 | 15.097 | .080 | 20.129 | . 143 |
| 45 | 25.937 | 5.002 | . 009 | 10.003 | . 036 | 15.004 | .080 | 20.006 | .142 |
| 5500 | 34-582 | 4.971 | . 009 | 9.942 | . 036 | 14.912 | . 080 | 19.883 | . 142 |
| 15 | 8.647 | 4.940 | . 009 | 9.879 | . 035 | 14.819 | . 080 | 19.759 | . 141 |
| 30 | 17.294 | 4.909 | . 009 | 9.817 | . 035 | 14.726 | . 079 | 19.634 | . 141 |
| 45 | 25.941 | 4.878 | . 009 | 9.755 | . 035 | 14.633 | . 079 | 19.510 | .140 |
| 5600 | 34.588 | 4.846 | . 009 | 9.693 | . 035 | 14.539 | . 079 | 19.386 | . 140 |
| 15 | 8.648 | 4.815 | . 009 | 9.630 | . 035 | 14.445 | . 079 | 19.260 | . 140 |
| 30 | 17.297 | 4.784 | . 009 | 9.567 | . 035 | 14.35 I | . 078 | 19.134 | . 139 |
| 45 | 25.946 | 4.752 | . 009 | 9.504 | . 035 | 14.256 | . 078 | 19.008 | . 39 |
| 5700 | 34.594 | 4.720 | . 009 | 9.44I | . 035 | 14.162 | . 078 | 18.882 | . 138 |
|  | 8.650 | 4.689 | . 009 | 9.377 | . 035 | 14.066 | . 077 | 18.754 | . 138 |
| 30 | 17.300 | 4.657 | . 009 | 9-314 | . 034 | 13.970 | . 077 | 18.627 | . 37 |
| 45 | 25.950 | 4.625 | . 009 | 9.250 | . 034 | 13.875 | . 077 | 18.500 | . 137 |
| 5800 | 34.600 | $4 \cdot 593$ | . 009 | 9.186 | . 034 | 13.779 | . 076 | 18.372 | . 136 |
| 15 | 8.651 | 4.561 | . 008 | 9.122 | . 034 | 13.683 | . 076 | 18.244 | . 135 |
| 30 | 17.303 | 4.529 | . 008 | 9.058 | . 034 | 13.586 | . 076 | 18.115 | . 35 |
| 45 | 25.954 | 4.497 | . 008 | 8.993 | . 034 | 13.490 | . 075 | 17.986 | . 34 |
| 5900 | 34-605 | 4.464 | . 008 | 8.929 | . 033 | 13.393 | :075 | 17.858 | . 134 |
| 15 | 8.653 | $4 \cdot 432$ | . 008 | 8.864 | . 033 | 13.296 | . 075 | 17.728 | -133 |
| 30 | 17.305 | 4.399 | . 008 | 8.799 | . 033 | 13.198 | . 075 | 17.597 | . 133 |
| 45 | 25.958 | 4.367 | . 008 | 8.734 | . 033 | 13.100 | . 074 | 17.467 | .132 |
| 6000 | 34.611 | 4.334 | . 008 | 8.669 | . 033 | 13.003 | . 074 | 17.337 | .131 |

Table 21.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $12 \delta^{2} \sqrt{20}$
[Derivatioc of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{5} 5^{\prime}$ longitude. |  | 30' longitude. |  | 45' longitude. |  | $1^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $60^{\circ} 00^{\prime}$ |  | 4.334 | . 008 | 8.669 | . 033 | 13.003 | . 074 | 17.337 | .131 |
| 15 | 8.654 | $4 \cdot 301$ | . 008 | 8.603 | . 032 | 12.904 | . 074 | 17.206 | .131 |
| 30 | 17.308 | 4.269 | . 008 | 8.537 | . 032 | 12.806 | . 073 | 17.074 | . 130 |
| 45 | 25.962 | 4.236 | . 008 | 8.471 | .032 | 12.707 | . 073 | 16.943 | . 129 |
| 6100 | 34.616 | 4.203 | . 008 | 08.406 | .032 | 12.608 | .072 | 16.81 I | . 128 |
| 15 | 8.655 | 4.170 | . 008 | 8.339 | . 032 | 12.509 | . 072 | 16.679 | . 128 |
| 30 | 17.311 | 4.136 | . 008 | 8.273 | .032 | 12.410 | . 072 | 16.546 | .127 |
| 45 | 25.966 | 4.103 | . 008 | 8.207 | .03I | 12.310 | . 071 | 16.413 | .126 |
| 6200 | 34.621 | 4.070 | . 008 | 8.140 | .03I | 12.210 | . 071 | 16.280 | .125 |
| 15 | 8.657 | 4.036 | . 008 | 8.073 | . 031 | 12.110 | . 071 | 16.146 | .125 |
| 30 | 17.313 | 4.003 | . 008 | 8.006 | .031 | 12.009 | . 070 | 16.012 | . 124 |
| 45 | 25.970 | 3.970 | . 008 | 7.939 | .03I | 11.909 | . 070 | 15.878 | .123 |
| 6300 | 34.626 | 3.936 | . 008 | 7.872 | .031 | 11.808 | . 069 | 15.744 | . 122 |
| 15 | 8.658 | 3.902 | . 008 | 7.804 | . 030 | 11.707 | . 069 | 15.609 | . 122 |
| 30 | 17.316 | 3.868 | . 007 | 7.737 | . 030 | 11.605 | . 068 | 15.474 | . 121 |
| 45 | 25.974 | 3.835 | . 007 | 7.669 | .030 | 11.504 | . 068 | 15.338 | . 120 |
| 6400 | 34.632 | 3.801 | . $007{ }^{\circ}$ | 7.602 | . 030 | 11.402 | . 067 | 15.203 | .119 |
| 15 | 8.659 | 3.767 | . 007 | 7.533 | . 029 | 11.300 | . 067 | 15.067 | . 119 |
| 30 | 17.318 | 3.733 | . 007 | 7.465 | . 029 | 11.198 | . 066 | 14.930 | . 118 |
| 45 | 25.977 | 3.698 | . 007 | 7.397 | . 029 | 11.096 | . 066 | 14.794 | .117 |
| 6500 | 34.636 | 3.664 | . 007 | 7.329 | . 029 | 10.993 | .065 | 14.658 | .116 |
| 15 | 8.660 | 3.630 | . 007 | 7.260 | . 028 | 10.890 | . 065 | 14.520 | .115 |
| 30 | 17.321 | 3.596 | . 007 | 7.191 | . 028 | 10.787 | . 064 | 14.383 | .114 |
| 45 | 25.981 | 3.56I | . 007 | 7.123 | . 028 | 10.684 | . 064 | 14.245 | . 113 |
| 6600 | 34.641 | $3 \cdot 527$ | . 007 | 7.054 | . 028 | 10.58 I | .063 | 14.108 | . 112 |
| 15 | 8.661 | 3.492 | . 007 | 6.984 | . 028 | 10.477 | . 063 | 13.969 | . 111 |
| 30 | 17.323 | 3.458 | . 007 | 6.915 | . 027 | 10.373 | . 062 | 13.830 | . 111 |
| 45 | 25.984 | 3.423 | . 007 | 6.846 | . 027 | 10.269 | . 062 | 13.692 | . 110 |
| 6700 | 34.646 | 3.388 | . 007 | 6.776 | . 027 | 10.165 | .061 | 13.553 | . 109 |
| 15 | 8.663 | 3.353 | . 007 | 6.706 | . 027 | 10.060 | . 061 | 13.413 | . 108 |
| 30 | 17.325 | 3.318 | . 007 | 6.637 | . 026 | 9.955 | . 060 | 13.273 |  |
| -45 | 25.988 | 3.283 | . 007 | 6.567 | . 026 | 9.850 | . 060 | I 3.134 | . 106 |
| 6800 | 34.650 | 3.248 | . 007 | 6.497 | . 026 | 9.746 | . 059 | 12.994 | . 105 |
| - 15 | 8.664 | 3.213 | . 007 | 6.427 | . 026 | 9.640 |  | 12.854 |  |
| 30 | 17.327 | 3.178 | . 006 | 6.356 | . 025 | 9.535 | . 058 | 12.713 | . 103 |
| 45 | 25.991 | 3.143 | . 006 | 6.286 | . 025 | 9.429 | . 058 | 12.572 | . 102 |
| 6900 | 34.655 | 3.108 | . 006 | 6.216 | . 025 | 9.323 | . 057 | 12.431 | . 101 |
|  |  | 3.072 | . 006 |  | . 025 | 9.217 |  |  | . 100 |
| 30 | 17.329 | 3.037 | . 006 | 6.074 | . 024 | 9.111 | . 056 | 12.148 |  |
| 45 | $25 \cdot 994$ | 3.002 | . 006 | 6.003 | . 024 | 9.005 | . 056 | 12.006 | . 098 |
| 7000 | 34.659 | 2.966 | . 006 | 5.932 | . 024 | 8.899 | . 055 | 11.865 | . 097 |

CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $12 \theta^{1}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 15 ${ }^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | $45^{\prime}$ longitude. |  | $x^{\circ}$ longitude. |  |
|  |  | x | $y$ | x | y | x | y | x | y |
|  | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. | Inches. |
| $70^{\circ} 00^{\prime}$ |  | 2.966 | . 006 | 5.932 | . 024 | 8.899 | . 055 | 11.865 | . 097 |
| 15 | 8.666 | 2.930 | . 006 | 5.861 | . 024 | 8.792 | . 055 | 11.722 | . 096 |
| 30 | 17.331 | 2.895 | . 006 | 5.790 | . 023 | 8.685 | . 054 | 11.580 | . 095 |
| 45 | 25.997 | 2.859 | . 006 | 5.718 | . 023 | 8.578 | . 053 | 11.437 | . 094 |
| 7100 | 34.663 | 2.824 | . 006 | 5.647 | . 023 | 8.471 | . 052 | 11.294 | . 093 |
| 15 | 8.667 | 2.788 | . 006 | 5.576 | . 023 | 8.363 | . 052 | I1.151 | . 092 |
| 30 | 17.333 | 2.752 | . 006 | 5.504 | . 022 | 8.256 | . 051 | 11.008 | . 091 |
| 45 | 26.000 | 2.716 | . 006 | 5.432 | . 022 | 8.148 | . 051 | 10.864 | . 090 |
| 7200 | 34.667 | 2.680 | .006 | $5 \cdot 360$ | . 022 | 8.040 | . 050 | 10.720 | . 089 |
| 15 | 8.668 | 2.644 | . 006 | 5.288 | . 022 | 7.932 | . 050 | 10.576 | . 088 |
| 30 | 17.335 | 2.608 | . 005 | 5.216 | . 021 | 7.824 | . 049 | 10.432 | . 087 |
| 45 | 26.003 | 2.572 | . 005 | 5.144 | .02I | 7.716 | . 249 | I0. 288 | . 086 |
| 7300 | 34.670 | 2.536 | . 005 | 5.072 | . 021 | 7.608 | . 048 | 10.144 | . 085 |
| 15 | 8.668 | 2.500 | . 005 | 4.999 | . 021 | 7.499 | . 048 | 9.998 | . 084 |
| 30 | 17.337 | 2.463 | . 005 | 4.927 | . 020 | 7.390 | . 047 | 9.854 | . 083 |
| 45 | 26.006 | 2.427 | . 005 | 4.854 | . 020 | 7.28I | . 046 | 9.708 | .08I |
| 7400 | 34.674 | 2.391 | . 005 | 4.782 | . 020 | 7.172 | . 045 | 9.563 | . 080 |
| 15 | 8.669 | 2.354 | . 005 | 4.709 | . 020 | 7.063 | . 044 | 9.417 | . 079 |
| 30 | 17.339 | 2.318 | . 005 | 4.636 | . 019 | 6.954 | . 044 | 9.272 | . 078 |
| 45 | 26.008 | 2.281 | . 005 | 4.563 | . 019 | 6.844 | . 043 | 9.126 | . 077 |
| 7500 | 34.677 | 2.245 | . 005 | $4 \cdot 490$ | . 019 | 6.735 | . 043 | 8.980 | . 076 |
| 15 | 8.670 | 2.208 | . 004 | 4.417 | . 019 | 6.625 | . 042 | 8.834 | . 074 |
| 30 | 17.340 | 2.172 | . 004 | $4 \cdot 343$ | . 018 | 6.515 | . 042 | 8.687 | . 073 |
| 45 | 26.010 | 2.435 | . 004 | 4.270 | . 018 | 6.405 | . 041 | 8.540 | . 072 |
| 7600 | 34.680 | 2.098 | . 004 | 4.197 | . 018 | 6.296 | . 040 | 8.394 | . 071 |
| 15 | 8.671 | 2.062 | . 004 | 4.123 | . 018 | 6.185 | . 040 | 8.247 | . 069 |
| 30 |  | 2.025 | . 004 | 4.050 | . 017 | 6.075 | . 039 | 8.100 | . 068 |
| 45 | 26.013 | 1.988 | . 004 | 3.976 | . 017 | 5.964 | . 038 | 7.952 | . 067 |
| 7700 | 34.684 | 1.951 | . 004 | 3.903 | . 017 | 5.854 | . 037 | 7.805 | . 066 |
| 15 | 8.672 | 1.914 | . 004 | 3.829 | . 017 | 5.743 | . 037 | 7.658 | . 065 |
| 30 | 17.343 | 1.877 | . 004 | 3.755 | . 016 | 5.632 | . 036 | 7.510 | . 064 |
| 45 | 26.015 | 1.840 | . 004 | 3.681 | .016 | 5.522 | . 036 | $7 \cdot 362$ | . 063 |
| 7800 | 34.686 | 1.804 | . 004 | 3.607 | . 015 | 5.411 | . 035 | 7.214 | . 062 |
| 15 | 8.672 | 1.766 | . 004 | 3.533 | .OI 5 | 5.300 | . 034 | 7.066 | . 060 |
| 30 | 17.344 | 1.729 | . 004 | 3.459 | . 015 | 5.188 | . 034 | 6.918 | . 059 |
| 45 | 26.017 | 1. 692 | . 004 | 3.385 | .OI4 | 5.077 | . 033 | 6.769 | . 058 |
| 7900 | 34.689 | 1.655 | . 004 | 3.310 | . 014 | 4.966 | . 032 | 6.621 | . 057 |
| 15 | 8.673 | 1.618 | . 003 | 3.236 | . 014 | 4.854 | . 031 | 6.472 | . 055 |
| $30$ | 17.346 | I. 581 | . 003 | 3.162 | . 013 | 4.742 | . 030 | 6.323 | . 054 |
| 45 | 26.018 | I. 544 | . 003 | 3.087 | . 013 | 4.63 I | . 030 | 6.174 | . 053 |
| 8000 | 34.691 | 1.506 | . 003 | 3.613 | . 013 | 4.519 | . 029 | 6.026 | . 052 |

Smithsonian Tables.

Table 22.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\delta \frac{1}{8} \frac{1}{50}$
[Derivation of table explained on pp. liii-lvi.]


Bmithsonian Tables.
[Derivation of table explained on pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]


Table 22.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE ${ }_{6 \frac{1}{8} \frac{1}{60} .}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ | $10^{\prime}$ | 15 | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $21^{\circ} 0^{\prime}$ | Inches. | Inches. | Inches. 10.768 | Inches.$16.151$ | Inches.$21.535$ | Inches.$26.919$ | Inches.$32.303$ |  | $21^{\circ}$ | $22^{\circ}$ |
|  | 68.787 | $5 \cdot 384$ |  |  |  |  |  |  |  |  |
| 10 | 11.466 | $5 \cdot 378$ | 10.755 | 16.133 | $\begin{aligned} & 21.5 I I \\ & 21.486 \end{aligned}$ | 26.889 | 32.266 |  |  |  |
| 20 | 22.932 | $5 \cdot 372$ | 10.743 | 16.115 |  | 26.858 | 32.230 | $5^{\prime}$ | Inches. 0.001 |  |
| 30 | 34.397 | $5 \cdot 366$ | ro.731 | 16.097 | 21.462 | 26.828 | 32.193 |  |  | $0.001$ |
| 40 | 45.863 | $5 \cdot 359$ | 10.719 | 16.078 | 21.438 | 26.797 | 32.156 |  | 0.001 |  |
| 50 | 57.329 | $5 \cdot 353$ | 10.707 | 16.060 | 21.413 | 26.767 | 32.120 | 10 | . .013 | . 013 |
| 2200 | 68.795 | $5 \cdot 347$ | 10.694 | 16.042 | 2I. 389 | 26.736 | 32.083 | 20 | .022.035.051 | $\begin{array}{r} .023 \\ .036 \end{array}$$.052$ |
| 10 | I I 467 | $5 \cdot 341$ | 10.682 | 16.022 | 21.363 | $\begin{aligned} & 26.704 \\ & 26.672 \end{aligned}$ | 32.045 | 30 |  |  |
| 20 | 22.934 | $5 \cdot 334$ | 10.669 | 16.003 | 21.338 |  | 32.006 |  |  |  |
| 30 | 34.401 | 5.328 | 10.656 | 15.984 | 21.312 | 26.64 L | 31.969 |  |  |  |
| 40 | 45.868 | $5 \cdot 322$ | 10.643 | 15.965 | 21.287 | 26.609 | 31.930 |  |  |  |
| 50 | 57.336 | $5 \cdot 315$ | 10.631 | 15.946 | 2 I .26 I | 26.577 | 31.892 |  |  |  |
| 2300 | 68.803 | $5 \cdot 309$ | 10.618 | 15.927 | 21.236 | 26.545 | 31.853 |  | $23^{\circ}$ | $24^{\circ}$ |
| 10 | 11.469 | $5 \cdot 302$ | 10.604 | I 5.907 | 21.209 | 26.511 | 31.813 |  |  |  |
| 20 | 22.937 | 5.296 | 10.591 | 15.887 | 21.182 | 26.478 | 31.774 | ${ }_{10}^{5}$ | 0.001 | 0.002 |
| 30 | 34.406 | 5.289 | 10.578 | 15.867 | 2 I .156 | 26.445 | 31.733 |  |  | . 006 |
| 40 | 45.874 | 5.282 | 10.565 | 15.847 | 21.129 | 26.412 | 31.694 | 15 |  | . 014 |
| 50 | 57.343 | 5.276 | 10.55 I | 15.827 | 21.102 | 26.378 | 31.654 | 20 | . 024 | . 025 |
| 2400 | 68.812 | 5.269 | 10.538 | 15.807 | 21.076 | 26.345 | 31.614 | $30$ | . 054 | $\begin{aligned} & .039 \\ & .056 \end{aligned}$ |
| 10 | 11.470 | 5.263 | 10.526 | 15.789 | 21.052 | 26.315 | $\begin{aligned} & \text { 3I. } 577 \\ & \text { 3I. } 535 \\ & \text { 3I. } 493 \\ & \text { 3I. } 450 \\ & \text { 3I.408 } \end{aligned}$ |  |  |  |
| 20 | 22.940 | 5.256 | 10.512 | 15.767 | 21.023 | 26.279 |  |  |  |  |
| 30 | 34.410 | 5.249 | 10.498 | 15.746 | 20.995 | 26.244 |  |  |  |  |
| 40 | 45.880 | 5.242 | 10.483 | 15.725 | 20.967 | 26.209 |  |  |  |  |
| 50 | 57.350 | 5.227 | 10.469 | 15.704 | 20.938 | 26.137 | 31.365 |  |  | $26^{\circ}$ |
| 2500 | 68.821 |  | 10.455 | 15.682 | 20.910 |  |  |  |  |  |  |
| 10 | 11.472 | 5.220 | 10.441 | 15.661 | 20.881 | 26.101 | 31.322 | 5 | 0.002 | 0.002 |
| 20 | 22.943 | 5.213 | 10.426 | 15.639 | 20.852 | 26.065 | 31. 279 | 10 | . 006 | . 007 |
| 30 | 34.415 | 5.206 | 10.412 | 15.618 | 20.824 | 26.029 | 31.235 | 15 | . 014 | .015 |
| 40 | 45.886 | 5.199 | 10.397 | 15.596 | 20.795 | 25.993 | 31.192 | 20 | . 026 | . 026 |
| 50 | $57 \cdot 35^{8}$ | 5.191 | 10.383 | 15.575 | 20.766 | 25.958 | 31.149 | $\begin{aligned} & 25 \\ & 30 \end{aligned}$ | $.058$ | $.059$ |
| 2600 | 68.830 | 5.184 | 10.369 | 15.553 | 20.737 | 25.922 | 31.106 |  |  |  |
| 10 | 11.473 | 5.177 | 10.354 | 15.531 | 20.708 | 25.884 | 31.061 |  |  |  |
| 20 | 22.946 | 5.169 | 10.339 | I 5.508 I 5.486 | 20.678 20.648 | 25.847 25.810 | 31.017 30.972 |  |  |  |
| 30 | 34.419 45.892 | 5.162 5.154 | 10.324 10.309 | 15.486 15.463 | 20.648 | 25.810 25.772 | 30.972 30.927 |  |  |  |
| 50 | $57 \cdot 365$ | 5.147 | 10.294 | 15.441 | 20.588 | 25.735 | 30.882 |  | $27^{\circ}$ | $28^{\circ}$ |
| 2700 | 68.838 | 5.140 | 10.279 | 15.419 | 20.558 | 25.698 | 30.838 | 5 | 0.002 | 0.002 |
| 10 | 11.475 | $5 \cdot 132$ | 10.264 | I 5.396 | 20.528 | 25.659 | 30.791 | 10 | .007 .015 | .007 .016 |
| 20 | 22.950 | 5.124 | 10.248 | 15.373 15.349 | 20.497 | 25.621 25.582 | 30.745 | 15 | . 015 | . 01028 |
| 30 40 | 34.424 45.899 | 5.116 | 10.233 10.218 | 15.349 15.326 | 20.466 20.435 | 25.582 25.544 | 30.699 30.653 | 25 | . 042 | . 043 |
| 50 | 45.899 57.374 | 5.101 | 10.202 | 15.3203 15.303 | 20.435 20.404 | 25.544 25.505 | 30.607 | 30 | .061 | . 063 |
| 2800 | 68.849 | 5.093 | ro.187 | 15.280 | 20.374 | 25.467 | 30.560 |  |  |  |

Table 22.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\overline{\text { E }} \frac{1}{8} छ \sigma$.
[Derivation of table explained on pp . liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED Parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5{ }^{\prime}$ | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $42^{\circ} \mathrm{OO}$ | Inches, | Inches. | Inches. | Inches.$12.871$ | Inches.$17.161$ | Inches.$21.451$ | Inches.$25.742$ |  | $42^{\circ}$ | $43^{\circ}$ |
|  | 69.007 | 4.290 | 8.581 |  |  |  |  |  |  |  |
| 10 | 11.503 | 4.279 | 8.558 | 12.837 | 17.116 | 21.395 | 25 |  |  |  |
| 20 | 23.006 | 4.268 | 8.535 | 12.803 | 17.071 | 21.338 | 25.606 |  |  |  |
| 30 | 34.510 | 4.256 | 8.513 | 12.769 | 17.025 | 21.282 | 25.538 | 5 | Inches. 0.002 | Inches. |
| 40 | 46.013 | 4.245 | 8.490 | 12.735 | 16.980 | 21.225 | 25.470 | 10 | . 0.08 | 0.002 .008 |
| 50 | 57.516 | 4.234 | 8.467 | 12.701 | 16.935 | 21.169 | 25.402 | 10 15 | .008 .019 | . 0108 |
| 4300 | 69.019 | 4.222 | 8.445 | 12.667 | 16.890 | 21.112 | 25.334 | 20 | .033 .052 | . 033 |
| 10 | 11.505 | 4.211 | 8.422 | 12.633 | 16.844 | 21.054 | 25.265 | 30 | . 075 | . 075 |
| 20 | 23.010 | 4.199 | 8.399 | 12.598 | 16.798 | 20.997 | 25.196 |  |  |  |
| 30 | 34.515 | 4.188 | 8.376 | 12.564 | 16.751 | 20.939 | 25.127 |  |  |  |
| 40 | 46.020 | 4.176 | 8.353 | 12.529 | 16.705 | 20.882 | 25.058 |  |  |  |
| 50 | 57.525 | 4.165 | 8.330 | 12.494 | 16.659 | 20.824 | 24.989 |  |  |  |
| 4400 | 69.030 | 4.153 | 8.307 | 12.460 | 16.613 | 20.767 | 24.920 |  | $44^{\circ}$ | $45^{\circ}$ |
| 10 | 11.507 | 4.142 | 8.283 | 12.425 | 16.566 | 20.708 | 24.849 |  |  |  |
| 20 | 23.014 | 4.130 | 8.260 | 12.390 | 16.519 | 20.649 | 24.779 | 5 | 0.002 | 0.002 |
| 30 | 34.522 | 4. 118 | 8.236 | 12.354 | 16.473 | 20.591 | 24.709 | 10 | . 008 | . 008 |
| 40 | 46.029 | 4.106 | 8.213 | 12.319 | 16.426 | 20.532 | 24.638 | 15 | . 019 | . 019 |
| 50 | 57.536 | 4.095 | 8.189 | 12.284 | 16.379 | 20.473 | 24.568 | 20 | . 034 | . 034 |
| 4500 | 69.043 | 4.083 | 8.166 | 12.249 | 16.332 | 20.415 | 24.498 | 25 30 | $\begin{aligned} & .052 \\ & .075 \end{aligned}$ | . 053 |
| 10 | 11.509 | 4.071 | 8.142 | 12.213 | 16.284 | 20.355 | 24.426 |  |  |  |
| 20 | 23.018 | 4.059 | 8.118 | 12.177 | 16.236 | 20.295 | 24.354 |  |  |  |
| 30 | 34.528 | 4.047 | 8.094 | 12.141 | 16.188 | 20.236 | 24.283 |  |  |  |
| 40 | 46.037 | 4.035 | 8.070 | 12.105 | 16.141 | 20.176 | 24.211 |  |  |  |
| 50 | 57.546 | 4.023 | 8.046 | 12.070 | 16.093 | 20.116 | 24.139 |  |  |  |
| 4600 | 69.055 | 4.011 | 8.023 | 12.034 | 16.045 | 20.056 | 24.068 |  | $46^{\circ}$ | $47^{\circ}$ |
| 10 | 11.511 | 3.999 | 7.998 | 11.997 | 15.997 | 19.996 | 23.995 | 5 | 0.002 | 0.002 |
| 20 | 23.023 | 3.987 | 7.974 | 11.961 | 15.948 | 19.935 | 23.922 | 10 | . 008 | . 008 |
| 30 | 34.534 | 3.975 | 7.950 | 11.925 | I 5.899 | 19.974 | 23.849 | 15 | . 019 | . 019 |
| 40 | 46.045 57.557 | 3.963 | 7.925 | 11.888 | 15.851 | 19.813 | 23.776 | 20 | . 034 | . 034 |
| 50 | 57.557 | 3.951 | 7.901 | 11.852 | 15.802 | 19.753 | 23.703 | 25 | . 053 | . 052 |
| 4700 | 69.068 | 3.938 | 7.877 | II.815 | I 5.754 | 19.692 | 23.630 | 30 | . 076 | . 075 |
| 10 | 11.513 | 3.926 | 7.852 | 11.778 | 15.704 | 19.630 | 23.556 |  |  |  |
| 20 | 23.027 | 3.914 | 7.827 | 11.741 | 15.655 | 19.569 | 23.482 |  |  |  |
| 30 | 34.540 | 3.981 | 7.803 | 11.704 | 15.606 | 19.507 | 23.408 |  |  |  |
| 40 | 46.053 | 3.889 | 7.778 | 11.667 | 15.556 | 19.445 | 23.334 |  |  |  |
| 50 | 57.567 | 3.877 | 7.753 | 11.630 | 15.507 | 19.383 | 23.260 |  | $48^{\circ}$ | $49^{\circ}$ |
| 4800 | 69.080 | 3.864 | 7.729 | 11.593 | 15.457 | 19.322 | 23.186 |  | 0.002 | 0.002 |
| 10 | 11.516 | 3.852 | 7.704 | 11.555 | 15.407 |  | 23.111 | 10 | 0.002 .008 | . 008 |
| 20 | 23.031 | 3.839 | 7.679 | 11.518 | 15.357 | 19.196 | 23.035 | 15 | . 019 | . 019 |
| 30 | 34.546 | 3.827 | 7.653 | 11.480 | 15.307 | 19.134 | 22.960 | 20 | . 033 | . 033 |
| 40 | 46.062 | 3.814 | 7.628 | 11.442 | 15.257 | 19.071 | 22.885 | 25 30 | . 052 | . 052 |
| 50 | 57.577 | 3.802 | 7.603 | 11.405 | 15.206 | 19.008 | 22.810 | 30 | . 075 | . 075 |
| 4900 | 69.093 | 3.789 | 7.578 | 11.367 | 15.156 | 18.945 | 22.734 |  |  |  |

Bmithsonian Tables.
[Derivation of table explained on pp. liii-lvi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ |  | $15^{\prime}$ |  | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $49^{\circ} 00^{\prime}$ | Inches. | Inches. | Irches. | Ynches. | Inches. | Inches. | Inches. |  |  | $50^{\circ}$ |
|  | 69.093 | 3.789 | 7.578 | 11.367 | 15.156 | 18.945 | 22.734 | 堅感 | $49^{\circ}$ | $50^{\circ}$ |
| 10 | 11.517 | 3.776 | 7.553 | 11.329 | 15.105 | 18.882 | 22.658 |  |  |  |
| 20 | 23.035 | 3.764 | 7.527 | 1 I .291 | 15.054 | 18.818 | 22.581 |  | Inches. | Inches. |
| 30 | 34.552 | 3.75 I | 7.502 | 11.253 | 15.003 | 18.754 | 22.505 | $5{ }^{\prime}$ | 0.002 | 0.002 |
| 40 | 46.070 | 3.738 | 7.476 | 1 I .214 | 14.952 | 18.690 | 22.429 | 10 | . 0008 | . 008 |
| 50 | 57.587 | 3.725 | $7 \cdot 451$ | 11.176 | 14.901 | 18.627 | $22.35{ }^{2}$ | 15 | .008 .019 | . 0108 |
| 5000 | 69.105 | 3.713 | 7.425 | 11.138 | 14.850 | 18.563 | 22.276 | 20 | .033 .052 | .033 |
| 10 | 11.520 | 3.700 | 7.399 | 11.099 | 14.799 | 18.499 | 22.198 | 30 | . 075 | . 075 |
| 20 | 23.039 | 3.687 | 7.374 | 11.060 | 14.747 | 18.434 | 22.121 |  |  |  |
| 30 | 34.558 | 3.674 | $7 \cdot 348$ | 11.021 | 14.695 | 18.369 | 22.043 |  |  |  |
| 40 | 46.078 57.598 | 3.665 3.648 | 7.322 7.296 | 10.983 | 14.644 | 18.305 | 21.965 21.888 |  |  |  |
| 50 | 57.598 | 3.648 | 7.296 | 10.944 | 14.592 | 18.240 |  |  |  |  |
| 5100 | 69.117 | 3.635 | 7.270 | 10.905 | 14.540 | 18.176 | 21.8 I! |  | $55^{\circ}$ | $52^{\circ}$ |
| 10 | II.521 | 3.622 | 7.244 | 10.866 | 14.488 | 18.110 | 21.732 |  |  |  |
| 20 | 23.043 | 3.609 | 7.218 | 10.827 | 14.436 | 18.045 | 21.653 | 5 | 0.002 | 0.002 |
| 30 | 34.564 | $3 \cdot 596$ | 7.191 | 10.787 | 14.383 | 17.979 | 21.574 | 10 | . 008 | . 008 |
| 40 50 | 46.086 57.607 | 3.583 3.570 | 7.165 7.139 | 10.748 10.709 | 14.330 14.278 | 17.913 | 21.496 21.417 | 15 | .019 | . 018 |
| 50 | 57.607 | 3.570 | 7.139 | 10.709 | 14.278 | 17.848 | 21.417 | 20 | . 033 | . 033 |
| 5200 | 69.128 | 3.556 | 7.113 | 10.669 | 14.226 | $17.78{ }^{2}$ | $21.33^{8}$ | 25 30 | . 075 | . 073 |
| 10 | 11.523 | 3.543 | 7.086 | 10.629 | 14.172 | 17.716 | 21.259 |  |  |  |
| 20 | 23.047 | 3.530 | 7.060 | 10.589 | 14.119 | 17.649 | 21.179 |  |  |  |
| 30 | 34.570 | 3.516 | 7.033 | 10.550 | 14.066 | 17.583 | 21.099 |  |  |  |
| 40 | 46.094 | 3.503 | 7.006 | 10.510 | 14.013 | 17.516 | 21.019 |  |  |  |
| 50 | 57.617 | 3.490 | 6.980 | 10.470 | 13.960 | 17.450 | 20.939 |  | $53^{\circ}$ | $54^{\circ}$ |
| 5300 | 69.140 | 3.477 | 6.953 | 10.430 | 13.906 | 17.383 | 20.860 |  |  |  |
| 10 | 11.525 | 3.463 | 6.926 | 10.389 | ${ }_{1} 3.852$ | 17.316 | 20.779 | 5 |  |  |
| 20 | 23.051 | 3.450 | 6.899 | 10.349 | r 3.798 | 17.248 | 20.698 | 10 | . 008 | .008 |
| 30 | 34.576 | 3.436 | 6.872 | 10.309 | 13.745 | 17.181 | 20.617 | 15 | . 018 | . 018 |
| 40 | 46.102 57.627 | 3.423 3.409 | 6.845 | 10.268 | 13.691 13637 | 17.114 17.046 | 20.536 | 20 | .032 .050 |  |
| 50 | 57.627 | 3.409 | 6.818 | 10.228 | 13.637 | 17.046 | 20.455 | 25 | $\begin{aligned} & .050 \\ & .073 \end{aligned}$ | .050 .072 |
| 5400 | 69.152 | 3.396 | 6.791 | 10.187 | 1 3.583 | 16.979 | 20.374 |  |  |  |
| 10 | 11.527 | 3.382 | 6.764 | 10.146 | 13.528 | 16.910 | 20.292 |  |  |  |
| 20 | 23.055 | $3 \cdot 368$ | 6.737 | 10.105 | 13.474 | 16.842 | 20.210 |  |  |  |
| 30 40 | 34.582 46.109 | $3 \cdot 355$ $3 \cdot 34$ | 6.709 6.682 | 10.064 | 13.419 13.364 | 16.774 16.706 | $\begin{aligned} & 20.128 \\ & 20.047 \end{aligned}$ | - |  |  |
| 40 50 | 46.109 57.636 | $3 \cdot 34 \mathrm{I}$ $3 \cdot 327$ | 6.682 6.655 | 10.023 9.982 | 13.364 13.310 | 16.706 16.637 | 20.047 19.964 |  | $55^{\circ}$ | $56^{\circ}$ |
| 5500 | 69.164 | 3.314 | 6.628 | 9.941 | 13.255 | 16.569 | 19.883 | 5 | 0.002 | 0.002 |
| 10 | 11.529 | $3 \cdot 300$ | 6.600 | 9.900 | 13.200 | 16.500 | 19.800 | 10 | .008 .018 | . 008 |
| 20 | 23.059 | 3.286 | 6.572 | 9.859 | 13.145 | 16.431 16.362 | $19.717$ | 20 | . 032 | . 031 |
| $30$ | 34.588 46.117 | 3.272 3.258 | 6.545 6.517 | 9.817 9.776 | 13.089 13.034 | 16.362 16.293 | $\begin{aligned} & 19.634 \\ & 19.551 \end{aligned}$ | 25 | . 049 | . 049 |
| 40 50 | 46.117 57.646 | 3.258 3.245 | 6.517 6.489 | 9.776 9.734 | 13.034 12.979 | 16.293 16.224 | $\begin{aligned} & 19.55 \mathrm{I} \\ & 19.468 \end{aligned}$ | 30 | . 07 I | . 070 |
| 5600 | 69.176 | 3.231 | 6.462 | 9.693 | 12.924 | 16.155 | 19.385 |  |  |  |

[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $10^{\prime \prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $56^{\circ} 00^{\prime}$ | Inches. | Inches.$3.23 \mathrm{I}$ | Inches. <br> 6.462 | Inches.$9.693$ | Inches. 12.924 | Inches. <br> 16.155 | Inches.$19.385$ |  | $56^{\circ}$ | $57^{\circ}$ |
|  | 69.176 |  |  |  |  |  |  |  |  |  |
| 10 | 11.531 | 3.217 | 6.434 | 9.651 | 12.868 | 16.085 | 19.301 |  |  |  |
| 20 | 23.063 | 3.203 | 6.406 | 9.609 | 12.812 | 16.015 | 19.217 |  | Inches. | Inches. |
| 30 | 34.594 | 3.189 | 6.378 | 9.567 | 12.756 | 15.945 | 19.134 | 5' | 0.002 | 0.002 |
| 40 | 46.125 | 3.175 | 6.350 | 9.525 | 12.700 | 15.875 | 19.050 18.966 | 10 | . 008 | . 008 |
| 50 | 57.656 | 3.161 | 6.322 | 9.483 | 12.644 | 15.805 | 18.966 | 15 | . 018 | . 017 |
| 5700 | 69.188 | 3.147 | 6.294 | 9.44 I | 12.588 | 15.735 | 18.882 | 20 | . 031 | .031 |
| 10 | 11.533 | 3.133 | 6.266 | 9.398 | 12.53 I | 15.664 | 18.797 | 30 | . 070 | . 069 |
| 20 | 23.066 | 3.119 | 6.237 | $9 \cdot 356$ | 12.475 | 15.594 | 18.712 |  |  |  |
| 30 | 34.599 | 3.104 | 6.209 | 9.314 | 12.418 | 15.523 | 18.627 |  |  |  |
| 40 | 46.132 | 3.090 | 6.181 | 9.271 | 12.362 | 15.452 | 18.542 |  |  |  |
| 50 | 57.666 | 3.076 | 6.152 | 9.229 | 12.305 | 15.381 | 18.457 |  |  |  |
| 5800 | 69.199 | 3.062 | 6.124 | 9.186 | 12.248 | 15.311 | 18.373 |  | $5^{\circ}$ | $59^{\circ}$ |
| 10 | 11.535 | 3.048 | 6.096 | 9.143 | 12.191 | 15.239 | 18.287 |  |  |  |
| 20 | 23.070 | 3.034 | 6.067 | 9.101 | 12.134 | 15.168 | 18.201 | 5 | 0.002 | 0.002 |
| 30 | 34.605 | 3.019 | 6.038 | 9.058 | 12.077 | 15.096 | 18.115 | 10 | . 008 | . 007 |
| 40 | 46.140 57.675 | 3.005 | 6.010 | 9.015 | 12.020 | 15.025 | 18.029 | 15 | . 017 | . 017 |
| 50 | 57.675 | 2.991 | 5.981 | 8.972 | 11.962 | 14.953 | 17.944 | 20 | . 030 | . 030 |
| 5900 | 69.210 | 2.976 | $5 \cdot 953$ | 8.929 | 11.905 | 14.882 | 17.858 | 30 | . 068 | . 067 |
| 10 | 11.537 | 2.962 | 5.924 | 8.885 | 11.847 | 14.809 | 17.771 |  |  |  |
| 20 | 23.074 | 2.947 | 5.895 | 8.842 | 11.790 | 14.737 | 17.684 |  |  |  |
| 30 | 34.610 | 2.933 | 5.866 | 8.799 | 11.732 | 14.665 | 17.597 |  |  |  |
| 40 | 46.147 | 2.918 | 5.837 | 8.755 | 11.674 | 14.592 | 17.510 |  |  |  |
| 50 | 57.684 | 2.904 | 5.808 | 8.712 | 11.616 | 14.520 | 17.424 |  | $60^{\circ}$ | $61^{\circ}$ |
| 6000 | 69.221 | 2.890 | 5.779 | 8.669 | I 1.558 | 14.448 | 17.337 |  |  |  |
| 10 | 11.539 | 2.875 | 5.750 | 8.625 | 11.500 | 14.375 | 17.249 | 5 | 0.002 | 0.002 |
| 20 | 23.077 | 2.860 | 5.721 | 8.581 | 11.44 I | 14.302 | 17.162 | 10 | . 007 | . 007 |
| 30 | 34.616 | 2.846 | 5.691 | 8.537 | 11.383 | 14.229 | 17.074 | 15 | . 016 | . 016 |
| 40 | 46.154 | 2.831 | 5.662 | 8.493 | 11.324 | 14.156 | 16.987 | 25 | . 029 | . 029 |
| 50 | 57.693 | 2.816 | 5.633 | 8.450 | 11.266 | 14.083 | 16.899 | $\begin{aligned} & 25 \\ & 30 \end{aligned}$ | $\begin{aligned} & .045 \\ & .065 \end{aligned}$ | .045 .064 |
| 6100 | 69.232 | 2.802 | 5.604 | 8.406 | 11.208 | 14.010 | 16.81 I |  |  |  |
| 10 | 11.540 | 2.787 | $5 \cdot 574$ | 8.361 | 11.148 | 13.936 | 16.723 |  |  |  |
| 20 | 23.081 | 2.772 | $5 \cdot 545$ | 8.317 | 11.090 | 13.862 | 16.634 |  |  |  |
| 30 | 34.621 | 2.758 | 5.115 | 8.273 | 11.030 | 13.788 | 16.546 |  |  |  |
| 40 | 46.162 | 2.743 | 5.486 | 8.229 | 10.972 | 13.715 | 16.457 |  | 62 ${ }^{\circ}$ |  |
| 50 | 57.702 | 2.728 | $5 \cdot 456$ | 8.184 | 10.912 | 13.641 | 16.369 |  |  | 63 |
| 6200 | 69.242 | 2.713 | 5.427 | 8.140 | 10.854 | 13.567 | 16.280 | 5 | 0.002 | 0.002 |
| 10 | 11.542 | 2.699 | 5.397 | 8.096 | 10.794 | 13.493 | 16.191 | 10 | . 007 | . 007 |
| 20 | 23.084 | 2.684 | $5 \cdot 367$ | 8.051 | 10.734 | 13.418 | 16.102 | 15 | . 0168 | . 015 |
| 30 | 34.626 | 2.669 | $5 \cdot 337$ | 8.006 | 10.675 | 13.344 | 16.012 | 25 | . 044 | . 043 |
| 40 50 | 46.168 57.710 | 2.654 2.639 | $5 \cdot 308$ 5.278 | 7.961 7.917 | 10.615 10.556 | 13.269 13.195 | 15.923 15.833 | 30 | . 063 | . 061 |
| 6300 | 69.253 | 2.624 | 5.248 | 7.872 | 10.496 | 13.120 | 15.744 |  |  |  |

Table 22.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\overline{88} \frac{1}{8} 6 \sigma^{\frac{1}{6}}$.
[Derivatios of table explained on pp. liii-lvi.]

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|}
\hline \multirow[b]{2}{*}{} \& \multirow[t]{2}{*}{} \& \multicolumn{6}{|c|}{ABSCISSAS OF DĖVELOPED PARALLEL.} \& \multicolumn{3}{|l|}{\multirow[b]{2}{*}{ORDINATES OF DEVELOPED PARALLEL.}} <br>
\hline \& \& 5 \& $10^{\prime}$ \& \& $20^{\prime}$ \& $25^{\prime}$ \& $30^{\prime}$ \& \& \& <br>
\hline \multirow[b]{2}{*}{$63^{\circ} 00^{\prime}$} \& Inches. \& Inc \& Inches. \& Inches. \& Inches. \& Inches. \& Inches. \& \multirow[t]{2}{*}{|} \& \multirow[t]{2}{*}{$63^{\circ}$} \& \multirow[t]{2}{*}{$64^{\circ}$} <br>
\hline \& 69.253 \& 2.624 \& 5.248 \& 7.872 \& 10.496 \& 13.120 \& 15.744 \& \& \& <br>
\hline 10 \& 11.544 \& 2.609 \& 5.218 \& 7.827 \& 10.436 \& 13.045 \& ${ }_{1} 5.654$ \& \& \& <br>
\hline 20 \& 23.087 \& 2.594 \& 5.188 \& 7.782 \& 10.376 \& $12.97{ }^{\circ}$ \& 15.564 \& \& Inches. \& Inches. <br>
\hline 30 \& 34.631 \& 2.579 \& 5.158 \& 7.737 \& 10.316 \& 12.895 \& 15.473 \& \multirow[t]{2}{*}{10 ${ }^{5}$} \& \multirow[t]{2}{*}{0.002
.007} \& \multirow[b]{2}{*}{0.002
.007} <br>
\hline 40 \& 46.175
57.718 \& 2.564 \& 5.128 \& 7.692 \& 10.256 \& 12.820 \& 15.383

5 \& \& \& <br>
\hline 50 \& 57.718 \& 2.549 \& 5.098 \& 7.647 \& 10.196 \& 12.745 \& 15.293 \& 15 \& . 015 \& . 015 <br>

\hline 6400 \& 69.262 \& 2.534 \& 5.068 \& 7.602 \& 10.136 \& 12.670 \& 15.203 \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 25 \\
& 30
\end{aligned}
$$} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& .043 \\
& .061
\end{aligned}
$$

\]} \& \[

$$
\begin{aligned}
& .041 \\
& .060
\end{aligned}
$$
\] <br>

\hline 10 \& 11.545 \& 2.519 \& 5.037 \& 7.556 \& 10.075 \& 12.594 \& 15.112 \& \& \& <br>
\hline 20 \& 23.091 \& 2.504 \& 5.007 \& 7.511 \& 10.014 \& 12.518 \& 15.022 \& \& \& <br>
\hline 30 \& 34.636 \& 2.488 \& 4.977 \& $7 \cdot 465$ \& 9.954 \& 12.442 \& 14.930 \& \& \& <br>
\hline 40 \& 46.182 \& 2.473 \& 4.947 \& 7.420 \& 9.893 \& 12.367 \& 14.840 \& \& \& <br>
\hline 50 \& 57.727 \& 2.458 \& 4.916 \& 7.374 \& 9.832 \& 12.291 \& 14.749 \& \& \& <br>
\hline 6500 \& 69.272 \& 2.443 \& 4.886 \& $7 \cdot 329$ \& 9.772 \& 12.215 \& 14.658 \& \multicolumn{2}{|r|}{$65^{\circ}$} \& $66^{\circ}$ <br>
\hline 10 \& I 1. 547 \& 2.428 \& 4.855 \& 7.283 \& 9.711 \& 12.139 \& 14.566 \& \multirow[b]{2}{*}{5} \& \& \multirow[b]{2}{*}{0.002} <br>
\hline 20 \& 23.094 \& 2.412 \& 4.825 \& 7.237 \& 9.650 \& 12.062 \& 14.474 \& \& 0.002
.006 \& <br>

\hline 30 \& 34.641 \& 2.397 \& 4.794 \& 7.191 \& 9.588 \& 11.986 \& 14.383 \& 10 \& . 006 \& $$
.006
$$ <br>

\hline 40
50 \& 46.188 \& 2.382
2.366 \& 4.764 \& 7.145 \& 9.527
9.466 \& 11.909

11.833 \& \multirow[t]{2}{*}{14.199} \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 20 \\
& 25
\end{aligned}
$$} \& .026 \& \multirow[t]{2}{*}{. 025} <br>

\hline 50 \& 57.735 \& 2.366 \& 4.733 \& 7.100 \& 9.466 \& 11.833 \& \& \& . 040 \& <br>
\hline 6600 \& 69.282 \& 2.351 \& 4.702 \& 7.054 \& 9.405 \& 11.756 \& 14.107 \& 30 \& . 058 \& . 056 <br>
\hline 10 \& 1 I .548 \& 2.336 \& \multirow[t]{2}{*}{4.672
4.641} \& \multirow[t]{2}{*}{7.007
6.961} \& \multirow[t]{2}{*}{9.343
9.282} \& \multirow[t]{2}{*}{11.679
11.602} \& 14.015 \& \& \& <br>
\hline 20 \& 23.097 \& 2.320 \& \& \& \& \& \multirow[t]{2}{*}{13.922
13.830} \& \& \& <br>
\hline 30 \& 34.646 \& 2.305 \& 4.610 \& \multirow[t]{2}{*}{6.915

6.869} \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 9.220 \\
& 9.158
\end{aligned}
$$} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 11.525 \\
& 11.448
\end{aligned}
$$
\]} \& \& \& \& <br>

\hline 40 \& 46.194 \& 2.290 \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 4 \cdot 579 \\
& 4 \cdot 548
\end{aligned}
$$} \& \& \& \& \multirow[t]{2}{*}{13.738

13.645} \& \& \multirow[b]{2}{*}{$67^{\circ}$} \& \multirow[b]{2}{*}{$68^{\circ}$} <br>

\hline 50 \& 57.742 \& 2.274 \& \& $$
\begin{aligned}
& .009 \\
& 6.823
\end{aligned}
$$ \& 9.097 \& 11.371 \& \& \& \& <br>

\hline 6700 \& 69.291 \& 2.259 \& 4.518 \& 6.776 \& 9.035 \& I 1.294 \& 13.553 \& \multirow[b]{2}{*}{5} \& \multirow[b]{2}{*}{$$
\begin{array}{r}
0.001 \\
.006
\end{array}
$$} \& \multirow[b]{2}{*}{0.001} <br>

\hline 10 \& 11.550 \& 2.243 \& 4.487 \& 6.730 \& 8.973 \& 11.217 \& 13.460 \& \& \& <br>
\hline 20 \& 23.100 \& 2.228 \& \multirow[t]{2}{*}{4.455
4.424} \& \multirow[t]{2}{*}{6.683

6.637} \& \multirow[t]{2}{*}{8.91 I} \& \multirow[t]{2}{*}{$$
\begin{aligned}
& \text { II.I } 39 \\
& \text { I } \mathbf{y} .06 \mathbf{n}
\end{aligned}
$$} \& 13.366 \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 15 \\
& 20
\end{aligned}
$$
\]} \& .014 \& . 006 <br>

\hline 30 \& 34.650 \& 2.212 \& \& \& \& \& \multirow[t]{2}{*}{$$
\begin{array}{r}
13.273 \\
13.180
\end{array}
$$} \& \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& .024 \\
& .038
\end{aligned}
$$
\]} \& \multirow[t]{2}{*}{. 023} <br>

\hline 40 \& 46.200 \& 2.197 \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 4.393 \\
& 4.362
\end{aligned}
$$} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 6.590 \\
& 6.543
\end{aligned}
$$

\]} \& 8.787 \& \[

$$
\begin{aligned}
& 11.061 \\
& 10.984
\end{aligned}
$$

\] \& \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 25 \\
& 30
\end{aligned}
$$
\]} \& \& <br>

\hline 50 \& 57.750 \& 2.181 \& \& \& 8.724 \& 10.906 \& 13.087 \& \& . 054 \& . 053 <br>
\hline 6800 \& 69.300 \& 2.166 \& 4.331 \& 6.497 \& 8.662 \& 10.828 \& 12.994 \& \& \& <br>
\hline 10 \& 11.552 \& 2.150 \& \multirow[t]{2}{*}{4.300

4.269} \& \multirow[t]{2}{*}{$$
\begin{aligned}
& 6.450 \\
& 6.403
\end{aligned}
$$} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 8.600 \\
& 8.538
\end{aligned}
$$

\]} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 10.750 \\
& 10.672
\end{aligned}
$$

\]} \& \multirow[t]{2}{*}{\[

$$
\begin{aligned}
& 12.900 \\
& 12.806
\end{aligned}
$$
\]} \& \& \& <br>

\hline 20 \& 23.103 \& 2.134 \& \& \& \& \& \& \& \& <br>
\hline 30 \& 34.654 \& 2.119 \& \multirow[t]{2}{*}{4.237
4.206} \& \multirow[t]{2}{*}{6.356
6.309} \& \multirow[t]{2}{*}{8.475

8.412} \& \multirow[t]{3}{*}{$$
\begin{aligned}
& 10.594 \\
& 10.516 \\
& 10.438
\end{aligned}
$$} \& \multirow[t]{3}{*}{\[

$$
\begin{aligned}
& 12.712 \\
& 12.619 \\
& 12.525
\end{aligned}
$$
\]} \& \& $69^{\circ}$ \& $70^{\circ}$ <br>

\hline 40 \& 46.206 \& 2.103 \& \& \& \& \& \& \& 69 \& $70^{\circ}$ <br>
\hline 50 \& 57.758 \& 2.088 \& 4.175 \& 6.263 \& 8.350 \& \& \& \& \& <br>
\hline 6900 \& 69.309 \& 2.072 \& 4.144 \& 6.216 \& 8.288 \& 10.360 \& 12.431 \& 5

10 \& $$
\begin{array}{r}
0.001 \\
.006
\end{array}
$$ \& \[

$$
\begin{array}{r}
0.001 \\
.005
\end{array}
$$
\] <br>

\hline 10 \& 11.553 \& 2.056 \& 4.112 \& 6.169 \& 8.225 \& 10.281 \& 12.337 \& 15 \& . 0102 \& . O 22 <br>

\hline 20 \& 23.106 \& 2.040 \& 4.081 \& 6.121 \& 8.162 \& 10.202 \& 12.242 \& 20 \& . 022 \& \multirow[t]{3}{*}{$$
\begin{aligned}
& .022 \\
& .034 \\
& .049
\end{aligned}
$$} <br>

\hline 30 \& 34.659 \& 2.025 \& 4.049 \& 6.074 \& 8.099 \& 10.124 \& 12.148 \& 30 \& . 051 \& <br>
\hline 40
50 \& 46.212
57.764 \& 2.009
I. 993 \& \multirow[t]{2}{*}{4.018
3.986} \& 6.027
5.980 \& 8.036
7.973 \& 10.045
9.966 \& 12.054
$\mathbf{I I} .959$ \& \& \& <br>
\hline 50 \& 57.764 \& I. 993 \& \& 5.980 \& 7.973 \& 9.966 \& 11.959 \& \& \& <br>
\hline 7000 \& 69.317 \& 1.977 \& 3.955 \& 5.932 \& 7.910 \& 9.888 \& 11.865 \& \& \& <br>
\hline
\end{tabular}

Smithsonian Tables.

## CO-ORDINATES FOR PROJECTION OF MAPS. SCALE бฐఫбで

[Derivation of table explained on p. liii-lvi.]


[^24]CO－ORDINATES FOR PROJECTION OF MAPS．SCALE |  |
| :---: |
| $\frac{1}{\delta} \sigma \pi$ |

［Derivation of table explained on $p$ ．liii－lvi．］

|  |  | ABSCISSAS OF DEVELOPED PARALEL． |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL． |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ |  | $15^{\prime}$ |  | $25^{\prime}$ | $30^{\prime \prime}$ |  |  |  |
| $77^{\circ} 00^{\prime}$ | Inches．$69.367$ | Itzches．$1.301$ | Inches．$2.602$ | Inches．$3.903$ | Inches．$5.204$ | Inches．$6.505$ | Inches． <br> 7.805 | $\begin{array}{\|c} \text { 苛品 } \\ \text { 品苞 } \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $77^{\circ}$ | $78^{\circ}$ |
|  |  |  |  |  |  |  |  |  |  |  |
| 10 | 11.562 | 1.284 | 2.569 | 3.854 | 5．138 | 6.423 | 7.707 |  |  |  |
| 20 | 23.124 | 1.268 | 2.536 | 3.804 | 5.072 | 6.34 I | 7.609 |  | Inches． |  |
| 30 | 34.686 | 1.252 | 2.503 | 3.755 | 5.006 | 6.258 | 7.510 | $5^{\prime}$ | 0．001 | $0.001$ |
| 40 | 46.248 57.810 | 1．235 | 2.470 2.438 | 3.706 | 4－941 | 6.176 | 7.4111 | 10 | ． 0.001 | 0.001 .003 |
| 50 | 57.810 | 1.219 | 2.438 | 3.656 | 4.875 | 6.094 | 7.313 | 15 | ． 008 | ． 008 |
| 7800 | 69.373 | 1.202 | 2.405 | 3.607 | 4．810 | 6.012 | 7．214 | 20 | ． 015 | ． 014 |
| 10 | 11.563 | 1.186 | $2 \cdot 372$ | $3 \cdot 558$ | 4.744 | 5.930 | 7.115 | 30 | ． 033 | .031 |
| 20 | 23.126 | 1． 169 | 2.339 | 3.508 | 4.678 | 5.847 | 7.016 |  |  |  |
| 30 | 34.689 | 1.153 | 2.306 | $3 \cdot 459$ | 4.6 I 2 | 5.765 | 6.918 |  |  |  |
| 40 | 46.252 | I．I 36 | 2.273 | 3.410 | 4.546 | 5.683 | 6.819 |  |  |  |
| 50 | 57.814 | 1.120 | 2.240 | 3.360 | 4.480 | 5.600 | 6.720 |  |  |  |
| 7900 | 69.37 | 1．104 | 2.207 | 3.31 I | 4 | 5.518 | 6.621 |  | $79^{\circ}$ | $80^{\circ}$ |
| 10 | 11.564 | 1.087 | 2.174 | 3．26I | $4 \cdot 348$ | $5 \cdot 43$ | 6.522 | 5 | 0.001 | 0.001 |
| 20 | 23.127 | 1.070 | 2.141 | 3.211 | 4.282 | $5 \cdot 352$ | 6.422 | 10 | ． 003 | ． 003 |
| 30 | 34.691 | 1.054 | 2.108 | 3.162 | 4.216 | 5.270 | 6.323 | 15 | ． 007 | ． 006 |
| 40 | 46.255 | 1.037 | 2.075 | 3．112 | 4.150 | 5.187 | 6.224 | 20 | ． 013 | ． OLI |
| 50 | 57．818 | 1.021 | 2.042 | 3.062 | 4.083 | 5．104 | 6.125 | 25 30 | .020 .028 | ． 018 |
| 8000 | 69.382 | 1.004 | 2.009 | 3.013 | 4.017 | 5.022 | 6.026 |  |  |  |

Smithsonian Tablee．

Table 23.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\frac{200 \% \delta \sigma^{\circ}}{}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED Parallel for - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Io'longitude. |  | $20^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | $4^{\circ}$ longitude. |  | 50' longitude. |  | $1^{\circ}$ Iongitude. |  |
|  |  | x | y | x | y | x | y | x | y | x | y | $\mathbf{x}$ | y |
|  | $m m$. | mm | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. |
| $0^{\circ} 00^{\prime}$ | ....... | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.1 | . 0 | 463.8 | . 0 | 556.6 | . 0 |
| 10 | 92.1 | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 37 I .1 | . 0 | 463.8 | . 0 | 556.6 | . 0 |
| 20 | 184.3 | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.1 | . 0 | 463.8 | . 0 | 556.6 | . 0 |
| 30 | 276.4 | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.0 | . 0 | 463.8 | . 0 | 556.6 | . 0 |
| 40 | 368.6 | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.0 | . 0 | 463.8 | . 0 | 556.6 | . |
| 50 | 460.7 | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.0 | . 0 | 463.7 | . 0 | 556.5 | . 1 |
| 100 |  | 92.8 | . 0 | 185.5 | . 0 | 278.3 | . 0 | 371.0 | . 0 | 463.7 | . 1 | 556.5 | . 1 |
| 10 | 92.1 | 92.7 | . 0 | 185.5 | . 0 | 278.2 | . 0 | 37 I .0 | . 0 | 463.7 | . 1 | 556.4 | . 1 |
| 20 | 184.3 | 92.7 | . 0 | 185.5 | . 0 | 278.2 | . 0 | 371.0 | . 0 | 463.7 | . 1 | 556.4 | . 1 |
| 30 | 276.4 | 92.7 | . 0 | 185.5 | . 0 | 278.2 | . 0 | 370.9 | . 0 | 463.7 | . 1 | 556.4 | . 1 |
| 40 | 368.6 | 92.7 | . 0 | 185.4 | . 0 | 278.2 | . 0 | 370.9 | . 0 | 463.6 | . 1 | 556.3 | . 1 |
| 50 | 460.7 | 92.7 | . 0 | 185.4 | . 0 | 278.2 | . 0 | 370.9 | . 1 | 463.6 | . 1 | 556.3 | . 2 |
| 200 | ..... | 92.7 | . 0 | 185.4 | . 0 | 278.1 | . 0 | 370.8 | - 1 | 463.6 | . 1 | 556.3 | . 2 |
| 10 | 92.1 | 92.7 | . 0 | 185.4 | . 0 | 278.1 | . 0 | 370.8 | . | 463.5 | . 1 | 556.2 | . 2 |
| 20 | 184.3 | 92.7 | . 0 | 185.4 | . 0 | 278.I | . 0 | 370.8 | . 1 | 463.4 | . 1 | 556.I | . 2 |
| 30 | 276.4 | 92.7 | . 0 | 185.3 | . 0 | 278.0 | . 0 | 370.7 | . 1 | 463.4 | . 1 | 556.0 | . 2 |
| 40 | 368.6 | 92.7 | . 0 | 185.3 | . 0 | 278.0 | . 0 | 370.6 | . 1 | 463.3 | . 2 | 556.0 | . 2 |
| 50 | 460.7 | 92.7 | . 0 | 185.3 | . 0 | 278.0 | . 1 | 370.6 | . I | 463.2 | .2 | 555.9 | . 2 |
| 300 | ..... | 92.6 | . 0 | 185.3 | . 0 | 277.9 | . 1 | 370.6 | . 1 | 463.2 | . 2 | 555.8 | . 2 |
| 10 | 92.1 | 92.6 | . 0 | 185.2 | . 0 | 277.9 | I | 370.5 | I | 463.1 | . 2 | 555.7 | . 3 |
| 20 | 184.3 | 92.6 | . 0 | 185.2 | . 0 | 277.8 | . 1 | 370.4 | . 1 | 463.0 | . 2 | 555.7 | . 3 |
| 30 | 276.4 | 92.6 | . 0 | 185.2 | . 0 | 277.8 | . 1 | 370.4 | . 1 | 463.0 | . 2 | $555 \cdot 5$ | . 3 |
| 40 | 368.6 | 92.6 | . 0 | 185.1 | . 0 | 277.7 | . 1 | 370.3 | . 1 | 462.8 | . 2 | 555.4 | . 3 |
| 50 | 46 c .7 | 92.6 | . 0 | 185.1 | . 0 | 277.7 | . 1 | 370.2 | . 1 | 462.8 | $\cdot 2$ | $555 \cdot 4$ | . 3 |
| 400 | ...... | 92.5 | . 0 | 185.1 | . 0 | 277.6 | . 1 | 370.2 | .2 | 462.7 | . 2 | 555.2 | $\cdot 3$ |
| 10 | 92.1 | 92.5 | . 0 | 185.0 | . 0 | 277.6 | . 1 | 370.1 | . 2 | 462.6 | .2 | 555.1 | . 3 |
| 20 | 184.3 | 92.5 | . 0 | 185.0 | . 0 | $277 \cdot 5$ | . 1 | 370.0 | . 2 | 462.5 | . 2 | 555.0 | . 3 |
| 30 | 276.4 | 92.5 | . 0 | 185.0 | . 0 | $277 \cdot 4$ | . 1 | 369.9 | .2 | 462.4 | . 2 | 554.9 | . 3 |
| 40 | 368.6 | 92.5 | . 0 | 184.9 | . 0 | 277.4 | . 1 | 369.8 | .$^{2}$ | 462.3 | $\cdot 3$ | 554.8 | . 4 |
| 50 | 460.7 | 92.4 | . 0 | 184.9 | . 0 | 277.3 | . 1 | 369.8 | . 2 | 462.2 | - 3 | 554.6 | . 4 |
| 500 | - | 92.4 | . 0 | 184.8 | . 0 | $277 \cdot 3$ | . 1 | 369.7 | .$^{2}$ | 462.1 | -3 | 554.5 | . 4 |
| 10 | 92.2 | 92.4 | . 0 | 184.8 | . 1 | 277.2 | . | 369.6 | . 2 | 462.0 | - 3 | 554.3 | . 4 |
| 20 | 184.3 | 92.4 | . 0 | 184.7 | 1 | 277.1 | . 1 | 369.5 | . 2 | 46 I .8 | . 3 | 554.2 | . 4 |
| 30 40 | 276.4 368.6 | 92.3 | . 0 | 184.7 | . 1 | 277.0 | . 1 | 369.4 | .2 | 461.7 | - 3 | 554.0 | . 4 |
| 40 | 368.6 | 92.3 | . 0 | 184.6 | . 1 | 276.9 | . 1 | 369.2 | .2 | 461.6 | - 3 | 553-9 | . 5 |
| 50 | 460.7 | 92.3 | . 0 | 184.6 | . 1 | 276.9 | . 1 | 369.2 | . 2 | 461.4 | . 3 | 553.7 | . 5 |
| 600 |  | 92.3 | . 0 | 184.5 | . 1 | 276.8 | . 1 | 369.0 | . 2 | 461.3 | -4 | 553.6 |  |
| 10 | 92.2 I 84 | 92.2 | . 0 | 184.5 | . 1 | 276.7 | . 1 | 368.9 | . 2 | 461.2 | . 4 | 553.4 | . 5 |
| 20 | 184.3 | 92.2 | . 0 | 184.4 | . 1 | 276.6 | . | 368.8 | . 2 | 461.0 | . 4 | 553.2 | . 5 |
| 30 | 276.4 | 92.2 | . 0 | 184.3 | -1 | 276.5 | - 1 | 368.7 | . 2 | 460.8 | -4 | 553.0 | . 5 |
| 40 | 368.6 460.7 | 92.1 | . 0 | 184.3 | . 1 | 276.4 | . 1 | 368.6 | . 2 | 460.7 | . 4 | 552.8 | . 6 |
| 50 | 460.7 | 92.1 | . 0 | 184.2 | . 1 | 276.3 | . 1 | 368.4 | . 2 | 460.6 | - 4 | 552.7 | . 6 |
| 700 |  | 92.1 | . 0 | 184.2 | $\cdot 1$ | 276.2 | . 1 | 368.3 | $\cdot 3$ | 460.4 | - 4 |  | . 6 |
| 10 | 92.2 184 | 92.0 | . 0 | 184.1 | . 1 | 276.1 | . 1 | 368.2 | . 3 | 460.2 | . 4 | 552.2 | . 6 |
| 20 | 184.3 276.4 | 92.0 | . 0 | 184.0 | . 1 | 276.0 | . 1 | 368.0 | - 3 | 460.0 | . 4 | 552.1 | . 6 |
| 30 | 276.4 | 92.0 | . 0 | 184.0 | - I | 275.9 | . 1 | 367.9 | - 3 | 459.9 | . 4 | 551.9 | . 6 |
| 40 50 | 368.6 460.7 | 91.9 91.9 | . 0 |  | . 1 | 275.8 | . 1 | 367.8 | - 3 | 459.7 | - 4 | 551.6 | . 6 |
| 50 | 460.7 | 9 P .9 | . 0 | 183.8 | .I | 275.7 | . 1 | 367.6 | $\cdot 3$ | 459.5 | . 5 | .551.4 | . 7 |
| 800 |  | 91.9 | . 0 | 183.7 | . 1 | 275.6 | . 2 | $367 \cdot 5$ | $\cdot 3$ | 459.4 | $\cdot 5$ | 551.2 | . 7 |

Table 23.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\frac{200600}{}$.
[Derivation of table explained on pp. liii-]vi.]

|  |  | co-ordinates of developed parallel for - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ${ }^{10}$ longitude. |  | 20' longitude. |  | 30 longitude. |  | 40 ${ }^{\prime}$ longitude. |  | 50 ${ }^{\prime}$ longitude. |  | ${ }_{1}{ }^{\circ}$ longitude. |  |
|  |  | x | y | x | y | $\times$ | y | $x$ | y | x | y | $\star$ | y |
|  | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | m. | mm. | mm |
| $8^{\circ} 00^{\prime}$ |  | 91.9 | . 0 | 183.7 | . 1 | 275.6 | . 2 | 367.5 | . 3 | 459.4 | $\cdot 5$ | 551.2 | . 7 |
| 10 | 9 | 91.8 | . 0 | 183.7 | . | 275.5 | . 2 | 367.3 | $\cdot 3$ | 459.2 | . 5 | 551.0 | . 7 |
| 20 | 184.3 | ${ }_{9}^{91.8}$ | . 0 | 183.6 | . 1 | 275.4 | . 2 | 367.2 | $\cdot 3$ | 459.0 | . 5 | 550.7 | . 7 |
| 30 | 276.5 368.6 | 91.8 | . 0 | 183.5 | . | 275.2 | .2 | 367.0 | $\cdot 3$ | 458.8 | . 5 | 550.5 | . 7 |
| 40 | 368.6 | 91.7 | . 0 | 183.4 | . | 275.1 | . 2 | 366.8 | $\cdot 3$ | 458.6 | . 5 | 550.3 | . 7 |
| 50 | 460.8 | 9 Pr 7 | . 0 | 183.3 | . 1 | 275.0 | .2 | 366.7 | $\cdot 3$ | 458.4 | $\cdot 5$ | 550.0 | . 7 |
| 900 |  | 91.6 | . 0 | 183.3 | . 1 | 274.9 | . 2 | 366.5 | -3 | 458.2 | . 5 | 549.8 | . 8 |
| 10 | 92 | 91.6 | . 0 | 183.2 | . 1 | 274.8 | . 2 | 366.4 | . 3 | 458.0 | . 5 | 549.5 | . 8 |
| 20 | 184.3 | 91.5 | . 0 | 183.1 | . 1 | 274.6 | .2 | 366.2 | $\cdot 3$ | 457.7 | . 5 | 549.2 | . 8 |
| 30 40 | 276.5 368.6 | 91.5 91.5 | . 0 | 183.0 182.9 | . 1 | 274.5 $274-4$ | . 2 | 366.0 365.8 | . 4 | 457.5 457.3 | . 5 | 549.0 | . 8 |
| 50 | 460.8 | 91.4 | . 0 | 182.8 | . 1 | 274.2 | . 2 | 365.6 | . 4 | 457.0 | . 6 | 548.5 | . 8 |
| 1000 |  | 91.4 | . 0 | 182.7 | . 1 | 274.1 | . 2 | 365.5 | . 4 | 456.8 | . 6 | 548.2 | . 8 |
| 10 | 98.2 | 91.3 | . 0 | 182.6 | . | 274.0 | . 2 | 365.3 | . 4 | 456.6 | . 6 | 547.9 | . 8 |
| 20 | 1843 | 91.3 | . 0 | 182.5 | . 1 | 273.8 | . 2 | 365.1 | -4 | 456.4 | . 6 | 547.6 | . 9 |
| 30 | 276.5 | 91.2 | . 0 | 182.4 | . 1 | 273.7 | 2 | 364.9 | 4 | 456.1 | . 6 | 547.3 | -9 |
| 50 | 460.8 | 91.1 | . 0 | 182.2 | . .1 | 273.4 | . 2 | 364.5 | . 4 | 455.6 | . 6 | 546.7 | . 9 |
| 1100 |  | 91.1 | . 0 | 182.1 | . 1 | 273.2 | . 2 | 364.3 | . 4 | 455.4 | . 6 | 546.4 | . 9 |
| 10 | 92.2 | 91.0 | . 0 | 182.0 | . | 273.1 | . 2 | 364.I | . 4 | 455.1 | . 6 | 546.1 | . 9 |
| 20 | 184.3 | 91.0 | . 0 | 181.9 | . | 272.9 | . 2 | 363.8 | - 4 | 454.8 | . 6 | 545.8 | . 9 |
| 30 | 276.5 | 90.9 | . 0 | 18 l 188 | . 1 | 272.7 | .$^{2}$ | 363.6 | . 4 | 454.6 | 7 | 545.5 | . 9 |
| 40 50 | 368.7 460.8 | 90.9 90.8 | . 0 | 181.7 <br> 181.6 | . 1 | 272.6 272.4 | . 2 | 363.4 363.2 | . 4 | $454 \cdot 3$ | . 7 | 545.2 | 1.0 |
| 1200 |  | 90.8 | . 0 | 181.5 | . 1 | 272.2 | . 2 | 363.0 | . 4 | 453.8 | . 7 | 544.5 | 1.0 |
| 10 | 92.2 | 90.7 | . 0 | 181.4 | . 1 | 272.1 | . 2 | 362.8 | . 4 | 453.4 | . 7 | 544.I | 1.0 |
| 20 | 184.4 | 90.6 | . 0 | 181.3 | . | 271.9 | . 2 | 362.5 | $\cdot 4$ | 453.2 | . 7 | 543.8 | 1.0 |
| 30 | 276.5 368.7 | 90.6 90.5 | . 0 | 181.1 | . | 271.7 | $\cdot 3$ | 362.3 | 4 | 452.8 | . 7 | 543.4 | 1.0 |
| 40 50 | 368.7 460.9 | 90.5 90.5 | . 0 | 181.0 180.9 | . 1 | 271.6 271.4 | $\stackrel{-3}{ } \cdot$ | 362.1 361.8 | . 4 | 452.6 452.3 | . 7 | 5432.8 | I. 0 I. I |
| 1300 |  | 90.4 | . 0 | 180.8 | . 1 | 271.2 | $\cdot 3$ | 361.6 | . 5 | 452.0 | . 7 | 542.4 | I.I |
| 10 | 92.2 | 90.3 | . 0 | 180.7 | . 1 | 271.0 | . 3 | 361.4 | . 5 | 451.7 |  | 542.0 | I.I |
| 20 | 184.4 | 90.3 | . 0 | 180.6 | . | 270.8 | . 3 | 361.1 | . 5 | 451.4 | . 8 | 541.7 | I.I |
| 30 | 276.6 | 90.2 | . 0 | 180.4 | . | 270.6 | $\cdot 3$ | 360.8 | . 5 | 451.0 | . 8 | 541.3 | I.I |
| 40 | 368.8 | 90.2 | . 0 | 188.3 | . 1 | 270.4 | $\cdot 3$ | 360.6 | - 5 | 450.8 | . 8 | 540.9 | I.I |
| 50 | 461.0 | 90.1 | . 0 | 180.2 | . | 270.3 | $\cdot 3$ | 360.4 | $\cdot 5$ | 450.4 | . 8 | 540.5 | I.I |
| 1400 |  | 90.0 | . 0 | 180.1 | . 1 | 270.1 | $\cdot 3$ | 360.1 | . 5 | 450.2 | . 8 | 540.2 | I.1 |
| 10 | 92.2 | 90.0 | . 0 | 179.9 | . 1 | 269.9 | $\cdot 3$ | 359.8 | . 5 | 449.8 | . 8 | 539.8 | 1.2 |
| 20 | 184.4 276.6 | 89.9 89.8 8 | . 0 | 179.8 | . 1 | 269.7 | $\cdot 3$ | 359.6 | . 5 | 449.5 44.2 |  | 5399.4 | 1.2 I. 2 12 |
| 40 | 276.6 368.8 | 889.8 | . 0 | 179.7 <br> 179.5 | . 1 | 269.5 269.3 | $\cdot \cdot 3$ | 359.3 | . 5 | 449.2 | . 8 | 539.0 | I. 2 |
| 50 | 461.0 | 89.7 | . 0 | 179.4 | . 1 | 269.1 | $\cdot 3$ | 358.8 | . 5 | 448.5 | . 8 | 538.2 | I. 2 |
| 1500 |  | 89.6 | . 0 | 179.3 | . | 268.9 | $\cdot 3$ | 358.5 | . 5 | 448.2 | . 8 | 537.8 | I. 2 |
| 10 | 92.2 | 89.6 | . 0 | 179.1 | . 1 | 268.7 | $\cdot 3$ | 358.2 | . 5 | 447.8 | . 8 | 537.4 | I. 2 |
| 20 | 184.4 | 89.5 | . 0 | 179.0 | . 1 | 268.5 | - 3 | 358.0 | . 6 | 447.4 | . 8 | 536.9 536.5 | 1.2 1.2 |
| 30 40 | 276.6 368.8 | 89.4 89.3 | . 0 | 178.8 178.7 | I | 268.3 268.0 | $\cdot \cdot 3$ | 357.7 357.4 | . 6 | 447.1 446.7 | . 9 | 536.5 | 1.2 1.3 |
| 50 | 461.0 | 89.3 | . 0 | 178.5 | . 1 | 267.8 | $\cdot 3$ | 357.1 | . 6 | 446.4 | . 9 | 535.6 | $1 \cdot 3$ |
| 1600 |  | 89.2 | . 0 | 178.4 | . 1 | 267.6 | . 3 | 356.8 | . 6 | 446.0 | . 9 | 535.2 | 1.3 |


| \%些要 |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ro' longitude. |  | 20' longitude. |  | $30^{\prime}$ longitude. |  | $40^{\prime}$ longitude. |  | 50\% longitude. |  | $5^{\circ} \mathrm{longitude}$. |  |
|  |  | $x$ | y | x | $y$ | x | y | x | y | x | $y$ | $\times$ | $y$ |
|  | mm. | mm. | nim. | mm. | m | mm | mm. | mun. | mm. | mm | min. | mm. | mm |
| $16^{\circ} 00^{\prime}$ | $\cdots$ | 89.2 | . 0 | 178.4 | . 1 | 267.6 | - 3 | 356.8 | . 6 | 446.0 | . 9 | 535.2 | 1.3 |
| 0 | 92.2 | 89. 5 | . 0 | 178.2 | . 1 | 267.4 | $\cdot 3$ | 356.5 | . 6 | 445.6 | . 9 | 534.7 | I. 3 |
| 20 | I 84.4 | 89.0 | . 0 | 178.1 | . 1 | 267.2 | - 3 | 356.2 | . 6 | 445.2 | . 9 | $534 \cdot 3$ | 1.3 |
| 30 | 276.6 | 89.0 | . 0 | 177.9 | . 1 | 266.9 | - 3 | 355.9 | . 6 | 444.8 | -9 | 533.8 | 1.3 |
| 40 | 368.8 | 88.9 | . 0 | 177.8 | . 1 | 266.7 | $\cdot 3$ | 355.6 | . 6 | 444.4 | . 9 | $533 \cdot 3$ | 1.3 |
| 50 | 461.0 | 88.8 | . 0 | 177.6 | . 1 | 266.5 | - 3 | $355 \cdot 3$ | . 6 | 444. 1 | . 9 | 532.9 | 1.4 |
| 1700 | .... | 88.7 | . 0 | 177.5 | . 2 | 266.2 | . 3 | 355.0 | . 6 | 443.7 | . 9 | 532.4 | 1.4 |
| 10 | 92.2 | 88.7 | . 0 | 177.3 | . 2 | 266.0 | . 3 | 354.6 | . 6 | 443.3 | . 9 | 532.0 | 1.4 |
| 20 | 184.4 | 88.6 | . 0 | 177.2 | . 2 | 265.7 | - 3 | 354.3 | . 6 | 442.9 | 1.0 | 531.5 | 1.4 |
| 30 | 276.7 | 88.5 | .o | 177.0 | . 2 | 265.5 | $\cdot 3$ | 354.0 | . 6 | 442.5 | 1.0 | 531.0 | 1.4 |
| 40 | 368.9 | 88.4 | . 0 | 176.8 | . 2 | 265.2 | . 4 | 353.6 | . 6 | 442.0 | 1.0 | 530.5 | 1.4 |
| 50 | 461.1 | 88.3 | . 0 | 176.7 | . 2 | 265.0 | . 4 | $353 \cdot 3$ | . 6 | 441.6 | 1.0 | 530.0 | I. 4 |
| 1800 | ..... | 88.3 | . 0 | 176.5 | .2 | 264.8 | . 4 | 353.0 | . 6 | 441.2 | 1.0 | 529.5 | 1.4 |
| 10 | 92.2 | 88.2 | . 0 | 176.3 | . 2 | 264.5 | . 4 | 352.6 | . 6 | 440.8 | 1.0 | 529.0 | 1.4 |
| 20 | 184.5 | 88.1 | . 0 | 176.2 | . 2 | 264.2 | . 4 | 352.3 | . 6 | 440.4 | 1.0 | 528.5 | 1.5 |
| 30 | 276.7 | 88.0 | . 0 | 176.0 | . 2 | 264.0 | - 4 | 352.0 | . 6 | 440.0 | I. 0 | 528.0 | 1.5 |
| 40 | 368.9 | 87.9 | . 0 | 175.8 | . 2 | 263.7 | . 4 | 351.6 | . 6 | 439.6 | 1.0 | 527.5 | 1. 5 |
| 50 | 461.2 | 87.8 | . 0 | 175.6 | . 2 | 263.5 | . 4 | 351.3 | . 7 | 439.1 | 1.0 | 526.9 | 1. 5 |
| 1900 |  | 87.7 | . 0 | 175.5 | . 2 | 263.2 | $\cdot 4$ | 351.0 | .7 | 438.7 | 1.0 | 526.4 | 1.5 |
| 10 | 92.2 | 87.6 | . 0 | 175.3 | .2 | 263.0 | . 4 | 350.6 | . 7 | 438.2 | 1.0 | 525.9 | 1.5 |
| 20 | 184.5 | 87.6 | . 0 | 175.1 | $\cdot 2$ | 262.7 | -4 | 350.2 | . 7 | 437.8 | 1.0 | 525.4 | 1.5 |
| 30 | 276.7 | 87.5 | . 0 | 174.9 | .2 | 262.4 | - 4 | $349 \cdot 9$ | . 7 | 437.4 | 1.I | 524.8 | 1.5 |
| 40 | 369.0 | 87.4 | . 0 | 174.8 | . 2 | 262.1 | . 4 | 349.5 | . 7 | 436.9 | I. 1 | 524.3 | 1. 5 |
| 50 | 461.2 | 87.3 | . 0 | 174.6 | . 2 | 261.9 | . 4 | 349.2 | $\cdot 7$ | 436.4 | I. I | 523.7 | 1.6 |
| 2000 | ....... | 87.2 | . 0 | 174.4 | . 2 | 261.6 | . 4 | 348.8 | $\cdot 7$ | 436.0 | 1.1 | 523.2 | 1.6 |
| 10 | 92.2 | 87.1 | . 0 | 174.2 | . 2 | 261.3 | . 4 | 348.4 | . 7 | 435.6 | 1.1 | 522.7 | 1.6 |
| 20 | 184.5 | 87.0 | . 0 | 174.0 | . 2 | 261.0 | . 4 | 348.0 | . 7 | 435.0 | I.I | 522.1 | 1.6 |
| 30 | 276.8 | 86.9 | . 0 | 173.8 | . 2 | 260.8 | . 4 | 347.7 | . 7 | 434.6 | I.I | 521.5 | I. 6 |
| 40 | 369.0 | 86.8 | . 0 | 173.7 | .2 | 260.5 | - 4 | $347 \cdot 3$ | . 7 | 434.2 | 1.1 | 521.0 | 1.6 |
| 50 | 461.2 | 86.7 | . 0 | 173.5 | . 2 | 260.2 | . 4 | 346.9 | . 7 | 433.6 | 1.1 | 520.4 | I. 6 |
| 2100 |  | 86.6 | . 0 | $173 \cdot 3$ | . 2 | 259.9 | . 4 | 346.6 | $\cdot 7$ | 433.2 | 1.1 | 519.8 | 1. 6 |
| 10 | 92.3 | 86.5 | . 0 | 173.1 | . 2 | 259.6 | - 4 | 346.2 | . 7 | 432.7 | 1.1 | 519.2 | 1.6 |
| 20 | 184.5 | 86.4 | . 0 | 172.9 | . 2 | 259.3 | . 4 | 345.8 | . 7 | 432.2 | I.1 | 518.6 | 1.6 |
| 30 | 276.8 | 86.3 | . 0 | 172.7 | . 2 | 259.0 | - 4 | 345.4 | . 7 | 431.7 | 1.2 | 518.0 | 1.7 |
| 40 | 369.0 | 86.2 | . 0 | 172.5 | . 2 | 258.8 | - 4 | 345.0 | . 7 | 43 F .2 | 1.2 | 517.5 | 1.7 |
| 50 | 461.3 | 86.1 | . 0 | 172.3 | . 2 | 258.4 | . 4 | 344.6 | . 7 | 430.8 | 1.2 | 516.9 | 1.7 |
| 2200 | ......... | 86.0 | . 0 | 172.1 | . 2 | 258.2 | - 4 | 344.2 | . 7 | 430.2 | I. 2 | 516.3 | 1.7 |
| 10 | 92.3 | 85.9 | . 0 | 171.9 | .2 | 257.8 | - 4 | 343.8 | . 8 | 429.8 | 1.2 | 515.7 | 1.7 |
| 20 | 184.5 | 85.8 | . 0 | 171.7 | . 2 | 257.6 | -4 | 343.4 | . 8 | 429.2 | I. 2 | 515.1 | 1.7 |
| 30 | 276.8 | 85.7 | . 0 | 171.5 | . 2 | 257.2 | - 4 | 343.0 | . 8 | 428.8 | 1.2 | 514.5 | 1.7 |
| 40 | 369.1 | 85.6 | . 0 | 171.3 | . 2 | 256.9 | - 4 | 342.6 | . 8 | 423.2 | 1.2 | 513.8 | 1.7 |
| 50 | 461.4 | 85.5 | . 0 | 171.1 | . 2 | 256.6 | . 4 | 342.2 | . 8 | 427.7 | 1.2 | 513.2 | 1.7 |
| 2300 | ........ | 85.4 | . 0 | 170.9 | . 2 | 256.3 | $\cdot 4$ | 341.8 | . 8 | 427.2 | 1.2 | 512.6 | 1.7 |
| 10 | 92.3 | 85.3 | . 0 | 170.7 | . 2 | 255.0 | - 4 | 341.3 | . 8 | 426.6 | 1.2 | 512.0 | 1.8 |
| 20 | 184.6 | 85.2 | . 0 | 170.4 | . 2 | 255.7 | - 4 | 340.9 | . 8 | 426.1 | 1.2 | 511.3 | 1.8 |
| 30 | 276.8 | 85.1 | . 0 | 170.2 | .2 | 255.3 | . 4 | 340.4 | . 8 | 425.6 | 1.2 | 510.7 | I. 8 |
| 40 | 369.1 | 85.0 | . 0 | 170.0 | . 2 | 255.0 | -4 | 340.0 | . 8 | 425.0 | 1.2 | 510 r | I. 8 |
| 50 | 461.4 | 84.9 | . 0 | 169.8 | . 2 | 254.7 | . 4 | 339.6 | . 8 | 424.5 | 1.2 | 509.4 | 1.8 |
| 2400 |  | 84.8 | . 0 | 169.6 | . 2 | $254 \cdot 4$ | . 4 | 339.2 | . 8 | 424.0 | 1.3 | 508.7 | Ј. 8 |

CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $20 \frac{1}{200 б 0 . ~}$
[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ro' longitude. |  | 20' longitude. |  | $30^{\prime}$ longitude. |  | 40' longitude. |  | 50' longitude. |  | $\Sigma^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y | x | y | x | y |
|  | mm. | $n$ | mm. | mm. | mon. | mm. | m | mm. | mm. | mm. | mm | mm. | mm . |
| $24^{\circ} 00^{\prime}$ |  | 84.8 | . 0 | 169.6 | . 2 | 254.4 | -4 | 339.2 | . 8 | 424.0 | 1.3 | 508.7 | 1.8 |
| 10 | 92.3 | 84.7 | . 0 | 169.4 | . 2 | 254.0 | . 5 | 338.7 | . 8 | 423.4 | I. 3 | 508.1 | 1.8 |
| 20 | 184.6 | 84.6 | . 0 | 169.1 | . 2 | 253.7 | . 5 | 338.3 | . 8 | 422.8 | 1.3 | 507.4 | 1.8 |
| 30 | 276.9 | 84.5 | . 0 | 168.9 | . 2 | 253.4 | . 5 | 337.8 | . 8 | 422.3 | I. 3 | 506.8 | 1.8 |
| 40 | 369.2 | 84.4 | . 0 | 168.7 | .2 | 253.0 | - 5 | 337.4 | . 8 | 42 I .8 | I. 3 | 506.I | I. 8 |
| 50 | 461.5 | 84.2 | . 0 | 168.5 | . 2 | 252.7 | - 5 | 337.0 | . 8 | 421.2 | 1.3 | 505.4 | 1.9 |
| 2500 | . . . . | 84.1 | . 1 | 168.3 | . 2 | 252.4 | - 5 | 336.5 | . 8 | 420.6 | I. 3 | 504.8 | I. 9 |
| 10 | 92.3 | 84.0 | . 1 | 168.0 | .2 | 252.0 | - 5 | 336.0 | . 8 | 420.0 | I. 3 | 504.I | 1.9 |
| 20 | 184.6 | 83.9 | . 1 | 167.8 | . 2 | 251.7 | - 5 | 335.6 | . 8 | 419.5 | 1.3 | 503.4 | 1.9 |
| 30 | 276.9 | 83.8 | . 1 | 167.6 | . 2 | 251.3 | - 5 | 335.I | . 8 | 418.9 | 1.3 | 502.7 | 1.9 |
| 40 | 369.2 | 83.7 | . 1. | $167 \cdot 3$ | . 2 | 251.0 | . 5 | 334.6 | . 8 | 418.3 | I. 3 | 502.0 | 1.9 |
| 50 | 46 I .6 | 83.6 | . 1 | 167.1 | . 2 | 250.6 | . 5 | 334.2 | . 8 | 417.8 | 1.3 | 501.3 | 1.9 |
| 2600 |  | 83.4 | . 1 | 166.9 | . 2 | 250.3 | - 5 | 333.7 | . 9 | 417.2 | 1.3 | 500.6 | 1.9 |
| 10 | 92.3 | 83.3 | . 1 | 166.6 | . 2 | 249.9 | . 5 | 333.2 | $\cdot 9$ | 416.6 | 1.3 | 499.9 | 1.9 |
| 20 | 184.6 | 83.2 | . 1 | 166.4 | . 2 | 249.6 | . 5 | 332.8 | . 9 | 416.0 | 1.3 | 499.1 | 1.9 |
| 30 | 277.0 | 83.1 | . 1 | 166.1 | . 2 | 249.2 | . 5 | 332.3 | . 9 | 415.4 | I. 3 | 498.4 | 1.9 |
| 40 | 369.3 | 82.9 | . 1 | 165.9 | . 2 | 248.8 | - 5 | 331.8 | . 9 | 414.8 | I. 4 | 497.7 | 2.0 |
| 50 | 46 I .6 | 82.8 | . 1 | 165.7 | $\cdot 2$ | 248.5 | . 5 | 331.3 | . 9 | 414.2 | 1.4 | 497.0 | 2.0 |
| 2700 | ........ | 82.7 | . 1 | 165.4 | . 2 | 248.1 | - 5 | 330.8 | -9 | 413.6 | 1.4 | 496.3 | 2.0 |
| 10 | 92.3 | 82.6 | . 1 | 165.2 | . 2 | 247.8 | . 5 | 330.4 | . 9 | 413.0 | I. 4 | 495.5 | 2.0 |
| 20 | 184.7 | 82.5 | .1 | 164.9 | . 2 | $247 \cdot 4$ | . 5 | 329.8 | . 9 | 412.3 | I. 4 | $49+8$ | 2.0 |
| 30 | 277.0 | 82.3 | . 1 | 164.7 | . 2 | 247.0 | - 5 | 329.4 | $\cdot 9$ | 411.7 | I. 4 | 494.0 | 2.0 |
| 40 | 369.3 | 82.2 | . 1 | 164.4 | . 2 | 246.7 | . 5 | 328.9 | . 9 | 411.1 | 1.4 | 493.3 | 2.0 |
| 50 | 461.6 | 82.1 | .1 | 164.2 | . 2 | 246.3 | . 5 | 328.4 | . 9 | 410.4 | 1.4 | 492.5 | 2.0 |
| 2800 |  | 82.0 | . 1 | 163.9 | . 2 | $245 \cdot 9$ | - 5 | 327.9 | . 9 | 409.8 | I. 4 | 49 I .8 | 2.0 |
| 10 | 92.4 | 81. 8 | . 1 | 163.7 | . 2 | 245.5 | . 5 | 327.4 | . 9 | 409.2 | 1.4 | 491.0 | 2.0 |
| 20 | 184.7 | 81.7 | $\cdot 1$ | 163.4 | . 2 | 245.1 | . 5 | 326.8 | . 9 | 408.6 | 1.4 | 490.3 | 2.0 |
| 30 | 277.0 | 81.6 | . 1 | 163.2 | $\stackrel{2}{2}$ | 244.7 | . 5 | 326.3 | . 9 | 407.9 | I. 4 | 489.5 | 2.0 |
| 40 | 369.4 | 81.5 | . 1 | 162.9 | . 2 | 244.4 | . 5 | 325.8 | . 9 | 407.3 | I. 4 | 488.8 | 2.0 |
| 50 | 461.8 | 81. 3 | $\cdot \mathrm{I}$ | 162.7 | . 2 | 244.0 | . 5 | 325.3 | $\cdot 9$ | 406.6 | 1.4 | 488.0 | 2.1 |
| 2900 |  | 81.2 | . 1 | 162.4 | . 2 | 243.6 | - 5 | 324.8 | . 9 | 406.0 | 1.4 | 487.2 | 2.1 |
| 10 | 92.4 | 8 I .1 | . 1 | 162.1 | . 2 | 243.2 | . 5 | 324.3 | . 9 | 405.4 | I. 4 | 486.4 | 2.1 |
| 20 | 184.7 | 80.9 | . 1 | 161.9 | .2 | 242.8 | . 5 | 323.8 | . 9 | 404.7 | I. 4 | 485.6 | 2.1 |
| 30 | 277.1 | 80.8 | . 1 | 16 r .6 | .2 | 242.4 | . 5 | 323.2 | . 9 | 404.0 | I. 4 | 484.8 | 2.1 |
| 40 | 369.4 | 80.7 | . 1 | 161.3 | . 2 | 242.0 | . 5 | 322.7 | . 9 | 403.4 | I. 4 | 484.0 | 2.1 |
| 50 | 461.8 | 80.5 | . 1 | 161.t | . 2 | 241.6 | . 5 | 322.2 | . 9 | 402.7 | 1.5 | 483.2 | 2.1 |
| 3000 |  | 80.4 | . 1 | 160.8 | . 2 | 241.2 | - 5 | 321.6 | $\cdot 9$ | 402.0 | I. 5 | 482.5 | 2.1 |
| 10 |  | 80.3 | . 1 | 160.5 | . 2 | 240.8 | . 5 | 32 I .1 | . 9 | 40 I .4 | 1.5 | 481.6 | 2.1 |
| 20 | 184.8 | $8 \mathrm{8o.I}$ | . 1 | 160.3 | . 2 | 2.40 .4 | . 5 | 320.6 | . 9 | 400.7 | 1. 5 | 480.8 | 2.1 |
| 30 | 277.1 | 80.0 | . 1 | 160. | . 2 | 240.0 | . 5 | 320.0 | . 9 | 400.0 | I. 5 | 480.0 | 2.1 |
| 40 | 369.5 | 79.9 | . 1 | 159.7 | . 2 | 239.6 | . 5 | 319.4 | . 9 | $399 \cdot 3$ | 1.5 | 479.2 | 2.1 |
| 50 | 461.9 | 79.7 | . 1 | I 59.5 | . 2 | 239.2 | . 5 | 318.9 | $\cdot 9$ | 398.6 | 1.5 | 478.4 | 2.1 |
| 3 I 00 |  | 79.6 | . 1 | 159.2 | . 2 | 238.8 | - 5 | 318.4 | 1.0 | 398.0 | I. 5 | 477.5 | 2.1 |
| 10 | 92.4 | 79.4 | . 1 | 158.9 | .2 | 238.4 | . 5 | 317.8 | 1.0 | 397.2 | I. 5 | 476.7 | 2.1 |
| 20 | 184.8 | 79.3 | . 1 | 158.6 | . 2 | 237.9 | . 5 | 317.2 | 1.0 | 396.6 | 1.5 | 475.9 | 2.2 |
| 30 | 277.2 | 79.2 | . 1 | 158.3 | . 2 | 237.5 | - 5 | 316.7 | 1.0 | 395.8 | 1.5 | 475.0 | 2.2 |
| 40 | 369.6 | 79.0 | . | 158.1 | ${ }^{2}$ | 237.1 | . 5 | 316.1 | 1.0 | 395-2 | 1.5 | 474.2 | 2.2 |
| 50 | 462.0 | 78.9 | . 1 | 157.8 | . 2 | 236.7 | . 5 | 315.6 | 1.0 | 394.4 | 1.5 | $473 \cdot 3$ | 2.2 |
| 3200 |  | $7^{8.8}$ | . 1 | 157.5 | . 2 | 236.2 | -5 | 315.0 | 1.0 | 393.8 | I. 5 | 472.5 | 2.2 |

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED Parallel for - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ro' longitude. |  | 20' longitude. |  | 3o' longitude. |  | 40 longitude. |  | 50' longitude. |  | $\mathrm{s}^{0} \mathrm{longitude}$. |  |
|  |  | x | y | x | y | $x$ | $y$ | x | y | $x$ | y | $\times$ | y |
|  | mm. | mm. | $m$ | mom | mm . | mm. | m | mm. | mm. | mm. | mm. | m | mm. |
| $32^{\circ} 00^{\prime}$ |  | 78.8 | - 1 | 157.5 | . 2 | 236.2 | . 5 | 315.0 | 1.0 | 393.8 | 1.5 | 472.5 | 2.2 |
| 10 | 92.4 | 78.6 | . 1 | ז 57.2 | . 2 | 235.8 | . 5 | 314.4 | 1.0 | 393.0 | I. 5 | 47 r .6 | 2.2 |
| 20 | 184.8 | 78.5 | . 1 | 156.9 | .2 | 235.4 | . 5 | 313.8 | 1.0 | 392.3 | I. 5 | 470.8 | 2.2 |
| 30 | 277.2 | 78.3 | . 1 | 156.6 | . 2 | 235.0 | . 5 | 313.3 | 1.0 | 391.6 | 1. 5 | 469.9 | 2.2 |
| 40 | 369.6 | 78.2 | . 1 | I 56.3 | ${ }^{2}$ | 234.5 | . 5 | 312.7 | 1.0 | 390.8 | I. 5 | 469.0 | 2.2 |
| 50 | 462.0 | 78.0 | . 1 | 156.0 | . 2 | 234.1 | . 5 | 312.5 | 1.0 | 390. 1 | 1.5 | 468.1 | 2.2 |
| 3300 |  | 77.9 | . 1 | 155.8 | . 2 | 233.6 | . 6 | 311.5 | 1.0 | 389.4 | I. 5 | 467.3 | 2.2 |
| 10 | 92.4 | 77.7 | . 1 | I 55.5 | . 2 | 233.2 | . 6 | 310.9 | 1.0 | 388.6 | 1.5 | 466.4 | 2.2 |
| 20 | 184.8 | 77.6 | .I | 1 55.2 | . 2 | 232.7 | . 6 | 310.3 | 1.0 | 387.9 | 1.5 | 465.5 | 2.2 |
| 30 | $277 \cdot 3$ | 77.4 | . 1 | 154.9 | . 2 | 232.3 | . 6 | 309.7 | 1.0 | 387.2 | 1. 6 | 464.6 | 2.2 |
| 40 | 369.7 | 77.3 | . 1 | I 54.6 | . 2 | 231.9 | . 6 | 309.2 | 1.0 | 386.4 | I. 6 | 463.7 | 2.2 |
| 50 | 462.1 | 77.1 | . 1 | I54.3 | . 2 | 231.4 | . 6 | 308.6 | 1.0 | $3^{8} 5 \cdot 7$ | I. 6 | 462.8 | 2.2 |
| 3400 |  | 77.0 | . 1 | 154.0 | $\cdot 3$ | 231.0 | . 6 | 308.0 | 1.0 | 384.9 | I. 6 | 461.9 | 2.3 |
| 10 | 92.4 | 76.8 | . 1 | 153.7 | -3 | 230.5 | . 6 | 307.4 | 1.0 | 384.2 | 1.6 | 461.0 | 2.3 |
| 20 | 184.9 | 76.7 | . 1 | 153.4 | $\cdot 3$ | 230.0 | . 6 | 306.7 | I.O | 383.4 | I. 6 | 460.1 | 2.3 |
| 30 | $277 \cdot 3$ | 76.5 | . 1 | 153.1 | $\cdot 3$ | 229.6 | . 6 | 306.1 | I. 0 | 382.6 | 1. 6 | 459.2 | 2.3 |
| 40 | 369.7 | 76.4 | . 1 | 152.8 | - 3 | 229.1 | . 6 | 305.5 | I. 0 | 38 r .9 | 1.6 | 458.3 | 2.3 |
| 50 | $462 . \mathrm{I}$ | 76.2 | . 1 | 152.4 | $\cdot 3$ | 228.7 | . 6 | 304.9 | 1.0 | $381 . \mathrm{I}$ | I. 6 | $457 \cdot 3$ | 2.3 |
| 3500 |  | 76.1 | . 1 | 152.1 | . 3 | 228.2 | . 6 | 304.3 | 1.0 | 380.4 | 1.6 | 456.4 | 2.3 |
| 10 | 92.4 | 75.9 | . 1 | 151.8 | . 3 | 227.8 | . 6 | 303.7 | 1.0 | 379.6 | 1.6 | 455.5 | 2.3 |
| 20 | 184.9 | 75.8 | . 1 | 151.5 | $\cdot 3$ | 227.3 | . 6 | 303.0 | I. 0 | 378.8 | 1.6 | 454.6 | 2.3 |
| 30 | 277.4 | 75.6 | . 1 | 151.2 | . 3 | 226.8 | . 6 | 302.4 | 1.0 | 378.0 | 1.6 | 453.6 | 2.3 |
| 40 | 369.8 | 75.4 | . 1 | 150.9 | $\cdot 3$ | 226.4 | . 6 | 301.8 | I. 0 | 377.2 | 1.6 | 452.7 | 2.3 |
| 50 | 462.2 | 75.3 | . 1 | 150.6 | $\cdot 3$ | 225.9 | . 6 | 301.2 | 1.0 | 376.5 | I. 6 | 451.8 | 2.3 |
| 36 co |  | 75.1 | . 1 | 150.3 | -3 | 225.4 | . 6 | 300.6 | 1.0 | 375.7 | 1.6 | 450.8 | 2.3 |
| 10 | 92.5 | 75.0 | . 1 | 150.0 | - 3 | 224.9 | . 6 | 299.9 | 1.0 | 374.9 | 1.6 | 449.9 | 2.3 |
| 20 | 184.9 | 74.8 | . | 149.6 | $\cdot 3$ | 224.5 | . 6 | 299.3 | 1.0 | 374.1 | r. 6 | 448.9 | 2.3 |
| 30 | 277.4 | 74.7 | . 1 | I 49.3 | - 3 | 224.0 | . 6 | 298.6 | 1.0 | 373.3 | 1.6 | 448.0 | 2.3 |
| 40 | 369.8 | 74.5 | . 1 | I 49.0 | $\cdot 3$ | 223.5 | . 6 | 298.0 | 1.0 | 372.5 | ェ. 6 | 447.0 | 2.3 |
| 50 | 462.3 | 74.3 | -1 | 148.7 | -3 | 223.0 | . 6 | 297.4 | 1.0 | 371.7 | I. 6 | 446.0 | 2.3 |
| 3700 | ......... | 74.2 | . 1 | 148.4 | -3 | 222.5 | . 6 | 296.7 | 1.0 | 370.9 | I. 6 | 445. 1 | 2.3 |
| 10 | 92.5 | 74.0 | . 1 | 148.0 | - 3 | 222.1 | . 6 | 296.1 | 1.0 | 370.1 | I. 6 | 444.I | 2.3 |
| 20 | 185.0 | 73.8 | . 1 | 147.7 | - 3 | 221.6 | . 6 | 295.4 | 1.0 | 369.2 | I. 6 | 443. 1 | 2.3 |
| 30 | $277 \cdot 4$ | 73.7 | . 1 | 147.4 | $\cdot 3$ | 221.1 | . 6 | 294.8 | 1.0 | 368.4 | I. 6 | 442.1 | 2.3 |
| 40 | 369.9 | 73.5 | . 1 | 147.1 | $\cdot 3$ | 220.6 | . 6 | 294.1 | 1.0 | 367.6 | I. 6 | 44 I .2 | 2.4 |
| 50 | 462.4 | 73.4 | . 1 | 146.7 | - 3 | 220.1 | . 6 | 293.4 | 1.0 | 366.8 | I. 6 | 440.2 | 2.4 |
| $3^{8} 00$ |  | 73.2 | . 1 | 146.4 | -3 | 219.6 | . 6 | 292.8 | 1.0 | 366.0 | 1.6 | 439.2 | 2.4 |
| 10 | 92.5 | 73.0 | . 1 | I46.I | - 3 | 219.1 | . 6 | 292.1 | 1.0 | 365.1 | 1.6 | 438.2 | 2.4 |
| 20 | 185.0 | 72.9 | . 1 | 145.7 | - 3 | 218.6 | . 6 | 297.4 | 1.1 | 364.3 | I. 6 | 437.2 | 2.4 |
| 30 | 277.5 | 72.7 | . 1 | 145.4 | -3 | 218.1 | . 6 | 290.8 | I.I | 363.5 | I. 6 | 436.2 | 2.4 |
| $40$ | 370.0 | 72.5 | . 1 | I45.I | - 3 | 217.6 | . 6 | 290.1 | 1.1 | 362.6 | I. 6 | 435.2 | 2.4 |
| 50 | 462.5 | 72.4 | . 1 | 144.7 | -3 | 217.1 | . 6 | 289.4 | 1.1 | 361.8 | 1.6 | 434.2 | 2.4 |
| 3900 | ......... | 72.2 | . 1 | 144.4 | $\cdot 3$ | 216.6 | . 6 | 288.8 | 1.1 | 361.0 | 1.7 |  |  |
| 10 | 92.5 | 72.0 | . 1 | 144.0 | - 3 | 216.1 | . 6 | 288.1 | 1.1 | 360.1 | 1.7 | 432.1 | 2.4 |
| 20 | 185.0 | 71.8 | . 1 | 143.7 | - 3 | 215.6 | . 6 | 287.4 | 1.1 | 359.2 | 1.7 | 431.1 | 2.4 |
| 30 40 | 277.5 370.0 | 71.7 71.5 | . I | 143.4 | - 3 | 215.0 |  | 286.7 | 1.1 | 358.4 | 1.7 | 430.1 | 2.4 |
| 40 50 | 370.0 462.6 | 71.5 71.3 | . I | 143.0 | - 3 | 214.5 | . 6 | 286.0 | I.I | 357.5 | 1.7 | 429.0 | 2.4 |
| 50 | 462.6 | 71.3 | . 1 | 142.7 | $\cdot 3$ | 214.0 | . 6 | 285.3 | I.I | 356.6 | 1.7 | 428.0 | 2.4 |
| 4000 |  | 71.2 | . 1 | 142.3 | $\cdot 3$ | 213.5 | . 6 | 284.6 | I.I | 355.8 | 1.7 | 427.0 | 2.4 |

[Derivation of table explained on pp. liii-lvi.]

| 茄 |  | CO-ORDINATES OF DEVELOPED Parallel for- |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ro' longitude. |  | 20' longitude. |  | 30 longitude. |  | 40' longitude. |  | 50\% longitude. |  | $1^{0}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y | x | y | x | $y$ |
|  | mm. | mm | m. | mm. | $m$. | mim. | mm. | mm. | m. | mm. | mm. | mm. | $m m$ |
| $40^{\circ} 00^{\prime}$ | - ....... | 71.2 | . 1 | 142.3 | $\cdot 3$ | 213.5 | . 6 | 284.6 | 1.1 | 355.8 | I. 7 | 427.0 | 2.4 |
| 10 | 92.5 | 71.0 | . 1 | 142.0 | - 3 | 212.9 | . 6 | 283.9 | 1.1 | 354.9 | I. 7 | 425.9 | 2.4 |
| 20 | $185 . \mathrm{I}$ | 70.8 | . 1 | 141.6 | - 3 | 212.4 | . 6 | 283.2 | 1.1 | 354.0 | 1.7 | 424.9 | 2.4 |
| 30 | 277.6 | 70.6 | . 1 | 141.3 | - 3 | 211.9 | . 6 | 282.6 | I.I | 353.2 | 1.7 | 423.8 | 2.4 |
| 40 | 370.1 | 70.5 | . 1 | 140.9 | - 3 | 211.4 | . 6 | 281.8 | I.I | 352.3 | 1. 7 | 422.8 | 2.4 |
| 50 | 462.6 | 70.3 | . 1 | 140.6 | - 3 | 210.8 | . 6 | 281.1 | I.I | 351.4 | 1.7 | 421.7 | 2.4 |
| 4100 | $\cdots$ | 70.1 | . 1 | 140.2 | -3 | 210.3 | . 6 | 280.4 | 1.1 | 350.6 | 1.7 | 420.7 | 2.4 |
| 10 | 92.5 | 69.9 | . 1 | 139.9 | - 3 | 209.8 | . 6 | 279.7 | I. 1 | 349.6 | I. 7 | 419.6 | 2.4 |
| 20 | 185.1 | 69.8 | . 1 | 139.5 | - 3 | 209.2 | . 6 | 279.0 | I.I | 348.8 | 1.7 | 418.5 | 2.4 |
| 30 | 277.6 | 69.6 | . 1 | 139.2 | - 3 | 208.7 | . 6 | 278.3 | I. 1 | 347.9 | 1.7 | 417.5 | 2.4 |
| 40 | 370.2 | 69.4 | . 1 | 138.8 | $\cdot 3$ | 208.2 | . 6 | 277.6 | I. 1 | 347.0 | I. 7 | 416.4 | 2.4 |
| 50 | 462.7 | 69.2 | . 1 | 138.4 | . 3 | 207.7 | . 6 | 276.9 | I.I | 346.1 | I. 7 | 415.3 | 2.4 |
| 4200 | - | 69.0 | . 1 | 138.1 | $\cdot 3$ | 207.1 | . 6 | 276.2 | I.I. | 345.2 | 1.7 | 414.2 | 2.4 |
| 10 | 92.6 | 68.9 | . 1 | 137.7 | $\cdot 3$ | 206.6 | . 6 | 275.4 | I.I | 344.3 | 1.7 | 413.2 | 2.4 |
| 20 | 185.1 | 68.7 | . 1 | 137.4 | $\cdot 3$ | 206.0 | . 6 | 274.7 | I. 1 | $343 \cdot 4$ | 1.7 | 412.1 | 2.4 |
| 30 | 277.7 | 68.5 | . 1 | 137.0 | - 3 | 205.5 | . 6 | 274.0 | 1.I | 342.4 | 1.7 | 410.9 | 2.4 |
| 40 | 370.2 | 68.3 | . 1 | I 36.6 | - 3 | 204.9 | . 6 | 273.2 | I.I | 341.5 | 1.7 | 409.9 | 2.4 |
| 50 | 462.8 | 68.1 | . 1 | 136.3 | $\cdot 3$ | 204.4 | . 6 | 272.5 | I.I | 340.6 | 1.7 | 408.8 | 2.4 |
| 4300 | ..... | 68.0 | . 1 | I35.9 | $\cdot 3$ | 203.8 | . 6 | 271.8 | I. 1 | 339.8 | 1.7 | 407.7 | 2.4 |
| 10 | 92.6 | 67.8 | . 1 | 135.5 | - 3 | 203.3 | . 6 | 271.0 | I.I | 338.8 | 1.7 | 406.6 | 2.4 |
| 20 | - 185.2 | 67.6 | . 1 | 135.2 | $\cdot 3$ | 202.7 | . 6 | 270.3 | I.I | 337.9 | 1.7 | 405.5 | 2.4 |
| 30 | 277.7 | 67.4 | . 1 | ${ }^{1} 34.8$ | - 3 | 202.2 | . 6 | 269.6 | I.I | 337.0 | 1.7 | 404.4 | 2.4 |
| 40 | 370.3 | 67.2 | . 1 | ${ }^{1} 34.4$ | - 3 | 201.6 | . 6 | 268.8 | I. 1 | 336.0 | 1.7 | $403 \cdot 3$ | 2.4 |
| 50 | 462.9 | 67.0 | . 1 | I 34.0 | $\cdot 3$ | 201.1 | . 6 | 268.1 | I.I | 335.I | 1.7 | 402.1 | 2.4 |
| 4400 | $\cdots$ | 66.8 | . 1 | 133.7 | $\cdot 3$ | 200.5 | . 6 | 267.4 | 1.1 | 334.2 | 1.7 | 401.0 | 2.4 |
| 10 | 92.6 | 66.6 | . 1 | 133.3 | $\cdot 3$ | 200.0 | . 6 | 266.6 | 1.1 | 33332 | 1.7 | 399.9 | 2.4 |
| 20 | 185.2 | 66.5 | I | ${ }_{1} 132.9$ | - 3 | 199.4 | . 6 | 265.8 | I.I | 332.3 | 1.7 | 398.8 | 2.4 |
| 30 | 277.8 | 66.3 | . | I 32.6 | - 3 | 198.8 | . 6 | 265.1 | I.I | 331.4 | 1.7 | 397.7 | 2.4 |
| 40 | 370.4 | 66.1 | . 1 | 132.2 | - 3 | 198.3 | . 6 | 264.4 | 1.1 | 330.4 | 1.7 | 396.5 | 2.4 |
| 50 | 463.0 | 65.9 | - | 131.8 | $\cdot 3$ | 197.7 | . 6 | 263.6 | 1.1 | 329.5 | 1.7 | $395 \cdot 4$ | 2.4 |
| 4500 |  | 65.7 | . 1 | 131.4 | . 3 | 197.1 | . 6 | 262.8 | 1.1 | 328.6 | 1.7 | 394.3 | 2.4 |
| 10 | 92.6 | 65.5 | . 1 | 135.0 | $\cdot 3$ | 196.6 | . 6 | 262.1 | 1.1 | 327.6 | 1.7 | 393.1 | 2.4 |
| 20 | 185.2 | 65.3 | . 1 | 130.6 | - 3 | 196.0 | . 6 | 261.3 | 1.1 | 326.6 | 1.7 | 391.9 | 2.4 |
| 30 | 277.8 | 65.1 | . 1 | 130.3 | - 3 | 195.4 | . 6 | 260.5 | 1.1 | 325.6 | I. 7 | 390.8 | 2.4 |
| 40 | 370.4 . | 64.9 | . 1 | 129.9 | $\cdot 3$ | 194.8 | . 6 | 259.8 | 1.1 | 324.7 | 1.7 | 388.6 | 2.4 |
| 50 | 463.0 | 64.7 | . 1 | 129.5 | $\cdot 3$ | 194.2 | . 6 | 259.0 | 1.1 | 323.7 | 1.7 | 388.4 | 2.4 |
| 4600 |  | 64.6 | . 1 | 129.1 | $\cdot 3$ | 193.6 | . 6 | 258.2 | I.I | 322.8 | 1. 7 | 387.3 | 2.4 |
| 10 | 92.6 | 64.4 | . 1 | 128.7 | . 3 | 193.1 | . 6 | 257.4 | I. 1 | 321.8 | 1.7 | 386.2 | 2.4 |
| 20 | 185.3 | 64.2 | . 1 | 128.3 | $\cdot 3$ | 192.5 | . 6 | 256.6 | 1.1 | 320.8 | 1.7 | 385.0 | 2.4 |
| 30 | 277.9 | 64.0 | $\cdot 1$ | 127.9 | $\cdot 3$ | 191.9 | . 6 | 255.9 | 1.1 | 31.8 | 1.7 | 383.8 | 2.4 |
| 40 | 370.5 | 63.8 | . 1 | 127.6 | . 3 | 191.3 | . 6 | 255.1 | I.I | 318.9 | 1.7 | 382.7 | 2.4 |
| 50 | 463.1 | 63.6 | . 1 | 127.2 | $\cdot 3$ | 190.7 | . 6 | 254.3 | I.I | 317.9 | 1.7 | 38 I .5 | 2.4 |
| 4700 |  | 63.4 | . 1 | 126.8 | $\cdot 3$ | 190.1 | . 6 | 253.5 | I.I | 316.9 | 1.7 | 380.3 | 2.4 |
| 10 | 92.6 | 63.2 | . 1 | 126.4 | . 3 | 189.5 | . 6 | 252.7 | 1.1 | 315.9 | 1.7 | 379.1 | 2.4 |
| 20 | 185.3 | 63.0 | . 1 | 126.0 | - 3 | 188.9 | . 6 | 251.9 | I.I | 314.9 | 1.7 | 377.9 | 2.4 |
| 30 | 277.9 | 62.8 | . 1 | 125.6 | - 3 | 188.3 | . 6 | 251.1 | I.I | 313.9 | 1.7 | 376.7 | 2.4 |
| 40 | 370.6 | 62.6 | . 1 | 125.2 | - 3 | 187.8 | . 6 | 250.4 | 1.1 | 313.0 | 1.7 | 375.5 | 2.4 |
| 50 | 463.2 | 62.4 | . 1 | I 24.8 | $\cdot 3$ | 187.2 | . 6 | 249.6 | 1.1 | 312.0 | 1.7 | 374.3 | 2.4 |
| 4800 |  | 62.2 | . 1 | 124.4 | $\cdot 3$ | 186.6 | . 6 | 248.8 | I.I | 311.0 | 1.7 | 373.1 | 2.4 |

Table 23.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $200{ }^{2}$
[Derivation of table explained on pp. liii-ivi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR- |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ıod longitude. |  | 20l longitude. |  | $3{ }^{0}$ longitude. |  | 40' longitude. |  | 50' longitude. |  | $x^{\circ}$ longitude. |  |
|  |  | $\mathbf{x}$ | y | $\mathbf{x}$ | y | $\mathbf{x}$ | y | x | y | x | $y$ | x | y |
|  | mm | m | mm. | m. | $m$ | mim. | mmb. | mm. | mm. | m. | mm. | mms. | ntm. |
| $4^{\circ} 00^{\prime}$ |  | 62.2 | . 1 | 124.4 | $\cdot 3$ | 186.6 | . 6 | 248.8 | 1.1 | 311.0 | 1.7 | 373.1 | 2.4 |
| 10 | 92.7 | 62.0 | . 1 | 124.0 | $\cdot 3$ | 186.0 | . 6 | 248.0 | I.I | 310.0 | 1.7 | 371.9 | 2.4 |
| 0 | 185.3 | 6 I .8 | . 1 | 123.6 | $\cdot 3$ | 185.4 | . 6 | 247.2 | I. 1 | 309.0 | 1.7 | 370.7 | 2.4 |
| 30 | 278.0 | 61.6 | . 1 | 123.2 | $\cdot 3$ | 184.7 | . 6 | 246.3 | I.I | 307.9 | 1.7 | 369.5 | 2.4 |
| 40 | 370.6 | 61.4 | . 1 | 122.8 | - 3 | 184.1 | . 6 | 245.5 | I.I | 306.9 | 1.7 | 368.3 | 2.4 |
| 50 | 463.3 | 61.2 | . 1 | 122.4 | . 3 | 183.5 | . 6 | 244.7 | 1.1 | 305.9 | 1.7 | 367.1 | 2.4 |
| 4900 | , | 61.0 | $\cdot \mathrm{I}$ | 122.0 | -3 | 182.9 | . 6 | 243.9 | I.I | 304.9 | 1.7 | 365.9 | 2.4 |
|  | 92.7 | 60.8 | . 1 | 121.6 | $\cdot 3$ | 182.3 | . 6 | 243.1 | I. 1 | 303.9 | 1.7 | 364.7 | 2.4 |
| 20 | 185.4 | 60.6 | . 1 | 12 | - 3 | 181.7 | . 6 | 242.3 | 1.1 | 302.8 | 1.7 | 363.4 | 2.4 |
| 30 | 278.0 | 60.4 | . 1 | 120.7 | . 3 | 18 I .1 | . 6 | 24 I .4 | I.I | 301.8 | 1.7 | 362.2 | 2.4 |
| 40 | 370.7 | 60.2 | . 1 | 120.3 | $\cdot 3$ | 180. 5 | . 6 | 240.6 | 1.1 | 300.8 | I. 7 | 361.0 | 2.4 |
| 50 | 463.4 | 60.0 | . 1 | 119.9 | $\cdot 3$ | 179.9 | . 6 | 239.8 | 1.1 | 299.8 | 1.7 | 359.8 | 2.4 |
| 5000 | 927 | 59.8 | . 1 | 119.5 | $\cdot 3$ | 179.2 | . 6 | 239.0 | I.I | 298.8 | 1.7 | 358.5 | 2.4 |
| 10 | 92.7 | 59.5 | 1. | 119.1 | . 3 | 178.6 | . 6 | 238.2 | I.I | 297.7 | 1.7 | 357.2 | 2.4 |
| 20 | 185.4 | 59.3 | . 1 | 118.7 | - 3 | 178.0 | . 6 | 237.3 | I. 1 | 2966 | 1.7 | 356.0 | 2.4 |
| 30 | 278.1 | 59.1 | . 1 | 118.2 | - 3 | 177.4 | . 6 | 236.5 | I. 1 | 295.6 | 1.7 | 354.7 | 2.4 |
| 40 | 370.8 463.4 | 58.9 | . 1 | 117.8 | - 3 | 176.8 | . 6 | 235.7 | I, I | 294.6 | 1.7 | 353.5 | 2.4 |
| 50 | 463.4 | 58.7 | . 1 | 117.4 | $\cdot 3$ | 176.1 | . 6 | 234.8 | I.I | 293.6 | 1.7 | 352.3 | 2.4 |
| 5100 | 92 | 58.5 | $\cdot \underline{1}$ | 117.0 | $\cdot 3$ | 175.5 | . 6 | 234.0 | I.I | 292.5 | 1.7 | 351.0 | 2.4 |
| 10 | 92.7 | 58.3 | . 1 | 116.6 | $\cdot 3$ | 174.9 | . 6 | 233.2 | I. 1 | 291.4 | 1.6 | 349.7 | 2.4 |
| 20 | 185.4 | 58.1 | . 1 | 116.2 | -3 | 174.2 | . 6 | 232.3 | 1.1 | 290.4 | 1.6 | 348.5 | 2.4 |
| 30 | 278.1 | 57.9 | . 1 | 115.7 | - 3 | 173.6 | . 6 | 231.5 | 1.1 | 289.4 | I. 6 | 347.2 | 2.4 |
| 40 | 370.8 | 57.6 | . 1 | 115 | $\cdot 3$ | 173.0 | . 6 | 230.6 | 1.1 | 288.2 | I. 6 | 345.9 | 2.4 |
| 50 | 463.6 | 57.4 | . 1 | 114.9 | $\cdot 3$ | 172.3 | . 6 | 229.8 | 1.1 | 287.2 | 1.6 | 344.6 | 2.4 |
| 5200 | 2 | 57.2 | -1 | 114.5 | $\cdot 3$ | 171.7 | . 6 | 228.9 | 1.0 | 286.2 | I. 6 | $343 \cdot 4$ | 2.4 |
| 10 | 92.7 | 57.0 | . 1 | 114.0 | - 3 | 171.1 | . 6 | 228.1 | 1.0 | 285.1 | 1.6 | 342.1 | 2.4 |
| 20 | 185.4 | 56.8 | . 1 | 113.6 | . 3 | 170.4 | . 6 | 227.2 | 1.0 | 284.0 | 1.6 | 340.8 | 2.4 |
| 30 | 278.2 | 56.6 | . 1 | 113.2 | $\cdot 3$ | 169.8 | . 6 | 226.4 | 1.0 | 283.0 | 1.6 | 339.5 | 2.4 2.3 |
| 40 | 370.9 463.6 | 56.4 56.2 | . 1 | 112.8 | $\cdot 3$ | 169.1 | . 6 | 225.5 | 1.0 | 281.9 | 1.6 | 338.3 | 2.3 |
| 50 | 463.6 | 56.2 | . 1 | 112.3 | $\cdot 3$ | 168.5 | . 6 | 224.6 | I. 0 | 280.8 | 1.6 | 337.0 | 2.3 |
| 5300 | …… | 56.0 | . 1 | 111.9 | -3 | 167.9 | . 6 | 223.8 | 1.0 | 279.8 | 1.6 |  |  |
| 10 | 92.7 | 55.7 | . 1 | 111.5 | $\cdot 3$ | 167.2 | . 6 | 222.9 | 1.0 | 278.6 | 1.6 | $335 \cdot 7$ 334.4 | 2.3 |
| 20 | 185.5 | 55.5 | I | 111.0 | $\cdot 3$ | 166.6 | . 6 | 222.1 | t. 0 | 277.6 | 1.6 | 333. 1 | 2.3 |
| 30 40 | 278.2 371.0 | 55.3 55.1 | . 1 | 110.6 | $\cdot 3$ | 165.9 | . 6 | 221.2 | 1.0 | 276.5 | 1.6 | 331. 8 | 2.3 |
| 40 50 | 371.0 | 55.1 | . 1 | 110.2 | - 3 | 165.2 | . 6 | 220.3 | 1.0 | 275.4 | 1.6 | 330.5 | 2.3 |
| 50 | 463.7 | 54.9 | .I | 109.7 | $\cdot 3$ | 164.6 | . 6 | 219.5 | 1.0 | 274.4 | 1.6 | 329.2 | 2.3 |
| 5400 |  | 54.6 | . 1 | 109.3 | $\cdot 3$ | 164.0 | . 6 | 218.6 | 1.0 | 273.2 | 1.6 | 327.9 | 2.3 |
| 10 | 92.8 | 54.4 | I | 108.9 | - 3 | 163.3 | . 6 | 217.7 | 1.0 | 272.1 | 1.6 | 326.6 | 2.3 |
| 20 | 185.5 | 54.2 | . 1 | 108.4 | $\cdot 3$ | 162.6 | . 6 | 216.8 | 1.0 | 271.0 | 1.6 | 325.3 | 2.3 |
| 30 | 278.3 | 54.0 | .I | 108.0 | $\cdot 3$ | 162.0 | . 6 | 216.0 | 1.0 | 269.9 | 1.6 | 323.9 | 2.3 |
| 40 | 371.0 463.8 | 53.8 | . 1 | 107.5 | $\cdot 3$ | 161.3 | . 6 | 215.1 | 1.0 | 268.8 | 1.6 | 322.6 | 2.3 |
| 50 | 463.8 | 53.6 | . 1 | 107.1 | - 3 | 160.6 | . 6 | 214.2 | 1.0 | 267.7 | 1. 6 | 321.3 | 2.3 |
| 5500 |  | 53.3 | . 1 | 106.7 | $\cdot 3$ | 160.0 | . 6 | 213.3 | 1.0 | 266.6 | I. 6 | 320.0 |  |
| 10 20 | 92.8 185 | 53.1 | . 1 | 106.2 | $\cdot 3$ | 159.3 | . 6 | 212.4 | 1.0 | 265.6 | 1.6 | 318.7 | 2.3 2.3 |
| 20 | 185.5 | 52.9 52.7 | $\cdot \mathrm{I}$ | 105.8 | $\cdot 3$ | 158.7 | . 6 | 211.6 | 1.0 | 264.4 | ז. 6 | 317.3 | 2.3 |
| 30 40 | 278.3 371.1 | 52.7 52.4 | . 1 | 105.3 | - 3 | 158.0 157.3 | . 6 | 210.7 | 1.0 | 263.4 | 1.6 | 316.0 | 2.3 |
| 50 | 463.8 | 52.4 52.2 | . 1 | 104.9 104.4 | $\cdot \cdot 3$ | 157.3 156.7 | . 6 | 209.8 208.9 | I. 1.0 | 262.2 | 1.6 | 314.6 | 2.3 |
| 5600 |  | 52.0 | . 1 | 104.0 | . 2 | I 56.0 | . 6 | 208.0 | 1.0 | 261.1 260.0 | 1.6 | 313.3 312.0 | 2.3 2.3 |

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | to longitude. |  | $20^{\prime}$ longitude. |  | $30^{\prime}$ longitude. |  | 40 ${ }^{\prime}$ longitude. |  | 50 longitude. |  | $\mathrm{I}^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | $\mathbf{x}$ | y | x | y | x | y |
|  | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. | nn | mm. | mm. |
| $5^{6} 00{ }^{\prime}$ |  | 52.0 | . 1 | 104.0 | . 2 | 156.0 | . 6 | 208.0 | 1.0 | 260.0 | 1.6 | 312.0 | 2.3 |
| 10 | 92.8 | 51.8 | . 1 | 103.6 | . 2 | I 55.3 | . 6 | 207.1 | 1.0 | 258.9 | 1.6 | 310.7 | 2.3 |
| 20 | 185.6 | 51.6 | . 1 | 103.1 | . 2 | 154.6 | . 6 | 206.2 | 1.0 | 257.8 | I. 6 | 309.3 | 2.2 |
| 30 | 278.4 | 51.3 | . 1 | 102.6 | . 2 | 154.0 | . 6 | 205.3 | 1.0 | 256.6 | 1.6 | 307.9 | 2.2 |
| 40 | 37 I .2 | 51.1 | . 1 | 102.2 | . 2 | I 53.3 | . 6 | 204.4 | 1.0 | 255.5 | 1.5 | 306.6 | 2.2 |
| 50 | 464.0 | 50.9 | . 1 | 101.8 | . 2 | I 52.6 | . 6 | 203.5 | 1.0 | 254.4 | 1.5 | 305.3 | 2.2 |
| 5700 | $\cdots$ | 50.6 | . 1 | 101.3 | .2 | 152.0 | . 6 | 202.6 | 1.0 | 253.2 | 1.5 | 303.9 | 2.2 |
| 10 | 92.8 | 50.4 | I | 100.8 | . 2 | 151.3 | . 6 | 201.7 | 1.0 | 252.1 | 1.5 | 302.5 | 2.2 |
| 20 | 185.6 | 50.2 | . 1 | 100.4 | . 2 | I 50.6 | . 6 | 200.8 | 1.0 | 251.0 | 1.5 | 301.1 | 2.2 |
| 30 | 278.4 | 50.0 | . 1 | 99.9 | . 2 | 149.9 | . 6 | 199.8 | 1.0 | 249.8 | I. 5 | 299.8 | 2.2 |
| 40 | 371.2 | 49.7 | . 1 | 99.5 | . 2 | 149.2 | . 6 | 199.0 | 1.0 | 248.7 | 1.5 | 298.4 | 2.2 |
| 50 | 464.0 | 49.5 | . 1 | 99.0 | . 2 | 148.5 | . 5 | 198.0 | 1.0 | 247.6 | 1.5 | 297.1 | 2.2 |
| 5800 |  | $49 \cdot 3$ | . 1 | 98.6 | . 2 | 147.8 | $\cdot 5$ | 197.1 | 1.0 | 246.4 | 1.5 | 295.7 | 2.2 |
| 10 | 92.8 | 49.0 | . 1 | 98.1 | . 2 | 147.2 | - 5 | 196.2 | 1.0 | 245.2 | 1.5 | 294.3 | 2.2 |
| 20 | 185.6 | 48.8 | . 1 | 97.6 | . 2 | 146.5 | . 5 | 195.3 | 1.0 | 244.1 | 1.5 | 292.9 | 2.2 |
| 30 | 278.5 | 48.6 | . 1 | 97.2 | . 2 | 145.8 | . 5 | 194.4 | 1.0 | 243.0 | 1.5 | 291.5 | 2.2 |
| 40 | 371.3 | 48.4 | . 1 | 96.7 | . 2 | I45. 1 | - 5 | $193-4$ | 1.0 | 241.8 | 1.5 | 290.2 | 2.2 |
| 50 | 464.I | 48.1 | . 1 | 96.3 | . 2 | 144.4 | $\cdot 5$ | 192.5 | 1.0 | 240.6 | 1.5 | 288.8 | 2.1 |
| 5900 |  | 47.9 | . 1 | 95.8 | . 2 | 143.7 | - 5 | 191.6 | 1.0 | 239.5 | 1.5 | 287.4 | 2.1 |
| 10 | 92.8 | 47.7 | . 1 | $95 \cdot 3$ | . 2 | 143.0 | . 5 | 190.7 | 1.0 | 238.4 | 1.5 | 286.0 | 2.1 |
| 20 | 185.7 | 47.4 | . 1 | 94.9 | .2 | 142.3 | - 5 | 189.7 | 1.0 | 237.2 | İ. 5 | 284.6 | 2.1 |
| 30 | 278.5 | 47.2 | . 1 | 94.4 | . 2 | 141.6 | . 5 | 188.8 | 1.0 | 236.0 | 1.5 | 283.2 | 2.1 |
| 40 | 371.3 | 47.0 | . 1 | 93.9 | . 2 | I40.9 | . 5 | 187.9 | . 9 | 234.8 | 1.5 | 281.8 | 2.1 |
| 50 | 464.2 | 46.7 | . 1 | 93.5 | . 2 | 140.2 | $\cdot 5$ | 186.9 | . 9 | 233.6 | 1.5 | 280.4 | 2.1 |
| 6000 |  | 46.5 | . 1 | 93.0 | . 2 | 139.5 | $\cdot 5$ | 186.0 | . 9 | 232.5 | 1.5 | 279.0 | 2.1 |
| 10 | 92.8 | 46.3 | . 1 | 92.5 | . 2 | 138.8 | . 5 | 185.0 | . 9 | 231.3 | I. 5 | 277.6 | 2.1 |
| 20 | 185.7 | 46.0 | . 1 | 92.1 | . 2 | 138.1 | . 5 | 184.1 | 9 | 230.2 | 1.4 | 276.2 | 2.1 |
| 30 | 278.6 | 45.8 | . 1 | 91.6 | . 2 | I 37.4 | . 5 | 183.2 | . 9 | 229.0 | I. 4 | 274.8 | 2.1 |
| 40 | 371.4 | 45.6 | . 1 | 91.1 | .2 | 136.7 | . 5 | 182.2 | . 9 | 227.8 | I. 4 | 273.4 | 2.1 |
| 50 | 464.2 | 45.3 | .I | 90.6 | . 2 | I 36.0 | . 5 | 181.3 | . 9 | 226.6 | I. 4 | 271.9 | 2.1 |
| 6100 | . . . . | 45.1 | . 1 | 90.2 | . 2 | 135.3 | $\cdot 5$ | 180.4 | . 9 | 225.4 | 1.4 | 270.5 | 2.1 |
| 10 | 92.9 | 44.8 | . 1 | 89.7 | . 2 | 134.6 | . 5 | 179.4 | .9 | 224.2 | 1.4 | 269.1 | 2.1 |
| 20 | 185.7 | 44.6 | . 1 | 89.2 | .2 | 133.9 | . 5 | 178.5 | . 9 | 223.1 | 1.4 | 267.7 | 2.1 |
| 30 | 278.6 | 44.4 | . 1 | 88.8 | . 2 | 133.1 | - 5 | 177.5 | $\cdot 9$ | 221.9 | I. 4 | 266.3 | 2.0 |
| 40 | 37 I .4 | 44.1 | . 1 | 88.3 | . 2 | 132.4 | . 5 | 176.6 | . 9 | 220.7 | I. 4 | 264.8 | 2.0 |
| 50 | 464.3 | 43.9 | . 1 | 87.8 | . 2 | 131.7 | . 5 | 175.6 | . 9 | 219.6 | I. 4 | 263.5 | 2.0 |
| 6200 |  | 43.7 | $\cdot 1$ | 87.3 | . 2 | 131.0 | $\cdot 5$ | 174.7 | . 9 | 218.4 | I. 4 | 262.0 | 2.0 |
| 10 | 92.9 | 43.4 | . 1 | 86.9 | . 2 | 130.3 | . 5 | 173.7 | . 9 | 217.2 | 1.4 | 260.6 | 2.0 |
| 20 | 185.7 | 43.2 | . 1 | 86.4 | . 2 | 129.6 | . 5 | 172.8 | . 9 | 216.0 | I. 4 | 259.1 | 2.0 |
| 30 | 278.6 | 43.0 | . 1 | 85.9 | . 2 | 128.8 | . 5 | 171.8 | . 9 | 214.8 | I. 4 | 257.7 | 2.0 |
| 40 | 371.5 | 42.7 | . 1 | 85.4 | . 2 | 128.1 | . 5 | 170.8 | . 9 | 213.6 | I. 4 | 256.3 | 2.0 |
| 50 | 464.4 | 42.5 | . 1 | 84.9 | .2 | 127.4 | . 5 | 169.9 | . 9 | 212.4 | 1.4 | 254.8 | 2.0 |
| 6300 |  | 42.2 | .I |  | .2 | 126.7 | - 5 | 168.9 | . 9 | 211.2 | 1.4 | 253.4 | 2.0 |
| 10 | 92.9 | 42.0 | . 1 | 84.0 | . 2 | 126.0 | . 5 | 168.0 | . 9 | 210.0 | 1.4 | 251.9 | 2.0 |
| 20 | 185.8 | 41.7 | . 1 | 83.5 | . 2 | 125.2 | . 5 | 167.0 | .9 | 208.8 | I. 4 | 250.5 | 2.0 |
| 30 | 278.7 | 41.5 | . 1 | 83.0 | . 2 | 124.5 | . 5 | 166.0 | . 9 | 207.5 | I. 3 | 249.0 | 1.9 |
| 40 | 371.6 | 41.3 | . 1 | 82.5 | .$^{2}$ | 123.8 | . 5 | 165.0 | . 9 | 206.3 | 1.3 | 247.6 | 1.9 |
| 50 | 464.4 | 41.0 | . 1 | 82.0 | . 2 | 123.1 | . 5 | 164.1 | .9 | 205.1 | 1.3 | 246.1 | 1.9 |
| 6400 |  | 40.8 | . 1 | 81.6 | . 2 | 122.3 | . 5 | 163.1 | .9 | 203.9 | 1.3 | 244.7 | 1.9 |

Table 23.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $20 . \frac{1}{20} \cdot$
[Derivation of table explained on pp. liii.-lviii.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | not longitude. |  | 20' longitude. |  | 30' longitude. |  | 40' longitude. |  | $50^{\prime}$ longitude |  | ${ }^{\circ} \mathrm{O}$ longitude. |  |
|  |  | $x$ | y | x | y | x | y | x | y | $\star$ | y | x | y |
|  | mm. | mm. | mm. | mm. | . | mme. | mm. | mm. | mm. | mm. | mm . | m | mm. |
| $64^{\circ} 0^{\prime}$ |  | 40.8 | . 1 | 81.6 | . 2 | 122.3 | $\cdot 5$ | 163.1 | $\cdot 9$ | 203.9 | 1.3 | 244.7 | I. 9 |
| 10 | 92.9 | 40.5 | .1 | 81.1 | .2 | 121.6 | . 5 | 162.2 | . 8 | 202.7 | 1.3 | 243.2 | r. 9 |
| 20 | 18.8 | 40.3 | . 1 | 80.6 | . 2 | 120.9 | . 5 | 161.2 | . 8 | 201.4 | 1.3 | 241.7 | 1.9 |
| 30 | 278.7 | 40.0 | . 1 | 80.1 | . 2 | 120.1 | . 5 | 160.2 | . 8 | 200.2 | 1.3 | 240.2 | 1.9 |
| 40 | 371.6 | 39.8 | . 1 | 79.6 | .2 | 119.4 | - 5 | 159.2 | . 8 | 199.0 | 1.3 | 238.8 | 1.9 |
| 50 | 464.5 | 39.6 | . 1 | 79.1 | . 2 | 118.7 | $\cdot 5$ | I 58.2 | . 8 | 197.8 | 1.3 | 237.4 | 1.9 |
| 6500 | . | 39.3 | . 1 | 78.6 | . 2 | 117.9 | $\cdot 5$ | 157.2 | . 8 | 196.6 | 1.3 | 235.9 | 1.9 |
| 10 | 92.9 | 39.1 | . 1 | 78.1 | . 2 | 117.2 | . 5 | 156.2 | . 8 | $195 \cdot 3$ | 1.3 | 234.4 | 1.9 |
| 20 | 185.8 | 38.8 | . 1 | 77.6 | . 2 | 116.5 | . 5 | 155.3 | . 8 | 194.1 | 1.3 | 232.9 | 1.8 |
| 30 | 278.7 | 38.6 | . 1 | 77.2 | . 2 | 115.7 | . 5 | 154.3 | . 8 | 192.9 | 1.3 | 231.5 | 1.8 |
| 40 | 371.6 | 38.3 | . 1 | 76.7 | . 2 | 115.0 | . 5 | 153.3 | . 8 | 191.6 | 1.3 | 230.0 | 1.8 |
| 50 | 464.6 | 38.1 | . 1 | 76.2 | .2 | 114.2 | . 5 | 152.3 | . 8 | 190.4 | 1.3 | 228.5 | 1.8 |
| 6600 | ....... | 37.8 | .1 | 75.7 | .2 | 113.5 | $\cdot 5$ | 151.4 | . 8 | 189.2 | 1.3 | 227.0 | 1.8 |
| 10 | 92.9 | 37.6 | . 0 | 75.2 | . 2 | 112.8 | . 4 | 150.4 | . 8 | 188.0 | 1.3 | 225.5 | 1.8 |
| 20 | 185.9 | 37.3 | . 0 | 74.7 | .2 | 112.0 | . 4 | 149.4 | . 8 | 186.7 | 1.2 | 224.0 | 1.8 |
| 30 | 278.8 | 37.1 | . 0 | 74.2 | . 2 | 111.3 | . 4 | 148.4 | . 8 | 185.4 | 1.2 | 222.5 | 1.8 |
| 40 | 371.7 | 36.8 | . 0 | 73.7 | .2 | 110.6 | - 4 | 147.4 | . 8 | 184.2 | 1.2 | 221.1 | 1.8 |
| 50 | 464.6 | 36.6 | . 0 | 73.2 | $\cdot 2$ | 109.8 | . 4 | 146.4 | . 8 | 183.0 | 1.2 | 219.6 | 1.8 |
| 6700 | . | 36.4 | . 0 | 72.7 | . 2 | 109.0 | - 4 | 145.4 | . 8 | 181.8 | 1.2 | 218.1 | 1.8 |
| 10 | 92.9 | 36.1 | . 0 | 72.2 | . 2 | 108.3 | . 4 | 144.4 | . 8 | 180.5 | 1.2 | 276.6 | 1.7 |
| 20 | 185.9 | 35.8 | . 0 | 71.7 | . 2 | 107.6 | . 4 | I 43.4 | . 8 | 179.2 | 1.2 | 215.1 | 1.7 |
| 30 | 278.8 | 35.6 | . 0 | 71.2 | . 2 | 106.8 | . 4 | 142.4 | . 8 | 178.0 | 1.2 | 213.6 | 1.7 |
| 40 | 371.8 | 35.4 | . 0 | 70.7 | .2 | 1060 | - 4 | 141.4 | . 8 | 176.8 | 1.2 | 212.1 | 1.7 |
| 50 | 464.7 | 35.1 | . 0 | 70.2 | . 2 | 105.3 | . 4 | 140.4 | . 8 | 175.5 | 1.2 | 210.6 | 1.7 |
| 6800 |  | 34.8 | . 0 | 69.7 | . 2 | 104.6 | . 4 | 139.4 | . 8 | 174.2 | 12 | 209.1 | 1.7 |
| 10 | 93.0 | 34.6 | . 0 | 69.2 | .2 | 103.8 | . 4 | 138.4 | . 7 | 173.0 | 1.2 | 207.6 | 1.7 |
| 20 | 185.9 | 34.4 | . 0 | 68.7 | . 2 | 103.0 | . 4 | 137.4 | . 7 | 171.8 | I. 2 | 206.1 | 1.7 |
| . 30 | 278.8 | 34.1 | . 0 | 68.2 | . 2 | 102.3 | . 4 | 136.4 | . 7 | 170.4 | I.I | 204.5 | 1.7 |
| 40 | 371.8 | 33.8 | . 0 | 67.7 | .2 | 101.5 | . 4 | 135.4 | . 7 | 169.2 | 1.1 | 203.0 | 1.7 |
| 50 | 464.8 | 33.6 | . 0 | 67.2 | . 2. | 100.8 | . 4 | 134.4 | . 7 | 168.0 | 1.1 | 201.5 | 1.6 |
| 6900 |  | $33 \cdot 3$ | . 0 | 66.7 | . 2 | 100.0 | . 4 | 133.4 | $\cdot 7$ | 166.7 | 1.1 | 200.0 | 1. 6 |
| 10 | 93.0 | 33.1 | . 0 | 66.2 | $\cdot 2$ | 99.3 | . 4 | 132.4 | . 7 | 165.4 | 1.1 | 198.5 | 1. 6 |
| 20 | 185.9 | 32.8 | . 0 | 65.7 | .2 | 98.5 | . 4 | 131.3 | . 7 | 164.2 | I.I | 197.0 | 1. 6 |
| 30 40 | 278.9 371.8 | 32.6 32.3 | . 0 | 65.2 | $\cdot 2$ | 97.7 | . 4 | 130.3 | . 7 | 162.9 | I.I | 195.5 | 1.6 |
| 40 50 | 371.8 464.8 | 32.3 | . 0 | 64.7 | . 2 | 97.0 | - 4 | 129.3 | - 7 | 161.6 | 1.1 | 194.0 | 1.6 |
| 50 | 464.8 | 32.1 | . 0 | 64.1 | $\cdot 2$ | 96.2 | $\cdot 4$ | 128.3 | $\cdot 7$ | 160.4 | I. 1 | 192.4 | I. 6 |
| 7000 |  | 31.8 | . 0 | 63.6 | . 2 | 95.5 | . 4 | 127.3 | . 7 | I 59.1 | 1.1 | 190.9 | I. 6 |
| 10 | 93.0 | 3 3 .6 | . 0 | 63.1 | .2 | 94.7 | . 4 | 126.2 | $\cdot 7$ | 157.8 | 1.1 | 189.4 | 1.6 |
| 20 | 185.9 | 31.3 | . 0 | 62.6 | . 2 | 93.9 | -4 | 125.2 | . 7 | 156.6 | 1 | 187.9 | 1.6 |
| 30 40 | 278.9 371.9 | 31.1 30.8 | . 0 | 62.1 61.6 | . 2 | 93.2 | -4 | 124.2 | . 7 | 155.3 | 1.1 | 186.4 | 1.5 |
| 40 50 | 371.9 464.9 | 30.8 30.5 | . 0 | 61.6 61.1 | . 2 | 92.4 91.6 | . 4 | 123.2 | $\cdot 7$ | 154.0 | 1.1 | 184.8 | 1. 5 |
| 50 | 464.9 | 30.5 | . 0 | 61.1 | . 2 | 91.6 | . 4 | 122.2 | $\cdot 7$ | 152.7 | 1.0 | 183.2 | 1.5 |
| 7100 | 930 | 30.3 | . 0 | 60.6 | . 2 | 90.9 | . 4 | 121.2 | $\cdot 7$ | 151.4 | 1.0 | 181.7 | I. 5 |
| 10 | 93.0 | 30.0 | . 0 | 60.1 | . 2 | 90.1 | . 4 | 120.2 | . 7 | 150.2 | 1.0 | 180.2 | 1.5 |
| 20 | 186.0 | 29.8 29.5 | . 0 | 59.6 | .2 | 89.3 | -4 | 119.1 | $\cdot 7$ | 148.9 | 1.0 | 178.7 | 1. 5 |
| 30 40 | 278.9 371.9 | 29.5 29.3 | . 0 | 59.0 58.5 | . 2 | 88.6 87.8 | . 4 | 118.1 | . 7 | 147.6 | 1.0 | 177.1 | 1.5 |
| 50 | 371.9 464.9 | 29.3 29.0 | . 0 | 58.5 58.0 | . 2 | 87.8 87.1 | -4 | 117.1 116.1 | . 6 | 146.4 | 1.0 | 175.6 | 1.5 I. 4 |
| 7200 |  | 28.8 | . 0 | 57.5 | . 2 | 86.3 | . 4 | 115.0 | . 6 | 143.8 | 1.0 | 172.6 | 1.4 |

[Derivation of table explained on pp. liii-lvi.]

|  |  | CO-ORDINATES OF DEVELOPED PARALLEL FOR - |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ro' longitude. |  | $20^{\prime}$ longitude. |  | 30' longitude. |  | 40' longitude. |  | 50 ${ }^{\prime}$ longitude. |  | $\Sigma^{\circ}$ longitude. |  |
|  |  | x | y | x | y | x | y | x | y | x | y | x | $y$ |
|  | mm. | $m m$. | m. | mm. | mm . | mm. | mm. | mm. | mm. | mm. | mm. | mm. | mm. |
| $72^{\circ} 00^{\prime}$ | ....... | 28.8 | . 0 | 57.5 | . 2 | 86.3 | . 4 | 115.0 | . 6 | 143.8 | 1.0 | 172.6 | 1.4 |
| 10 | 93.0 | 28.5 | . 0 | 57.0 | . 2 | 85.5 | - 4 | II 4.0 | . 6 | 142.5 | 1.0 | 171.0 | 1.4 |
| 20 | 186.0 | 28.2 | . 0 | 56.5 | . 2 | 84.7 | . 3 | 113.0 | . 6 | 141.2 | 1.0 | 169.4 | 1.4 |
| 30 | 279.0 | 28.0 | . 0 | 56.0 | . 2 | 83.9 | $\cdot 3$ | 111.9 | . 6 | 139.9 | 1.0 | 167.9 | 1.4 |
| 40 | 372.0 | 27.7 | . 0 | 55.5 | . 2 | 83.2 | $\cdot 3$ | 110.9 | . 6 | 138.6 | 1.0 | 166.4 | 1.4 |
| 50 | 465.0 | 27.5 | . 0 | 54.9 | . 2 | 82.4 | $\cdot 3$ | 109.9 | . 6 | 137.4 | I.O | 164.8 | 1.4 |
| 7300 | ........ | 27.2 | . 0 | 54.4 | .2 | 8r. 6 | $\cdot 3$ | 108.8 | . 6 | 136.0 | . 9 | 163.3 | 1.4 |
| 10 | 93.0 | 27.0 | . 0 | 53.9 | . 1 | 80.8 | $\cdot 3$ | 107.8 | . 6 | ${ }^{1} 34.8$ | . 9 | 16 I .7 | 1.4 |
| 20 | 186.0 | 26.7 | . 0 | 53.4 | . 1 | 80.1 | - 3 | 106.8 | . 6 | 133.4 | . 9 | 160.1 | 1.3 |
| 30 | 279.0 | 26.4 | . 0 | 52.9 | . 1 | 79.3 | $\cdot 3$ | 105.7 | . 6 | 132.2 | . 9 | 158.6 | 1.3 |
| 40 | . 372.0 | 26.2 | . 0 | 52.3 | . 1 | 78.5 | $\cdot 3$ | 104.7 | . 6 | I 30.8 | . 9 | 157.0 | 1.3 |
| 50 | 465.0 | 25.9 | . 0 | 51.8 | . 1 | 77.7 | $\cdot 3$ | 103.6 | . 6 | 129.6 | . 9 | 155.5 | 1.3 |
| 7400 | ........ | 25.6 | . 0 | 51.3 | . 1 | 77.0 | $\cdot 3$ | 102.6 | . 6 | 128.2 | . 9 | 153.9 | 1.3 |
| 10 | 93.0 | 25.4 | . 0 | 50.8 | . 1 | 76.2 | $\cdot 3$ | 101.6 | . 6 | 127.0 | . 9 | 152.3 | I. 3 |
| 20 | 186.0 | 25.1 | . 0 | 50.3 | . 1 | 75.4 | - 3 | 100.5 | . 6 | 125.6 | -9 | I 50.8 | 1.3 |
| 30 | 279.0 | 24.9 | . 0 | 49.7 | . 1 | 74.6 | $\cdot 3$ | 99.5 | . 6 | 124.4 | . 9 | 149.2 | 1.3 |
| 40 | 372.0 | 24.6 | . 0 | 49.2 | . 1 | 73.8 | $\cdot 3$ | 98.4 | . 6 | 123.0 | . 9 | 147.7 | 1.2 |
| 50 | 465.0 | 24.4 | . 0 | 48.7 - | . 1 | 73.0 | $\cdot 3$ | 97.4 | . 5 | 121.8 | . 9 | 146.I | 1.2 |
| 7500 |  | 24.1 | . 0 | 48.2 | .1 | 72.3 | $\cdot 3$ | 96.4 | $\cdot 5$ | 120.4 | . 8 | 144.5 | 1.2 |
| 10 | 93.0 | 23.8 | . 0 | 47.7 | . 1 | 71.5 | $\cdot 3$ | $95 \cdot 3$ | . 5 | 119.2 | . 8 | 143.0 | 1.2 |
| 20 | 186.0 | 23.6 | . 0 | 47.1 | . 1 | 70.7 | $\cdot 3$ | 94.2 | . 5 | 117.8 | . 8 | 141.4 | 1.2 |
| 30 | 279.1 | 23.3 | . 0 | 46.6 | . 1 | 69.9 | - 3 | 93.2 | . 5 | 116.5 | . 8 | 139.8 | 1.2 |
| 40 | 372.1 | 23.0 | . 0 | 46.1 | I | 69.1 | $\cdot 3$ | 92.2 | $\cdot 5$ | 115.2 | . 8 | $1{ }^{1} 8.2$ | 1.2 |
| 50 | 465.I | 22.8 | . 0 | $45 \cdot 5$ | . 1 | 68.3 | $\cdot 3$ | 91.1 | . 5 | 113.8 | . 8 | $13^{6.6}$ | 1.1 |
| 7600 | ....... | 22.5 | . 0 | 45.0 | .I | 67.5 | $\cdot 3$ | 90.0 | $\cdot 5$ | 112.6 | . 8 | 135.1 | 1.1 |
| 10 | 93.0 | 22.2 | . 0 | 44.5 | . 1 | 66.8 | - 3 | 89.0 | . 5 | III. 2 | . 8 | 133.5 | 1.1 |
| 20 | 186.I | 22.0 | . 0 | 44.0 | . 1 | 65.9 | $\cdot 3$ | 87.9 | . 5 | 109.9 | . 8 | 131.9 | 1.1 |
| 30 | 279.1 | 21.7 | . 0 | 43.4 | . 1 | 65.2 | $\cdot 3$ | 86.9 | $\cdot 5$ | 108.6 | 8 | 130.3 | 1.1 |
| 40 | 372.1 | 21.5 | . 0 | 42.9 | . 1 | 64.4 | - 3 | 85.8 | . 5 | 107.3 | . 8 | 128.8 | 1.1 |
| 50 | 465.1 | 21.2 | . 0 | 42.4 | . 1 | 63.6 | $\cdot 3$ | 84.8 | $\cdot 5$ | 106.0 | $\cdot 7$ | 127.1 | 1.1 |
| 7700 |  | 20.9 | . 0 | 41.9 | . 1 | 62.8 | $\cdot 3$ | 83.7 | $\cdot 5$ | 104.6 | $\cdot 7$ | 125.6 | 1.1 |
| 10 | 93.0 | 20.7 | . 0 | 41.3 | . 1 | 62.0 | $\cdot 3$ | 82.7 | . 5 | 103.4 | $\cdot 7$ | 124.0 | 1.1 |
| 20 | 186.I | 20.4 | . 0 | 40.8 | . 1 | 61.2 | $\cdot 3$ | 81.6 | . 5 | 102.0 | . 7 | 122.4 | 1.0 |
| 30 | 279.1 | 20.1 | . 0 | 40.3 | . 1 | 60.4 | $\cdot 3$ | 80.6 | $\cdot 5$ | 100.7 | $\cdot 7$ | 120.8 | 1.0 |
| 40 | 372.2 | 19.9 | . 0 | 39.8 | I | 59.6 | $\cdot 3$ | 79.5 | -4 | 99.4 | .7 | 119.3 | 1.0 |
| 50 | 465.2 | 19.6 | . 0 | 39.2 | . 1 | 58.8 | $\cdot 3$ | 78.4 | . 4 | 98.0 | $\cdot 7$ | 117.7 | 1.0 |
| 7800 |  | 19.4 | . 0 | 38.7 | . 1 | 58.0 | . 2 | 77.4 | -4 | 96.8 | . 7 | 116.1 | 1.0 |
| 10 | 93.0 | 19.1 | . 0 | 38.2 | . 1 | 57.2 | .2 | 76.3 | -4 | 95.4 | $\cdot 7$ | 114.5 | 1.0 |
| 20 | 186.1 | 18.8 | . 0 | 37.6 | . 1 | 56.5 | .2 | $75 \cdot 3$ | - 4 | 94.1 | .7 | 112.9 | 1.0 |
| 30 | 279.1 | 18.6 | . | 37.1 | . 1 | 55.7 | .2 | 74.2 | . 4 | 92.8 | $\cdot 7$ | 111.4 | 1.0 |
| 40 | 372.2 | 18.3 | . 0 | 36.6 | . 1 | $54: 9$ | . 2 | 73.2 | . 4 | 91.4 | . 6 | 109.7 108.1 | . 9 |
| 50 | 465.2 | 18.0 | . 0 | 36.0 | . 1 | 54.1 | . 2 | 72.1 | . 4 | 90.1 | . 6 | 108.1 | -9 |
| 7900 |  | 17.8 | . 0 | $35 \cdot 5$ | . 1 | 53.3 | . 2 | 71.0 | . 4 | 88.8 | . 6 | 106.6 | . 9 |
| 10 | 93.0 | 17.5 | . 0 | 35.0 | . 1 | 52.5 | 2 | 70.0 | . 4 | 87.4 | . 6 | 104.9 | . 9 |
| 20 | 186.1 | 17.2 | . 0 | 34.5 | 1 | 51.7 | . 2 | 68.9 | -4 | 86.2 | . 6 | 103.4 | . 9 |
| 30 | 279.2 | 17.0 | . 0 | $33 \cdot 9$ | 1 | 50.9 | . 2 | 67.8 | . 4 | 84.8 83.4 | . 6 |  | . 8 |
| 40 | 372.2 465.2 | 16.7 16.4 | . 0 | 33.4 32.9 | . 1 | 50.1 49.3 | . 2 | 66.8 65.7 | -4 | 83.4 82.2 | . 6 | 100.1 98.6 | . 8 |
| 8000 |  | 16.2 | . 0 | 32.3 | . 1 | 48.5 | . 2 | 64.6 | -4 | 80.8 | . 6 | 97.0 | . 8 |

［Derivation of table explained on pp．liii－lvi．］

|  |  | ABSCISSAS OF DEVELOPED PARALLEL． |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL． |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5＇ | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
|  | mm． | mm． | mm． | mm． | mm． | mm． | mm． | 發通 |  |  |
| $0^{\circ} 00^{\prime}$ | －．． | 116.0 | 231.9 | 347.9 | 463.8 | 579.8 | 695.8 | 号虫 | $0^{\circ}$ | $1^{\circ}$ |
| 10 | 230.4 | I 16.0 | 231.9 | 347.9 | 463.8 | 579.8 | 695.8 | 9．E．E |  |  |
| 20 | 460.7 | 116.0 | 231.9 | 347.8 | 463.8 | 579.8 | $695 \cdot 7$ |  |  |  |
| 30 | 691.0 | 116.0 | 231.9 | 347.8 | 463.8 | 579.8 | 695.7 |  |  |  |
| 40 | 921.4 | 116.0 | 231.9 | 347.8 | 463.8 | 579.8 | 695.7 | 5 | m． 0.0 | $m m$. 0.0 |
| 50 | 1151.8 | 115.9 | 231.9 | 347.8 | 463.8 | 579.7 | 695.6 | 10 | 0.0 0.0 | 0．0 |
| 100 |  | 115.9 | 231.9 | 347.8 | 463.8 | 579.7 | 695.6 | 15 | 0.0 | 0.0 |
| 10 | 230.4 | I 115.9 | 231.9 | 347.8 | 463.7 | 579.6 | 695.6 | 20 | 0.0 | 0.0 |
| 20 | 460.7 | 115.9 | 231.8 | 347.8 | 463.7 | 579.6 | 695.5 | 25 | 0.0 | 0.0 |
| 30 | 691．0 | 115.9 | 231.8 | 347.7 | 463.6 | 579.6 | 695.5 | 30 | 0.0 | 0.1 |
| 40 | 921.4 | 115.9 | 231.8 | 347.7 | 463.6 | 579.6 | 695.5 |  |  |  |
| 50 | 1151.8 | 115.9 | 231.8 | 347.7 | 463.6 | 579.5 | $695 \cdot 4$ |  |  |  |
| 200 | ．．．．． | II 5.9 | 231.8 | 347.7 | 463.6 | 579.4 | $695 \cdot 3$ |  | $2^{\circ}$ | $3^{\circ}$ |
| 10 | 230.4 | 115.9 | 231.8 | 347.6 | 463.5 | 579.4 | $695 \cdot 3$ |  |  |  |
| 20 | 460.7 | 115.9 | 231.7 | 347.6 | 463.4 | 579.3 | 695.2 | 5 | 0.0 | 0.0 |
| 30 40 | 691.0 | I 15.8 | 231.7 | $347 \cdot 5$ | 463.4 | 579.2 | 695.0 | 10 | 0.0 | 0.0 |
| 40 | 921.4 | 115.8 | 231.7 | 347.5 | $463 \cdot 3$ | 579.2 | 695.0 | 15 | 0.0 | 0.0 |
| 50 | 1151.8 | I 15.8 | 231.6 | $347 \cdot 5$ | 463.3 | 579.1 | 694.9 | 20 | 0.0 | 0.1 |
| 300 |  | 115.8 | 231.6 | 347.4 | 463.2 | 579.0 | 694.8 | 25 30 | 0.1 0.1 | 0.1 0.2 |
| 10 | 230.4 | 115.8 | 231.6 | $347 \cdot 3$ | 463.1 | 578.9 | 694.7 | 30 | 0.1 | 0.2 |
| 20 | 460.7 | 115.8 | 231.5 | 347.3 | 463.0 | 578.8 | 694.6 |  |  |  |
| 30 | 691.1 | 115.7 | 231.5 | 347.2 | 463.0 | 578.7 | 694.4 |  |  |  |
| 40 | 921.4 | II 15.7 | 231.4 | 347.2 | 462.9 | 578.6 | $694 \cdot 3$ |  | $4^{0}$ | $5^{\circ}$ |
| 50 | 1151.8 | 115.7 | 231.4 | 347．1 | 462.8 | 578.5 | 694.2 |  | 4 | 5 |
| 400 |  | II 5.7 | 231.4 | 347.0 | 462.7 | 578.4 | 694． 1 | 5 | 0.0 | 0.0 |
| 10 | 230.4 | 115.7 | 23 r .3 | 347.0 | 462.6 | 578.2 | 693.9 | 10 | 0.0 | 0.0 |
| 20 | 460.7 | 115.6 | 231.3 | 346.9 | 462.5 | 578.2 | 693.8 | 15 | 0.1 | 0.1 |
| 30 | 691.1 | 115.6 | 231.2 | 346.8 | 462.4 | 578.0 | 693.6 | 20 | 0.1 | 0.1 |
| 40 | 92 I .4 | 115.6 | 23 I .1 | 346.7 | 462.3 | 577.8 | 693.4 | 25 | o． 1 | 0.2 |
| 50 | 1151.8 | 115.6 | 231.1 | 346.6 | 462.2 | 577.8 | 693.3 | 30 | 0.2 | 0.3 |
| 500 | ．．．．．．．． | 115.5 | 231.0 | 346.6 | 462.1 | 577.6 | 693.1 |  |  |  |
| 10 20 | 230.4 460.7 | II 15.5 | 231.0 | 346.5 | 462.0 | 577.4 | 692.9 |  |  |  |
| 30 | 460.7 691.1 | 115.5 II 5.4 | 230.9 230.8 | 346.4 346.3 | 461.8 | 577.3 577.1 | 692.8 |  | $6^{\circ}$ | $7^{\circ}$ |
| 40 | 921.5 | II 5.4 | 230.8 | 346.2 | 461.6 | 577.0 | 692.3 |  |  |  |
| 50 | I 151.8 | 115.4 | 230.7 | 346．I | 461.4 | 576.8 | 692.2 | 5 10 | 0.0 0.0 | 0.0 0.0 |
| 600 |  | 115.3 | 230.7 | 346.0 | 46 r .3 | 576.6 | 692.0 | 15 | 0.1 | 0.1 |
| 10 | 230.4 | 115.3 | 230.6 | 345.9 | 461.2 | 576.4 | 691.7 | 20 | 0.1 | 0.2 |
| 20 | 460.8 691.1 | 115.2 | 230.5 | 345.8 | 461.0 | 576.2 | 691.5 | 25 | 0.2 | 0.3 |
| 30 | 691.1 | 115.2 | 230.4 | 345.7 | 460.9 | 576.1 | 691.3 | 30 | 0.3 | 0.4 |
| 40 | 921.5 | 115.2 | 230.4 | 345.5 | 460.7 | 575.9 | 691.1 |  |  |  |
| 50 | 1151．9 | 115.1 | 230.3 | 345.4 | 460.6 | 575.7 | 690.8 |  |  |  |
| 700 |  | 115.1 | 230.2 | $345 \cdot 3$ | 460.4 | 575.5 | 690.6 |  | $8^{\circ}$ |  |
| 10 | 230.4 | 115.1 | 230.1 | 345.2 | 460.2 | 575.3 | 690.4 |  |  |  |
| 20 | 460.8 | 115.0 | 23.0 | 345.0 | 460.0 | 575.0 | 690.1 | 5 | 0.0 |  |
| 30 40 40 | 691.1 921.5 | 115.0 114.9 | 229.9 | 344.9 | 459.9 | 574.8 | 689.8 | 10 | 0.0 |  |
| 40 50 | 921.5 1151.9 | 114.9 114.9 | 229.9 229.8 | 344.8 344.6 | 459.7 459.5 | 574.6 | 689.6 | I 5 | 0.1 |  |
| 50 | 1151.9 | 114.9 | 229.8 | 344.6 | 459.5 | 574.4 | 689.3 | 20 | 0.2 |  |
| 800 |  | I 14.8 | 229.7 | 344.5 | 459.4 | 574.2 | 689.0 | 25 30 | 0.3 0.4 |  |

[Derivation of table explaioed on pp. liii-lvi.]

table 24.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE Eणठण.
[Derivation of table explained on pp. liii--lvi.]

|  |  | AbSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ |  | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
|  | mm. | mm. | mm. | mm. | mm. | mm. | mm. | \% |  |  |
| $16^{\circ} 00^{\prime}$ | $\ldots$ | 111.5 | 223.0 | $334 \cdot 5$ | 446.0 | 557.4 | 668.9 | 宮品 | $16^{\circ}$ | $17^{0}$ |
| 10 | 230.5 | 111.4 | 222.8 | 334.2 | 445.6 | 557.0 | 668.3 | +, |  |  |
| 20 | 461.1 | III. 3 | 222.6 | 333.9 | 445.2 | 556.5 | 667.8 |  |  |  |
| 30 | 691.6 | 111.2 | 222.4 | 333.6 | 444.8 | 556.0 | 667.2 |  | m, |  |
| 40 | 922.1 | III.1 | 222.2 | 333.3 | 444.4 | 555.6 | 666.7 |  |  | mm. |
| 50 | 1152.6 | III.O | 222.0 | 333.1 | 444.I | 555.1 | 666.1 | 10 | 0.0 0.1 | 0.0 0.1 |
| 1700 | ...... | 110.9 | 221.8 | 332.8 | 443.7 | 554.6 | 665.5 | 15 | 0.2 | 0.2 |
| 10 | 230.6 | 110.8 | 221.6 | 332.5 | $443 \cdot 3$ | 554.1 | 664.9 | 20 | 0.4 | 0.4 |
| 20 | 461.1 | 110.7 | 221.4 | 332.2 | 442.9 | 553.6 | 664.3 | 25 30 | 0.6 0.8 | 0.6 |
| 30 | 691.6 | 110.6 | 221.2 | 331.9 | 442.5 | 553.1 | 663.7 |  | 0.8 | 0.8 |
| 40 | 922.2 | 110.5 | 221.0 | 331.6 | 442.1 | 552.6 | 663.1 |  |  |  |
| 50 | 1152.8 | 110.4 | 220.8 | 331.3 | 441.7 | 552.1 | 662.5 |  |  |  |
| 1800 |  | 110.3 | 220.6 | 331.0 | 441.3 | 551.6 | 661.9 |  | $18^{\circ}$ | $19^{\circ}$ |
| 10 | 230.6 | 110.2 | 220.4 | 330.6 | 440.8 | 551.0 | 661.3 |  |  |  |
| 20 | 46 t .1 | 110.1 | 220.2 | 330.3 | 440.4 | 550.6 | 660.7 | 5 | 0.0 | 0.0 |
| 30 | 691.7 | 110.0 | 220.0 | 330.0 | 440.0 | 550.0 | 660.0 | 10 | 0.1 | 0.1 |
| 40 | 922.3 | 109.9 | 219.8 | 329.7 | 439.6 | 549.4 | 659.3 | 15 | 0.2 | 0.2 |
| 50 | I 152.8 | 109.8 | 219.6 | 329.4 | 439.2 | 549.0 | 658.7 | 20 | 0.4 | 0.4 |
| 1900 | ...... | 109.7 | 219.4 | 329.0 | 438.7 | 548.4 | 658.1 | 25 30 | 0.6 0.9 | 0.6 0.9 |
| 10 | 230.6 | 109.6 | 219.1 | 328.7 | 438.3 | 547.8 | 657.4 | 3 |  | 0.9 |
| 20 | 461.2 | 109.5 | 218.9 | 328.4 | 437.8 | 547.3 | 656.8 |  |  |  |
| 30 | 691.8 | 109.4 | 218.7 | 328.0 | 437.4 | 546.8 | 656.1 |  |  |  |
| 40 | 922.4 | 109.2 | 218.5 | 327.7 | 436.9 | 546.1 | 655.4 |  | $20^{\circ}$ | $21^{\circ}$ |
| 50 | 1153.0 | 109. 1 | 218.2 | 327.4 | 436.5 | 545.6 | 654.7 |  |  | 21 |
| 2000 |  | 109.0 | 218.0 | 327.0 | 436.0 | 545.0 | 654.1 | 5 | 0.0 | 0.0 |
| 10 | 230.6 | 108.9 | 217.8 | 326.7 | 435.6 | 544.4 | 653.3 | 10 | 0.1 | 0.1 |
| 20 | 461.2 | 108.8 | 217.5 | 326.3 | 435.1 | 543.8 | 652.6 | 15 | 0.2 | 0.3 |
| 30 | 691.9 | 108.7 | 217.3 | 326.0 | 434.6 | 543.3 | 652.0 | 20 | 0.4 | 0.5 |
| 40 | 922.5 | 108.5 | 217.1 | 325.6 | 434.2 | 542.7 | 651.2 | 25 | 0.7 | 0.7 |
| 50 | II 53.1 | 108.4 | 216.8 | $325 \cdot 3$ | $433 \cdot 7$ | 542.1 | 650.5 | 30 | 1.0 | 1.0 |
| 2100 | .130.6 | 108.3 | 216.6 | 324.9 | 433.2 | 541.5 | 649.8 |  |  |  |
| 10 | 230.6 | 108.2 | 216.4 | 324.5 | 432.7 | 540.9 | 649.1 |  |  |  |
| 20 | $46 \mathrm{I} \cdot 3$ | 108.1 | 216.1 | 324.2 | 432.2 | 540.3 | 648.4 |  | $22^{\circ}$ | $23^{\circ}$ |
| 30 | 692.0 | 107.9 | 215.9 | 323.8 | 431.7 | 539.6 | 647.6 |  |  |  |
| 40 | 922.6 | 107.8 | 215.6 | 323.4 | 431.2 | 539.0 | 646.9 |  |  |  |
| 50 | II 53.2 | 107.7 | 215.4 | 323.1 | 430.8 | 538.4 | 646.1 | $\begin{array}{r}5 \\ 10 \\ \hline\end{array}$ | 0.0 0.1 0.3 | 0.0 0.1 |
| 2200 | ....... | 107.6 | 215.1 | 322.7 | 430.3 | 537.8 | 645.4 | 15 | 0.3 | 0.3 |
| 10 | 230.7 | 107.4 | 214.9 | 322.3 | 429.8 | 537.2 | 644.6 | 20 | 0.5 | 0.5 |
| 20 | 461.4 | 107.3 | 214.6 | 321.9 | 429.2 | 536.6 | 643.9 | 25 | 0.7 | 0.8 |
| 30 | 692.0 | 107.2 | 214.4 | 321.6 | 428.8 | 536.0 | 643.1 | 30 | I.I | I.I |
| 40 | 922.7 | 107.1 | 214.1 | 321.2 | 428.2 | 535.3 | 642.4 |  |  |  |
| 50 | 1153.4 | 106.9 | 213.9 | 320.8 | 427.7 | 534.6 | 641.6 |  |  |  |
| 2300 | ........ | 106.8 | 213.6 | 320.4 | 427.2 | 534.0 | 640.8 |  | $24^{\circ}$ |  |
| 10 | 230.7 | 106.7 | 213.3 | 320.0 | 426.6 | 533.3 | 640.0 |  |  |  |
| 20 | 461.4 | 106.5 | 213.1 | 319.6 | 426.1 | 532.6 | 639.2 | 5 | 0.0 |  |
| 30 40 | 692.1 922.8 | 106.4 | 212.8 | 319.2 318.8 | 425.6 | 532.0 | 638.4 | 10 | 0.1 |  |
| 40 50 | 922.8 I 153.6 | 106.3 | 212.5 212.3 | 318.8 318.4 | 425.0 | 531.3 | 637.6 | 15 | 0.3 |  |
|  | 1153.6 | 10.1 | 212.3 | 318.4 | 424.5 | 530.6 | 636.8 | 20 | 0.5 |  |
| 2400 |  | 106.0 | 212.0 | 318.0 | 424.0 | 530.0 | 636.0 | 25 30 | 0.8 1.1 |  |

[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED Parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 | $10^{\prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
|  | mm. | mm. | mm. | mm. ${ }^{\text {. }}$ | $m m$. | mm. | mm. | 怱家 |  |  |
| $24^{\circ} 0^{\prime}$ |  | 106.0 | 212.0 | 318.0 | 424.0 | 530.0 | 636.0 | -60 | $24^{\circ}$ | $25^{\circ}$ |
| 10 | 230.7 | 105.9 | 21.7 | 317.6 | 423.4 | 529.3 | 635.2 | , |  |  |
| 20 | 461.5 | 105.7 | 21.4 | 317.2 | 422.9 | 528.6 | 634.3 |  |  |  |
| 30 | 692.2 | 105.6 | 211.2 | 316.7 | 422.3 | 527.9 | $633 \cdot 5$ |  | mm. | mm. |
| 40 | 923.0 | 105.4 | 210.9 | 316.3 | 42 I .8 | 527.2 | 632.6 | $5^{\prime}$ | 0.0 | 0.0 |
| 50 | 1153.7 | 105.3 | 210.6 | 315.9 | 42 I .2 | 526.5 | 631.8 | 10 | 0.0 | 0.1 |
| 2500 |  | 105.2 | 210.3 | 315.5 | 420.6 | 525.8 | 631.0 | 15 | 0.3 | 0.3 |
| 10 | 230.8 | 105.0 | 210.0 | 315.0 | 420.0 | 525.0 | 630.1 | 20 | 0.5 0.8 | 0.5 0.8 |
| 20 | 461.5 | 104.9 | 209.7 | 314.6 | 419.5 | 524.4 | 629.2 | 30 | I.I | 1.2 |
| 30 | 692.3 | 104.7 | 209.4 | 314.2 | 418.9 | 523.6 | 628.3 |  |  |  |
| 40 | 923.1 | 104.6 | - 209.2 | 313.7 | 418.3 | 522.9 | 627.5 |  |  |  |
| 50 | 1153.8 | 104.4 | 208.9 | 313.3 | 417.7 | 522.2 | 626.6 |  |  |  |
| 2600 | ...... | 104.3 | 208.6 | 312.9 | 417.2 | 521.4 | 625.7 |  | $26^{\circ}$ | $27^{\circ}$ |
| 10 | 230.8 | 104.I | 208.3 | 312.4 | 416.6 | 520.7 | 624.8 |  |  |  |
| 20 | 461.6 | 104.0 | 208.0 | 312.0 | 416.0 | 520.0 | 623.9 | 5 | 0.0 | 0.0 |
| 30 | 692.4 | 103.8 | 207.7 | 311.5 | 415.4 | 519.2 | 623.0 | 10 | 0.1 | 0.1 |
| 40 | 923.2 | 103.7 | 207.4 | 31 I .15 | 414.8 | 518.4 | 622.1 | 15 | 0.3 | 0.3 |
| 50 | 1154.0 | 103.5 | 207.1 | 310.6 | 414.2 | 517.7 | 621.2 | 20 | 0.5 0.8 | 0.5 0.8 |
| 2700 |  | 103.4 | 206.8 | 310.2 | 413.6 | 517.0 | 620.3 | 30 | 1.2 | 1.2 |
| 10 | 230.8 | 103.2 | 206.5 | 309.7 | 413.0 | 516.2 | 619.4 |  |  |  |
| 20 | 461.7 | 103.I | 206.2 | 309.2 | 412.3 | 515.4 | 618.5 |  |  |  |
| 30 | 692.5 | 102.9 | 205.8 | 308.8 | 411.7 | 514.6 | 617.5 616.6 |  |  |  |
| 40 | 923.3 1154.2 | 102.8 | 205.5 205.2 | 308.3 | 411.1 | ${ }_{513.8} 5$ | 616.6 615.7 |  | $28^{\circ}$ | $29^{\circ}$ |
| 2800 | . | 102.5 | 204.9 | 307.4 | 409.8 | 512.3 | 614.8 | 5 | 0.0 | 0.0 |
| 10 | 230.9 | 102.3 | 204.6 | 306.9 | 409.2 | 511.5 | 613.8 | 10 | 0.1 | 0.1 |
| 20 | 461.7 | 102.1 | 204.3 | 306.4 | 408.6 | 510.7 | 612.8 | 15 | 0.3 | 0.3 |
| 30 | 692.6 | 102.0 | 204.0 | 305.9 | 407.9 | 509.9 | 611.9 | 20 | 0.6 | 0.6 |
| 40 | - 923.5 | 101.8 | 203.6 | 305.5 | 407.3 | 509.1 | 610.9 | 25 | 0.9 | 0.9 |
| 50 | 1154.4 | 101.7 | 203.3 | 305.0 | 406.6 | 508.3 | 610.0 | 30 | 1.3 | 1.3 |
| 2900 | ........ | 101.5 | 203.0 | 304.5 | 406.0 | 507.5 | 609.0 |  | - |  |
| 10 | 230.9 | 101. 3 | 202.7 | 304.0 | 405.4 | 506.7 | 608.0 |  | $30^{\circ}$ | $31^{\circ}$ |
| 20 | 46 I .8 | 101.2 | 202.3 | 303.5 | 404.7 | 505.8 | 607.0 |  |  |  |
| 30 | 692.7 | 101.0 | 202.0 | 303.0 | 404.0 | 505.0 | 606.0 |  |  |  |
| 40 | 923.6 1154.5 | 100.8 | 201.7 | 302.5 | 403.4 | 504.2 | 605.0 604.1 | 5 | 0.0 | 0.0 |
| 50 | 1154.5 | 100.7 | 201.4 | 302.0 | 402.7 | 503.4 | 604.1 | 10 | 0.1 | 0.1 |
| 3000 |  | 100.5 | 201.0 | 301.5 | 402.0 | 502.6 | 603.1 | 15 | 0.3 | 0.3 0.6 |
| 10 | 230.9 | 100.3 | 200.7 | 301.0 | 401.4 | 501.7 | 602.0 | 25 | 0.9 | 0.9 |
| 20 | 46 I .9 | 100.2 | 200.3 | 300.5 | 400.7 | 500.8 | 601.0 | 30 | I. 3 | 1.3 |
| 30 | 692.8 | 100.0 | 200.0 | 300.0 | 400.0 | 500.0 | 599.9 |  |  |  |
| 40 | 923.8 | 99.8 | 199.6 | 299.5 | 399.3 | 499.I | 598.9 |  |  |  |
| 50 | 1154.7 | 99.6 | 199.3 | 299.0 | 398.6 | 498.2 | 597.9 |  | $32^{\circ}$ |  |
| 3100 |  | 99.5 | 199.0 | 298.4 | 397.9 | 497.4 | 596.9 |  |  |  |
| 10 | 231.0 | 99.3 | 198.6 | 297.9 | 397.2 | 496.5 | 595.8 |  |  |  |
| 20 | 461.9 | 99.1 | 198.3 | 297.4 | 396.5 | 495.6 | 594.8 | 5 | 0.0 |  |
| 30 | 692.9 | 99.0 | 197.9 | 296.9 | 395.8 | 494.8 | 593.8 | 10 | 0.2 |  |
| 40 | 923.9 | 98.8 | 197.6 | 296.3 | 395.I | 493.9 | 592.7 | 15 | $0.3$ |  |
| 50 | 1154.8 | 98.6 | 197.2 | 295.8 | 394.4 | 493.0 | 591.6 | 20 | 0.6 0.9 |  |
| 3200 |  | 98.4 | 196.9 | 295.3 | 393.7 | 492.2 | 590.6 | 30 | 1.4 |  |

Table 24.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $80 \frac{1}{8} 00^{\circ}$
[Derivatioc of table explained on pp. liii-lvi.]

[Derivation of table explained on pp. liii-lvi.]


Table 24.
CO-ORDINATES FOR PROJECTION OF MAPS. SCALE $\overline{\sigma 0} \delta \sigma \sigma$.
[Derivation of table explained on pp. liii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ |  | $15^{\prime}$ |  | 25 | $30^{\prime}$ |  |  |  |
|  | mm. | mim. | mm. | mm. | mm. | mm. | mm. |  | $4^{\circ}$ |  |
| $48^{\circ} 00^{\prime}$ | ........ | 77.7 | 155.5 | 233.2 | 311.0 | 388.7 | 466.4 |  | $48^{\circ}$ | $49^{\circ}$ |
| 10 | 231.6 | 77.5 | $\pm 55.0$ | 232.5 | 310.0 | 387.4 | 464.9 | - |  |  |
| 20 | 463.3 | 77.2 | 154.5 | 231.7 | 308.9 | 386.2 | 463.4 |  |  |  |
| 30 | 695.0 | 77.0 | 154.0 | 230.9 | 307.9 | 384.9 | 461.9 |  | mm | mm. |
| 40 | 926.6 | 76.7 | 153.5 | 230.2 | 306.9 | 383.6 | 460.4 | 5 | 0.0 | 0.0 |
| 50 | 1158.2 | 76.5 | 152.9 | 229.4 | 305.9 | 382.4 | 458.8 | 10 | 0.2 | 0.2 |
| 4900 |  | 76.2 | 152.4 | 228.7 | 304.9 | 381.1 | 457.3 | 15 | 0.4 0.7 | 0.4 0.7 |
| 10 | 231.7 | 76.0 | 151.9 | 227.9 | 303.8 | 379.8 | 455.8 | 25 | 0.7 1.0 | 0.7 1.0 |
| 20 | 463.4 | 75.7 | 151.4 | 227.1 | 302.8 | 378.6 | $454 \cdot 3$ | 25 30 | 1.0 |  |
| 30 | 695.1 | 75-4 | 150.9 | 226.4 | 301.8 | 377.2 | 452.7 |  | 1.5 | . 5 |
| 40 | 926.8 | 75.2 | 150.4 | 225.6 | 300.8 | 376.0 | 45 I . 1 |  |  |  |
| 50 | 1158.4 | 74.9 | 149.9 | 224.8 | 299.8 | 374.7 | 449.6 |  |  |  |
| 5000 |  | 74-7 | 149.4 | 224.0 | 298.7 | 373.4 | 448.1 |  | $50^{\circ}$ | $51^{\circ}$ |
| 10 | 231.7 | 74.4 | 148.8 | 223.3 | 297.7 | 372.1 | 446.5 |  |  |  |
| 20 | 463.5 | 74.2 | 148.3 | 222.5 | 296.6 | 370.8 | 445.0 | 5 | 0.0 | 0.0 |
| 30 | 695.2 | 73.9 | 147.8 | 221.7 | 295.6 | 369.5 | $443 \cdot 4$ | 10 | 0.2 | 0.2 |
| 40 | 926.9 | 73.6 | 147.3 | 220.9 | 294.6 | 368.2 | 441.8 | 15 | 0.4 | 0.4 |
| 50 | 1158.6 | 73.4 | 146.8 | 220.1 | 293.5 | 366.9 | 440.3 | 20 | 0.7 | 0.7 |
| 5100 | ...... | 73.1 | 146.2 | 219.4 | 292.5 | 365.6 |  | 25 30 | 1.0 1.5 | 1.0 1.5 |
| 10 | 231.8 | 72.9 | 145.7 | 218.6 | 291.4 | 364.3 | 437.2 | 30 | 1.5 | 1.5 |
| 20 | 463.5 | 72.6 | 145.2 | 217.8 | 290.4 | 363.0 | $435 \cdot 5$ |  |  |  |
| 30 | 695.3 | 72.3 | 144.7 | 217.0 | 289.3 | 361.6 | $434.0^{\circ}$ |  |  |  |
| 40 | 927.1 1158.8 | 72.1 | 144.1 | 216.2 | 288.3 | 360.4 | 432.4 |  | $52^{\circ}$ | $53^{\circ}$ |
| 50 | 1158.8 | 71.8 | 143.6 | 215.4 | 287.2 | 359.0 | 430.8 |  |  |  |
| 5200 |  | 71.5 | 143.1 | 214.6 | 286.2 | 357.7 | 429.2 | 5 | 0.0 | 0.0 |
| 10 | 231.8 | 71.3 | 142.5 | 213.8 | 285.1 | 356.4 | 427.6 | 10 | 0.2 | 0.2 |
| 20 | 463.6 | 71.0 | 142.0 | 213.0 | 284.0 | 355.0 | 426.1 | 15 | 0.4 | 0.4 |
| 30 | 695.4 | 70.7 | 141.5 | 212.2 | 283.0 | 353.7 | 424.4 | 20 | 0.7 | 0.6 |
| 40 | $\begin{array}{r}927.2 \\ \hline 159.0\end{array}$ | 70.5 | 140.9 | 211.4 | 281.9 | 352.4 | 422.8 | 25 | 1.0 | 1.0 |
| 50 | 1159.0 | 70.2 | 140.4 | 210.6 | 280.8 | 351.0 | 42 I .3 | 30 | 1.5 | 1.5 |
| 5300 | $\cdots$ | 69.9 | 139.9 | 209.8 | 279.8 | 349.7 | 419.6 |  |  |  |
| 10 | 231.8 | 69.7 | 139.3 | 209.0 | 278.7 | 348.4 | 418.0 |  |  |  |
| 20 | 463.7 | 69.4 | 138.8 | 208.2 | 277.6 | 347.0 | 416.4 |  | $54^{\circ}$ | $55^{\circ}$ |
| 30 | 695.6 | 69.1 | 138.3 | 207.4 | 276.5 | 345.6 | 414.8 |  |  |  |
| 40 | 927.4 | 68.8 | 137.7 | 206.6 | $275 \cdot 4$ | 344.2 | 413.1 |  |  |  |
| 50 | 1159.2 | 68.6 | 137.2 | 205.7 | 274.3 | 342.9 | 411.5 | 5 | 0.0 0.2 | 0.0 0.2 |
| 5400 |  | 68.3 | 136.6 | 204.9 | 273.2 | 341.6 | 409.9 | 15 | 0.4 | 0.4 0.6 |
| 10 | 231.9 | 68.0 | 136.1 | 204.1 | 272.2 | 340.2 | 408.2 | 20 | 0.6 | 0.6 1.0 |
| 20 | 463.8 | 67.8 | 135.5 | 203.3 | 271.0 | 338.8 | 406.6 | 25 | 1.0 | 1.0 1.4 |
| 30 | 695.7 | 67.5 | 135.0 | 202.4 | 269.9 | 337.4 | 404.9 | 30 | 1.4 | 1,4 |
| 40 | $\begin{array}{r}927.6 \\ \hline\end{array}$ | 67.2 | 134.4 | 201.6 | 268.8 | 336.0 | $403 \cdot 3$ |  |  |  |
| 50 | 1159.4 | 66.9 | 133.9 | 200.8 | 267.8 | 334.7 | 401.6 |  |  |  |
| 5500 | 2319 | 66.7 | 133.3 | 200.0 | 266.6 | 333.3 | 400.0 |  | $56^{\circ}$ |  |
| 10 | 231.9 | 66.4 | 132.8 | 199.1 | 265.5 | 331.9 | 398.3 |  |  |  |
| 20 | 463.9 | 66.1 | 132.2 | 198.3 | 264.4 | 330.5 | 396.6 | 5 | 0.0 |  |
| 30 40 | 695.8 | 65.8 | 131.7 | 197.5 | 263.3 | 329.2 | 395.0 | 10 | 0.2 |  |
| 40 50 | 927.7 1159.6 | 65.6 65.3 | 131.1 130.5 | 196.6 | 262.2 | 327.8 | 393.3 | 15 | 0.4 |  |
| 50 | 1159.6 | 65.3 | 130.5 | 195.8 | 261.1 | 326.4 | 391.6 | 20 | 0.6 |  |
| 5600 |  | 65.0 | 130.0 | 195.0 | 260.0 | 325.0 | 389.9 | 25 30 | 1.0 1.4 |  |

［Derivation of table explained oa pp．liii－lvi．］

|  |  | ABSCISSAS OF DEVELOPED PARALLEL． |  |  |  |  |  | ORDINATES OF DEVELOPED PARALLEL． |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ |  | $15{ }^{\prime}$ |  | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $56^{\circ} 00^{\prime}$10 | $m m$ ． | mm． | $m m$ | mm． | mm． | mm． | mm． | 步芭 | $56^{\circ}$ |  |
|  | ．．．．．． | 65.0 | 130.0 | 195.0 | 260.0 | 325.0 | 389.9 | －${ }_{0}^{\text {号苞 }}$ | $56^{\circ}$ | $57^{\circ}$ |
|  | 232.0 | 64.7 | I 29.4 | 194．1 | 258.8 | 323.6 | 388.3 | 9， |  |  |
| 20 | 463.9 | 64.4 | 128.9 | 193.3 | 257.7 | 322.2 | 386.6 |  |  |  |
| 30 | $695 \cdot 9$ | 64.2 | 128.3 | 192.4 | 256.6 | 320.8 | 384.9 |  | mm． | mm． |
| 40 | 927．9 | 63.9 | 127.7 | 191.6 | 255.5 | 319.4 3180 | 383.2 | 5 | 0.0 | 0.0 |
| 50 | 1159.8 | 63.6 | 127.2 | 190.8 | 254.4 | 318.0 | 381.5 | 10 | 0.2 | 0.2 |
| 5700IO | $\cdots$ | 63.3 | 126.6 | 189.9 | 253.2 | 316.6 | 379.9 | 15 | 0.4 0.6 | 0.3 0.6 |
|  | 232.0 | 63.0 | 126.0 | 189.1 | 252.1 | 315.1 | 378.1 | 25 | I． 0 | 1.0 <br>  <br> 1.0 |
| 20 | 464.0 | 62.7 | 125.5 | 188.2 | 251.0 | 313.7 312.3 | 376.4 374.8 | 30 | 1.4 | 1.4 |
| 30 | 696.0 | 62.5 | 124.9 | 187.4 186.5 | 249.8 | 312.3 310.8 | 374.8 373.0 |  |  |  |
| 40 | 928.0 1160.0 | 62.2 | 124.3 | 186.5 | 248.7 | 310.8 309.4 | 373.0 |  |  |  |
| 5800 |  |  |  |  |  |  |  |  | $5^{\circ}$ | $59^{\circ}$ |
|  |  | $6 \mathrm{6I} .6$ | 123.2 | 184.8 | 246.4 | 308.0 | 369.6 |  | 5 |  |
| 10 | 232.0 | $6 \mathrm{~L} \cdot 3$ | 122.6 | 183.9 | 245.2 | 306.6 | 367.9 |  |  |  |
| 20 | 464.1 | 61.0 | 122.0 | 183.1 | 244.1 | 305.1 | 365.1 | 5 | 0.0 | 0.0 |
| 30 | 696.1 | 60.7 | 121.5 | 182.2 | 242.9 | 303.6 | －364．4 | 10 | 0.2 | 0.1 |
| 40 | 988.2 | 60.4 | 120.9 | 181.4 | 241.8 | 302.2 | 362.7 | 15 | 0.3 | 0.3 |
| 50 | 1160.2 | 60.2 | 120.3 | 180.5 | 240.6 | 300.8 | 361.0 | 20 | 0.6 1.0 | 0.6 0.9 |
| 59 10 10 |  | 59.9 | 119.7 | 179.6 | 239.5 | 299.4 | 359.2 | 30 | 1.4 | 1.3 |
| 10 | 232.1 | 59.6 | 119.2 | 178.7 | 238.3 | 297.9 | 357.5 |  |  |  |
| 20 | 464.2 | 59.3 | 118.6 | 177.9 | 237.2 | 296.4 | $355 \cdot 7$ |  |  |  |
| 30 | － 696.2 | 59.0 | 118.0 | 177.0 | 236.0 | 295.0 | 354.0 |  |  |  |
| 40 | 928.3 | 58.7 | 117.4 | 176.1 | 234.8 | 293.6 | 352.3 |  | $60^{\circ}$ | $61^{\circ}$ |
| 50 | 1160.4 | 58.4 | 116.8 | 175.3 | 233.7 | 292.1 | 350.5 |  |  |  |
| 6000 | …… | 58.1 | 116.3 | 174.4 | 232.5 | 20.6 | 348.8 | 5 | 0.0 | 0.0 |
| 10 | 232.1 | 57.8 | 115.7 | 173.5 | 231.4 | 289.2 | 347.0 345.2 | 10 | 0.1 0.3 | 0.1 0.3 |
| 20 | 464.2 | 57.5 | 115.1 | 172.6 | 230.2 | 287.7 286.2 | 345.2 343.4 | 15 20 | 0.3 0.6 | 0.3 0.6 |
| 30 | 696.4 | 57.2 | 114.5 | 171.7 | 229.0 | 286.2 284.8 | 343.4 341.7 | 25 | 0.9 | 0.9 |
| 40 | 928.5 1160.6 | 57.0 | 113.9 113.3 | 170.8 170.0 | 227.8 226.6 | 284.8 283.3 | 341.7 340.0 | 30 | 1．3 | 1． 3 |
| 6100 |  | 56.4 | 112.7 | 169.1 | 225.4 | 281.8 | 338.2 |  |  |  |
| 10 | 232.2 | 56.1 | 112.1 | 168.2 | 224.2 | 280.3 | 336.4 334.6 |  | $62^{\circ}$ | $63^{\circ}$ |
| 20 | 4643 | 55.8 | 111.5 | 167.3 | 223.1 | 278.8 277.4 | 334.6 <br> 332.8 |  |  |  |
| 30 40 | 696.4 928.6 | 55.5 55.2 | 110.9 | 166.4 165.5 | 221.9 | 277.4 275.8 | 332.8 331.0 |  |  |  |
| 50 50 | 928.6 1160.8 | $55 \cdot 2$ 54.9 | 110.3 109.8 | 165.5 | 220.7 219.5 | 275.8 274.4 | 331.0 329.3 | 10 | $0.0-$ | 0.0 0.1 |
| 6200 | ．．．．．． | 54.6 | 109.2 | 163.7 | 218.3 | 272.9 | 327.5 | 15 | 0.3 0.6 | 0.3 0.5 |
| 10 | 232.2 | 54.3 | 108.6 | 162.8 | 217.1 | 271.4 | 325.7 | 25 | 0.9 | 0.9 0.9 |
| 20 | 464.4 | 54.0 | 108.0 | 161.9 | 215.9 | 269.9 | 323.9 | 30 | I． 3 | 1.2 |
| 30 | 696.6 | 53.7 | 107.4 | 161.0 | 214.7 | 268.4 | 322.1 320.3 |  |  |  |
| 40 | 928.8 | 53.4 | 106.8 | 160.1 | 213.5 | 266.9 | 320.3 |  |  |  |
| 50 | 1161.0 | 53.1 | 106.2 | I 59.2 | 212.3 | 265.4 | 318.5 |  | $64^{\circ}$ |  |
| 6300 |  | 52.8 | 105.6 | 158.3 | 211.1 | 263.9 | 316.7 |  |  |  |
| 10 | 232.2 | 52.5 | 105.0 | 157.4 | 209.9 | 262.4 | 314.9 |  |  |  |
| 20 | 464.4 | 52.2 | 104.4 | 156.5 | 208.7 | 260.9 | 313.1 | 5 | 0.0 |  |
| 30 | 696.7 | 51.9 | 103.8 | 155.6 | 207.5 | 259.4 | 311.3 | 10 | 0.1 |  |
| 40 | 928.9 | 51.6 | 103.1 | 154.7 | 206.3 | 257.8 256.4 | 309.4 307.6 | 15 20 | 0.3 0.5 |  |
| 50 | 1161.1 | 51.3 | 102.5 | 153.8 | 205.1 | 256.4 | 307.6 | 20 25 | 0.5 0.8 |  |
| 6400 |  | 51.0 | 101.9 | 1 52.9 | 203.9 | 254.8 | 305.8 | 30 | 1.2 |  |

Table 24.
CO－ORDINATES FOR PROJECTION OF MAPS．SCALE
［Derivation of table explained on pp．liii－1vi．］

|  |  | ABSCISSAS OF DEVELOPED PARALLEL． |  |  |  |  |  | ORDINATES OF DEVELOPED PaRALLEL． |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5{ }^{\prime}$ | $10^{\prime \prime}$ | $15^{\prime}$ | $20^{\prime}$ | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
|  | mm． | mm ． | mm． | mm． | mm． | mm． | $m m$ ． | 云家 |  |  |
| $64^{\circ} 00^{\prime}$ |  | 51.0 | 101.9 | 152.9 | 203.9 | 254.8 | 305.8 | 最忽 |  |  |
| 10 | 232.2 | 50.7 | 101.3 | 152.0 | 202.6 | 253.3 | 304.0 | H．${ }^{\text {ch }}$ |  |  |
| 20 | 464.5 | 50.4 | 100.7 | 151.1 | 201.4 | 251.8 | 302.2 |  |  |  |
| 30 | 696.8 | 50.1 | 100.1 | 150.2 | 200.2 | 250.3 | 300.4 |  | mm． | mm． |
| 40 | 929.0 | 49.8 | 99.5 | 149.2 | 199.0 | 248.8 | 298.5 | $5^{\prime}$ | 0.0 | 0.0 |
| 50 | 116 t .2 | 49.4 | 98.9 | 148.3 | 197.8 | 247.2 | 296.6 | 10 | 0.1 | 0.0 |
| 6500 | ． | 49.1 | 98.3 | 147.4 | 196.6 | 245.7 | 294.8 | 15 | 0.3 0.5 | 0.3 0.5 |
| 10 | 232.3 | 48.8 | 97.7 | 146.5 | 195.3 | 244.2 | 293.0 | 20 25 | 0.5 | 0.5 0.8 |
| 20 | 464.6 | 48.5 | 97.1 | 145.6 | 194.1 | 242.6 | 291.2 | 30 | 1.2 |  |
| 30 | 696.9 | 48.2 | 96.4 | 144.7 | 192.9 | 241.1 | 289.3 |  |  |  |
| 40 | 929．1 | 47.9 | 95.8 | 143.7 | 191.6 | 239.6 | －287．5 |  |  |  |
| 50 | 1161.4 | 47.6 | 95.2 | 142.8 | 190.4 | 238.0 | 285.7 |  |  |  |
| 6600 |  | 47.3 | 94.6 | 141.9 | 189.2 | 236.5 | 283.8 |  | $66^{\circ}$ | $67^{\circ}$ |
| 10 | 232.3 | 47.0 | 94.0 | 141.0 | 188.0 | 235.0 | 281.9 |  |  |  |
| 20 | 464.6 | 46.7 | 93.4 | 140.0 | 186.7 | 233.4 | 280.1 | 5 | 0.0 | 0.0 |
| 30 | 697.0 | 46.4 | 92.7 | 139.1 | 185.5 | 231.8 | 278.2 | 10 | 0.1 | 0.1 |
| 40 | 929.3 | 46.1 | 92.1 | 138.2 | 184.2 | 230.3 | 276.4 | 15 | 0.3 | 0.3 |
| 50 | 1161.6 | 45.8 | 9 c .5 | 137.2 | 183.0 | 228.8 | 274.5 | 20 | 0.5 0.8 | 0.5 0.8 |
| 6700 | －．．．．${ }^{\text {a }}$ | $45 \cdot 4$ | 90.9 | 136.3 | 181.8 | 227.2 | 272.6 | 30 | I． 1 | I．I |
| 10 | 232.4 | 45． 1 | 90.3 | 135.4 | 180.5 | 225.6 | 270.8 |  |  |  |
| 20 | 464.7 | 44.8 | 89.6 | 134.4 | 179.2 | 224.0 | 268.9 |  |  |  |
| 30 | 697.0 | 44.5 | 89.0 | 133.5 | 178.0 | 222.5 | 267.0 |  |  |  |
| 40 | 929.4 | 44.2 | 88.4 | 132.6 | ${ }^{1} 76.8$ | 221.0 | 265.1 |  | $68^{\circ}$ | $69^{\circ}$ |
| 50 | 1161.8 | 43.9 | 87.7 | 131.6 | 175.5 | 219.4 | 263.2 |  |  |  |
| 6800 |  | 43.6 | 87.1 | 130.7 | 174.2 | 217.8 | 261.4 | 5 | 0.0 | 0.0 |
| 10 | 232.4 | 43.2 | 86.5 | 129.8 | 173.0 | 216.2 | 259.5 | 10 | 0.1 | 0.1 |
| 20 | 464.8 | 42.9 | 85.9 | 128.8 | 17 r .7 | 214.6 | 257.6 | 15 | 0.3 | 0.3 |
| 30 | 697.1 | 42.6 | 85.2 | 127.9 | 170.5 | 213.1 | 255.7 | 20 | 0.5 | 0.5 |
| 40 | 929.5 | 42.3 | 84.6 | 126.9 | 169.2 | 211.6 | 253.9 | 25 | 0.7 | 0.7 |
| 50 | 1161.9 | 42.0 | 84.0 | 126.0 | 168.0 | 210.0 | 251.9 | 30 | I．I | 1.0 |
| 6900 | ．．．．．．．． | 41.7 | 83.4 | 125.0 | 166.7 | 208.4 | 250.1 |  |  |  |
| 10 | 232.4 464.8 | 41.4 41.0 | 82.7 82.1 | 124.1 123.2 | 165.4 | 206.8 | 248.2 |  |  |  |
| 20 | 464.8 | 41.0 | 82.1 | 123.2 | 164.2 | 205.2 | 246.3 |  | $70^{\circ}$ | $7{ }^{0}$ |
| 30 40 | 697.2 929.6 | 40.7 40.4 | 81.5 80.8 | 122.2 I 21.2 | 162.9 161.6 | 203.6 | 244.4 |  |  |  |
| 50 | 1162.0 | 40.1 | 80.2 | 120.3 | 160.4 | 200.5 | 240.6 | 5 | 0.0 0.1 | 0.0 0.1 |
| 7000 | ． | 39.8 | 79.6 | 119.3 | 159.1 | 198.9 |  | 15 | 0.2 | 0.2 |
| 10 | 232.4 | 39.5 | 78.9 | 118.4 | 159.1 <br> 158 | 198.9 197.3 | 238.7 236.8 | 20 25 | 0.4 | 0.4 |
| 20 | 464.9 | 39.1 | 78.3 | 117.4 | 156.6 | 195.7 | 234.8 | 25 30 | 0.7 1．0 | 0.7 0.9 |
| 30 | $697 \cdot 3$ | 38.8 | 77.6 | 116.5 | 155.3 | 194.1 | 232.9 | 30 |  |  |
| 40 | 929.7 | 38.5 | 77.0 | 115.5 | 154.0 | 192.6 | 231.1 |  |  |  |
| 50 | 1162.2 | 38.2 | 76.4 | 114.6 | 152.8 | 191.0 | 229.1 |  |  |  |
| 7100 | ．．．．．． | 37.9 | 75.7 | 113.6 | 151.5 | 189.4 | 227.2 |  | $72^{0}$ |  |
|  | 232.5 | 37.6 | 75.1 | 112.6 | 150.2 | 187.8 | 225.3 |  |  |  |
| 20 | 464.9 697.4 | 37.2 | 74.5 | 111.7 | 148.9 | 186.2 | 223.4 | 5 | 0.0 |  |
| 30 40 | 697.4 929.8 | 36.9 36.6 | 73.8 | 110.7 | 147.6 | 184.5 | 22 I .4 | 10 | 0.1 |  |
| 40 50 | 929.8 1162.3 | 36.6 36.3 | 73.2 72.5 | 109.7 | 146.3 145.0 | 182.9 | 219.5 | 15 | 0.2 |  |
| 50 7200 | 1162 | 36 | 72.5 | 108.8 | 145.0 | 181.3 | 217.6 | 20 | 0.4 0.6 |  |
| 7200 |  | 35.9 | 71.9 | 107.8 | 143.8 | 179.7 | 215.6 | 30 | 0.9 |  |

[Derivation of table explained on pp. hiii-lvi.]

|  |  | ABSCISSAS OF DEVELOPED PARALLEL. |  |  |  |  |  | ORDINATES OF DEVELOPED Parallel. |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $5^{\prime}$ | $10^{\prime}$ | 15 |  | $25^{\prime}$ | $30^{\prime}$ |  |  |  |
| $72^{\circ} 00^{\prime}$ | mm. | mm. | mm. | mm. | mm. | mm. | mm. |  |  |  |
|  |  | 35.9 | 71.9 | 107.8 | 143.8 | 179.7 | 215.6 | 皆昜 | 72 | $73^{\circ}$ |
| 10 | 232.5 | 35.6 | 71.2 | 106.9 | 142.5 | 178.1 | 213.7 | . ${ }^{0}$ |  |  |
| 20 | 465.0 | $35 \cdot 3$ | 70.6 | 105.9 | 141.2 | 176.5 | 211.8 |  |  |  |
| 30 | 697.4 | 35.0 | 70.0 | 104.9 | 139.9 | 174.9 | 209.9 |  | $m$ | mm. |
| 40 | 929.9 | 34.6 | 69.3 | 104.0 | 138.6 | 173.2 | 207.9 | 5 ' | 0.0 | 0.0 |
| 50 | 1162.4 | 34.3 | 68.7 | 103.0 | 137.3 | 171.6 | 206.0 | 10 | 0.1 | 0.1 |
| 7300 | ........ | 34.0 | 68.0 | 102.0 | 136.0 | 170.0 | 204.1 | 15 20 | 0.2 0.4 | 0.2 0.4 |
| 10 | 232.5 | 33.7 | 67.4 | 101.0 | I 34.7 | 168.4 | 202.1 | 20 | 0.4 0.6 | 0.4 0.6 |
| 20 | 465.0 | 33.4 | 66.7 | 100.1 | 133.4 | 166.8 | 200.2 | 30 | 0.9 | 0.9 |
| 30 | 697.5 | 33.0 | 66.1 | 99.1 | 132.2 | 165.2 | 198.2 |  |  |  |
| 40 | 930.0 | 32.7 | 65.4 | 98.1 | 130.8 | 163.6 | 196.3 |  |  |  |
| 50 | 1162.6 | 32.4 | 64.8 | 97.1 | 129.5 | 161.9 | 194.3 |  | $74^{\circ}$ | $75^{\circ}$ |
| 7400 |  | 32.1 | 64.1 | 96.2 | 128.2 | 160.3 | 192.4 |  |  |  |
| 10 | 232.5 | 31.7 | 63.5 | 95.2 | 127.0 | 158.7 | 190.4 |  |  |  |
| 20 | 465.1 | 3 I .4 | 62.8 | 94.2 | 125.6 | 157.0 | 188.5 | 5 | 0.0 | 0.0 |
| 30 | 697.6 | 31.1 | 62.2 | 93.2 | 124.3 | 155.4 | 186.5 | 10 | 0.1 | 0.1 |
| 40 | 930.1 | 30.8 | 61.5 | 92.3 | 123.0 | 153.8 | 184.6 | 15 | 0.2 | 0.2 |
| 50 | 1162.6 | 30.4 | 60.9 | 91.3 | 121.8 | 152.2 | 182.6 | 20 | 0.4 0.6 | 0.3 0.5 |
| 750010 | ...... | 30.1 | 60.2 | 80.3 | 120.4 | 150.6 | 180.7 | 30 | 0.8 | 0.8 |
|  | 232.6 | 29.8 | 59.6 | 89.3 | 119.1 | 148.9 | 178.7 |  |  |  |
| 20 | 465.I | 29.4 | 58.9 | 88.4 | 117.8 | 147.2 | 176.7 |  |  |  |
| 30 40 | 697.6 | 29.1 28.8 | 58.3 57.6 | 87.4 86.4 | 116.5 115.2 | 145.6 | 174.8 172.8 |  | $76^{\circ}$ | $77^{\circ}$ |
| 50 | $\begin{array}{r}1162.8 \\ \hline\end{array}$ | 28.5 | 56.9 | 85.4 | 113.9 | 142.4 | 170.8 |  |  |  |
| 7600 |  | 28.1 | 56.3 | 84.4 | 112.6 | 140.7 | 168.8 | 5 10 | 0.0 0.1 | 0.0 0.1 |
| 10 | 232.6 | 27.8 | 55.6 | 83.4 | 111.2 | 139.0 | 166.9 | 15 | 0.2 | 0.2 |
| 20 | 465.1 | 27.5 | 55.0 | 82.4 | 109.9 | 137.4 | 164.9 | 20 | 0.3 | 0.3 |
| 30 40 | 697.7 930.3 | 27.2 26.8 | 54.3 | 81.4 80.5 | 108.6 | 135.8 134.2 | 162.9 | 25 | 0.5 | 0.5 |
| 40 50 | 930.3 1162.8 | 26.8 26.5 | 53.7 53.0 | 79.5 | 107.3 | 134.2 132.5 | 159.0 | 30 | 0.7 | 0.7 |
| 7700 |  | 26.2 | 52.3 | 78.5 | 104.7 | 130.8 | 157.0 |  |  |  |
| 10 | 232.6 | 25.8 | 51.7 | 77.5 | 103.4 | 129.2 | 155.0 |  | $78^{\circ}$ | $79^{\circ}$ |
| 20 | 465.2 | 25.5 | 51.0 | 76.5 | 102.0 | 127.6 | 153.1 |  |  |  |
| 30 | 697.8 | 25.2 | 50.4 | $75 \cdot 5$ | 100.7 | 125.9 | 151.1 |  |  |  |
| 40 | 930.4 | 24.8 | 49.7 | 74.6 | 99.4 | 124.2 | 149.1 | 5 | 0.0 | 0.0 |
| 50 | 1163.0 | 24.5 | 49.0 | 73.6 | 98.1 | 122.6 | 147.1 | io | 0.1 | 0.1 |
| 7800 |  | 24.2 | 48.4 | 72.6 | 96.8 | 121.0 | 145.1 | 15 20 | 0.2 0.3 | 0.1 |
| 10 | 232.6 | 23.9 | 47.7 | 71.6 | $95 \cdot 4$ | 119.3 | 143.2 | 25 | 0.4 | 0.4 |
| 20 | 465.2 | 23.5 | 47.1 | 70.6 | 94.1 | 117.6 | 141.2 | 30 | 0.6 | 0.6 |
|  | 697.8 | 23.2 | 46.4 | 69.6 | 92.8 | 116.0 | I 39.2 |  |  |  |
| 40 | 930.4 | 22.9 | $45 \cdot 7$ | 68.6 | 91.4 | 114.3 | 137.2 |  |  |  |
|  | 1163.0 | 22.5 | 45.1 | 67.6 | 90.1 | 112.6 | 135.2 |  | $80^{\circ}$ |  |
| 7900 | ....... | 22.2 | 44.4 | 66.6 | 88.8 | 111.0 | 133.2 |  |  |  |
| 10 | 232.6 | 21.9 | 43.7 | 65.6 | 87.5 | 109.4 | 131.2 |  |  |  |
| 20 | 465.2 | 21.5 | 43.1 | 64.6 | 86.1 | 107.6 | 129.2 | 5 | 0.0 |  |
| 30 | 697.9 | 21.2 | 42.4 | 63.6 | 84.8 | 106.0 | 127.2 | 10 | 0.1 |  |
| 40 | 930.5 1163.1 | 20.9 20.5 | 41.7 | 62.6 61.6 | 83.5 82.1 | 104.4 | 125.2 123.2 | 15 20 | 0.1 0.2 |  |
| 50 | 1163.1 | 20.5 | 41.1 | 61.6 | 82.1 | 102.6 | 123.2 | 20 | 0.2 0.4 |  |
| 8000 |  | 20.2 | 40.4 | 60.6 | 80.8 | 101.0 | 121.2 | 30 | 0.5 |  |

Table 25.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF $10^{\circ}$ EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle Latitude of Quadrilateral. | Area in Square Miles. |
| :---: | :---: |
| $0^{\circ}$ | 474653 |
| 5 | 472895 |
| 10 | 46763I |
| 15 | 458891 |
| 20 | 446728 |
| 25 | 431213 |
| 30 | 412442 |
| 35 | 390533 |
| 40 | 365627 |
| 45 | 337890 |
| 50 | 307514 |
| 55 | 274714 |
| 60 | 239730 |
| 65 | 202823 |
| 70 | 164279 |
| 75 | 124400 |
| 80 | 83504 |
| 85 | 41924 |

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Table 26.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF $1^{\circ}$ EXTENT LATITUDE AND LONGITUDE,
[Derivation of table explained on pp. 1-lii.]

\begin{tabular}{|c|c|c|c|c|c|}
\hline Middle latitude of quadrilateral. \& Area in square miles. \& Middle latitude of quadrilateral. \& Area in square miles. \& Middle latitude of quadrilateral. \& Area in square miles <br>
\hline $\bigcirc^{\circ} 00^{\prime}$ \& 4752.33 \& ${ }^{26}{ }^{\circ} 0^{\prime}$ \& 4282.50 \& $5^{20}{ }^{\circ} 0^{\prime}$ \& 2950.58 <br>
\hline - 30 \& 4752.16
4751.63 \& 26
27

00 \& 4264.51
4246.20 \& 5230 \& 2917.85
2884.88 <br>
\hline $\begin{array}{ll}1 & 00 \\ 1 & 30\end{array}$ \& 4751.63
4750.75 \& 27
27
27 \& 424.20
422.56 \& 53
53 \& 2851.68 <br>
\hline 200 \& 4749.52 \& 28 - \& 4208.61 \& 5400 \& 2818.27 <br>
\hline 230 \& 4747.93 \& 2830 \& 4189.33 \& 5430 \& 2784.62 <br>
\hline 300 \& 4746.00 \& 2900 \& 4169.34 \& 55 \& 2750.76 <br>
\hline 330 \& 4743.71 \& 2930 \& 4149.83 \& 5530 \& 2716.67 <br>
\hline 400 \& 4741.07 \& $30 \times$ \& 4129.60 \& 5600 \& 2682.37 <br>
\hline 430 \& 4738.08 \& 3030 \& 4109.06 \& 5630 \& 2647.85 <br>
\hline 50 \& 4734.74 \& 31.00 \& 4088.21 \& 57 \& ${ }^{26113.13}$ <br>
\hline 530 \& 4731.04 \& 3130 \& 4067.05 \& 5730 \& 2578.19 <br>
\hline 6 00 \& 4727.00 \& 3200 \& 4045.57 \& 58 00 \& 2543.05 <br>
\hline 630 \& 4722.61 \& 3230 \& 4023.79 \& 5830 \& 2507.70 <br>
\hline 7 - \& 4717.86 \& 3300 \& 4001.69 \& 59 -0 \& 2472.16 <br>
\hline 730 \& 4712.76 \& 3330 \& 3979.30 \& 5930 \& 2436.42 <br>
\hline 8 ¢ \& 4707.32 \& 34 O0 \& 3956.59 \& 6000 \& 2400.48 <br>
\hline 830 \& 4701.52 \& 3430 \& 3933.59 \& 6030 \& 2364.34 <br>
\hline $9 \bigcirc$ \& 4695.38 \& 3500 \& 3910.28
3886.67 \& 6100 \& 2338.02 <br>
\hline 930 \& 4688.89 \& 3530 \& 3886.67 \& 6130 \& 2291.51 <br>
\hline 1000 \& 4682.05 \& 36 00 \& 3862.76 \& 62 - \& 2254.82 <br>
\hline 1030 \& 4674.86 \& 3630 \& 3838.56 \& 6230 \& 2217.94 <br>
\hline 1100 \& 4667.32 \& 37 ¢0 \& 3814.06 \& 63 оo \& 2180.89 <br>
\hline 1130 \& 4659.43 \& 3730 \& 3789.26 \& 6330 \& 2143.66 <br>
\hline 1200 \& 4651.20 \& 38 - \& 3764.18 \& 64 -0 \& 2106.26 <br>
\hline 1230 \& 4642.63 \& 3830 \& 3738.80 \& 6430 \& 2068.68 <br>
\hline 13

I \& 4633.71 \& 3900 \& 3713.14
3687.18 \& 6500 \& 2030.94 <br>
\hline 1330 \& 4624.44 \& 3930 \& 3687.18 \& 6530 \& 1993.04 <br>
\hline 1400 \& 4614.82 \& $40 \sim$ \& 3660.95 \& 66 00 \& 1954.97 <br>
\hline 1430 \& 4604.87 \& 4030 \& 3634.42 \& 6630 \& 1916.75 <br>
\hline 1500 \& 4594.57 \& $41 \times$ \& 3697.62 \& 67 ¢ \& <br>
\hline 1530 \& 4583.92 \& 4130 \& 3580.54 \& 6730 \& 1839.84 <br>
\hline 1600 \& 4572.94 \& $42 \infty$ \& 3553.17 \& 68 - \& 180 r .16 <br>
\hline 1630 \& 4561.61 \& 4230 \& 3525.54 \& 6830 \& 1762.33 <br>
\hline 1700 \& 4549.94 \& 4300 \& 3497.62 \& 69 oo \& 1723.36 <br>
\hline 1730 \& 4537.93 \& 4330 \& 3469.44 \& 6930 \& 1684.24 <br>
\hline 1800 \& 4525.59 \& $440^{\circ}$ \& 3440.98 \& 70 00 \& 1645.00 <br>
\hline 1830 \& 4512.90 \& 4430 \& 3412.26 \& 7030 \& 1605.62 <br>
\hline 1900
19 \& 4499.87 \& 45 \& 3383.27 \& 7100 \& 1566.10 <br>
\hline 1930 \& 4486.51 \& 4530 \& 3354.01 \& 7130 \& 1526.46 <br>
\hline 2000 \& 4472.81 \& 46 -0 \& 3324.49 \& 7200 \& 1486.70 <br>
\hline 2030 \& 4458.78 \& 4630 \& 3294.76 \& 7230 \& 1446.81 <br>
\hline $\begin{array}{ll}21 \\ 21 & 00 \\ & 30\end{array}$ \& 4444.4I \& 47 Oo \& 3264.68 \& 73 -0 \& 1406.81 <br>
\hline 2130 \& 4429.7 I \& 4730 \& 3234.39 \& 7330 \& 1366.69 <br>
\hline 2200 \& 4414.67 \& 48 - \& 3203.84 \& 74 00 \& 1326.46 <br>
\hline 2230 \& 4399.30 \& 4830 \& 3173.04 \& 7430 \& 1286.12 <br>
\hline 23
23
23 \& 4383.60 \& 49 oo \& 3141.99 \& 7500 \& 1245.68 <br>
\hline $233^{\circ}$ \& 4367.57 \& 4930 \& 3110.69 \& 7530 \& 1205.13 <br>
\hline \& 4351.21 \& 5000 \& 3079.15 \& 76 00 \& 1164.49 <br>
\hline 2430
2500 \& 4334.52
4317.51 \& 50
50
50 \& 3047.37 \& 7630 \& 1123.75 <br>
\hline 2530 \& 431700.51
43 \& 51
51 \& 3015.34

2983.08 \& | 77 |
| :--- |
| 77 |
| 00 | \& 1082.91 <br>

\hline \& \& \& \& \& 1041.99 <br>
\hline
\end{tabular} LATITUDE AND LONGITUDE.

[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $78^{\circ} 00^{\prime}$ | 1000.99 | $82^{\circ} 00^{\prime}$ | 670.27 | $86^{\circ} 0^{\prime}$ | 336.02 |
| 7830 | 959.90 | 8230 | 628.64 | 8630 | 294.08 |
| 79 \% | 918.73 | 830 | 586.97 | 87 ¢ | 252.11 |
| 7930 | 877.49 | 8330 | 545.24 | 8730 | 210.12 |
| 80 - | 836.18 | 8400 |  | 8800 | 168.12 |
| 8030 | 794.79 | 8430 | 46 I .66 | 8830 | 126.10 |
| 8100 | 753.34 | 8500 | 419.8 I | 89 oo | 84.07 |
| 8130 | 711.83 | 8530 | 377.93 | 8930 | 42.04 |

Smithsonian Tables.

Table 27.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF 30' EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ} 00^{\prime}$ | 1188.10 | $13^{\circ} 00^{\prime}$ | 1158.44 | $26^{\circ} 00^{\prime}$ | 1070.64 |
| -15 | 1188.08 | 1315 | 1157.29 | 2615 | 1068.40 |
| 030 045 | 1188.05 1188.00 | 1330 13 | 1156.12 1154.93 | 26 2645 | 1066.14 1063.86 |
| 100 | 1187.92 | 1400 | 1153.72 | 2700 | 1061.56 |
| 115 | 1187.82 | 1415 | 1152.48 | 2715 | 1059.24 |
| 130 | 1187.70 | 1430 | 1151.23 | 2730 | 1056.90 |
| 145 | 1187.56 | 1445 | 1149.95 | 2745 | 1054.54 |
| 200 | 1187.39 | 1500 | 1148.65 | 28 - | 1052.16 |
| 215 | 1187.20 | 1515 | 1147.33 | 2815 | 1049.76 |
| 230 | 1186.99 | 1530 | 1145.99 | 2830 | $1047 \cdot 34$ |
| 245 | 1186.76 | 1545 | 1144.63 | 2845 | 1044.90 |
| 300 | 1186.51 | 1600 | 1143.25 | 2900 | 1042.44 |
| 315 | 1186.24 | 1615 | 1141.84 | 2915 | 1039.97 |
| 330 | 1185.95 | 1630 | 1140.41 | 2930 | 1037.47 |
|  | 1185.62 | 1645 | 1138.96 | 2945 | 1034.95 |
| 400 | 1185.28 | 1700 | 1137.50 | $30 \times$ | 1032.41 |
| 415 | 1184.92 | 1715 | 1136.00 | 3015 30 | 1029.85 |
| 430 | 1188.53 | 1730 | 1134.49 | 3030 | 1027.27 |
| 445 | 1184.13 | 1745 | 1132.96 | 3045 | 1024.68 |
| 500 | 1183.70 | 1800 | 1131.41 | 3100 | 1022.06 |
| $\begin{array}{ll}515 \\ 5 & \\ 50\end{array}$ | 1183.24 1182.77 | 1815 | 1129.83 | 3115 | 1019.43 |
| 5 5 45 | 1182.77 1182.28 | 18 <br> 18 <br> 15 | 1128.24 1126.62 | 31 31 31 | 1016.77 1014.10 |
| 6 -0 | 1181.76 | 1900 | 1124.98 | 3200 | 1011.40 |
| 615 | 1181.22 | 1915 | 1123.32 | 3215 | 1008.69 |
| 630 | 1180.66 | 1930 | 1121.64 | 3230 | 1005.96 |
| 645 | 1180.08 | 1945 | 1119.93 | 3245 | 1003.20 |
| 700 | 1179.48 | 2000 | 1118.21 | 3300 | 1000.43 |
| 715 | 1178.85 | 2015 | 1116.47 | 3315 | 997.64 |
| 730 | 1178.20 | 2030 | 1114.71 | 3330 | 994.83 |
| 745 | 1177.53 | 2045 | 1112.92 | 3345 | 992.00 |
| 8 о0 | 1176.84 | 2100 | 1111.11 | 3400 | 989.16 |
| 8 8 15 | 1176.13 | 2115 | 1109.28 | 3415 | 986.29 |
| 88 | 1175.39 | 2130 | 1107.44 | 3430 | 983.41 |
| 845 | 1174.63 | 2145 | 1105.57 | 3445 | 980.50 |
| 900 | 1173.86 | 2200 | 1103.68 |  |  |
| 915 930 | 1173.06 1172.23 | 2215 | 1101.77 | 3515 | 974.64 |
| 930 | 1172.23 | 2230 | 1099.84 | 3530 | 971.68 |
| 945 | 1171.39 | 2245 | 1097.88 | 3545 | 968.70 |
| 1000 | 1170.52 | 2300 | 1095.91 |  |  |
| $\begin{array}{ll}10 & 15 \\ 10 & 30\end{array}$ | 1169.63 | 2315 | 1093.92 | 3615 | 962.68 |
| 1030 | 1168.73 | 2330 | 109 r .90 | 3630 | 959.65 |
| 1045 | 1167.80 | 2345 | 1089.87 | 3645 | 956.60 |
| 1100 | 1166.84 | 2400 | ${ }_{1087} 8.81$ |  |  |
| 1115 | 1165.86 | 2415 | 1085.74 | 3715 | 950.43 |
| $\begin{array}{ll}11 & 30 \\ 1145\end{array}$ | 1164.86 | 2430 | 1083.64 | 3730 | $947.3{ }^{2}$ |
| 1145 | 1163.85 | 24 45 | 1081.52 | 3745 | 944.21 |
| 1200 | 1162.81 | 25 -0 | 1079.39 |  |  |
| 1215 | 1161.75 | 2515 | 1077.23 | 3815 | 947.88 |
| 1230 1245 | 1160.67 1159.56 | 2530 | 1075.05 | 3830 | 934.71 |
| 1245 | 1159.56 | 2545 | 1072.85 | 3845 | 931.51 |

[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $39^{\circ} 00^{\prime}$ | 928.29 | $52^{\circ} 00^{\prime}$ | 737.65 | $65^{\circ} 00^{\prime}$ | 507.74 |
| 3915 | 925.06 | 5215 | 733.57 | 6515 | 503.01 |
| 3930 | $92 \mathrm{I}, 80$ | 5230 | 729.47 | 6530 | 498.26 |
| 3945 | 918.53 | 5245 | $725 \cdot 36$ | 6545 | 493.51 |
| 40 0 | 915.25 | 5300 | 721.23 | 6600 | 488.75 |
| 4015 | 91.94 | 5315 | 717.08 | 665 | 483.97 |
| 4030 | 908.61 | 5330 | 712.93 | 6630 | 479.19 |
| 4045 | 905.27 | 5345 | 708.76 | 6645 | 474.40 |
| 4100 | 901.91 | 5400 | 704.57 | 6700 | 469.60 |
| 4115 | 898.54 | 5415 | 700.38 | 6715 | 464.78 |
| 4130 | 895.14 | 5430 | 696.16 | 6730 | 459.96 |
| 4145 | 891.73 | 5445 | 691.94 | 6745 | 455.13 |
| 4200 | 888.30 | 5500 | 687.70 | 68 00 | 450.29 |
| 4215 | 884.85 | 5515 | 683.44 | 68 15 | $445 \cdot 45$ |
| 4230 | 88 I .39 | 5530 | 679.17 | 6830 | 440.59 |
| 4245 | 877.91 | 5545 | 674.89 | 6845 | $435 \cdot 7^{2}$ |
| 4300 | 874.41 | 56 ¢ | 670.60 | 6900 | 430.84 |
| 4315 | 870.90 | 5615 | 666.29 | 6915 | 425.96 |
| 4330 | 867.37 | 5630 | 661.97 | 6930 | 421.06 |
| 4345 | 863.82 | 5645 | 657.64 | 6945 | 416.16 |
| 4400 | 860.25 | 5700 | 653.29 | 7000 | 417.25 |
| 4415 | 856.67 | 57.15 | 648.93 | 7015 | 406.34 |
| 4430 | 853.07 | 5730 | $644 \cdot 55$ | 7030 | 4 OI .41 |
| 4445 | 849.46 | 5745 | 640.17 | 7045 | 396.47 |
| 4500 | 845.82 | 580 | 635.77 | 7100 | 391.53 |
| 4515 | 842.18 | 58 I5 | $63 \mathrm{I} \cdot 36$ | 715 | 386.58 |
| 4530 | 838.51 | 5830 | 626.93 | 7130 | 381.62 |
| 4545 | 834.83 | 5845 | 622.49 | 7145 | 376.65 |
| 4600 | 831.13 | 5900 | 618.05 | 7200 | 37 I .68 |
| 4615 | 827.42 | 5915 | 6 F 3.59 | 7215 | 366.70 |
| $46 \quad 30$ $46 \quad 45$ | 823.68 819.94 | 5930 <br> 59 <br> 95 | 609.11 604.62 | 7230 7245 | 361.71 356.71 |
| 4645 | 819.94 | 5945 | 604.62 | 7245 | 356.71 |
| 4700 47 | 8 I 6.18 | 6000 | 600.13 | 7300 | 351.71 |
| 4715 | 812.40 | 6015 | 595.62 | 7315 | 346.69 |
| 4730 | 808.60 | 6030 | 591.09 | 73 30 | 341.68 |
| 4745 | 804.79 | 6045 | 586.56 | 7345 | 336.65 |
| 4800 | 800.97 | 6100 | 582.01 | 7400 | 331.62 |
| 4815 | 797.13 | $6 \mathrm{6I} 15$ | 577.45 | 7415 | 326.58 |
| 4830 | 793.27 | 6130 | 572.88 | $\begin{array}{r}74 \\ 74 \\ \hline 15\end{array}$ | 321.53 316.48 |
| 4845 | 789.39 | 6 L 45 | 568.30 | 7445 | 316.48 |
| 4900 | 785.50 | 6200 | 563.71 | 7500 | 311.42 |
| 4915 | 78 I .60 | 6215 | 559.11 | 7515 | 306.36 |
| 4930 | 777.68 | 6230 | 554.49 5498 | $\begin{array}{r}75 \\ 75 \\ \hline 15\end{array}$ | 301.28 296.21 |
| 4945 | 773.74 | 6245 | 549.86 | 7545 | 296.21 |
| 5000 | 769.79 | 6300 | 545.23 | 7600 | 291.12 |
| 5015 | 765.83 | 6315 | 540.58 | $\begin{array}{lll}76 & 15 \\ 76 & \end{array}$ | 286.04 |
| 5030 | 761.85 | 6330 | 535.92 | 7630 | 280.94 275.84 |
| 5045 | 757.85 | 6345 | 53 L .25 | 7645 | 275.84 |
| 5100 | 753.84 | 6400 | 526.57 | 7700 | 270.73 |
| 5115 | 749.82 | $\begin{array}{ll}64 & 15 \\ 64 & 30\end{array}$ | 521.88 |  | 265.62 |
|  | 745.78 741.72 | 6430 6445 | 517.17 512.46 | 7730 7745 | 260.50 255.38 |

Table 27.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF $30^{\prime}$ EXTENT IN LATITUDE AND LONCITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $78^{\circ} 00^{\prime}$ | 250.25 | $82^{\circ} 00^{\prime}$ | 167.57 | $86^{\circ} 00^{\prime}$ | 84.01 |
| 7815 | 245.12 | 8215 | 162.37 | 8615 | 78.76 |
| 7830 | 239.88 | 8230 | 157.16 | 8630 | 73.52 |
| 7845 | 234.83 | 8245 | 151.95 | 8645 | 68.27 |
| 7900 | 229.68 | 8300 | 146.74 | 8700 | 63.03 |
| 7915 | 224.53 | 8315 | 141.53 | 8715 | 57.78 |
| 7930 79 | 219.37 | 83 83 | 136.31 | 8730 | 52.53 |
| 7945 | 214.21 | 8345 | 131.09 | 8745 | 47.28 |
| 8000 | 209.05 | 8400 | 125.87 | 8800 | 42.03 |
| 8015 | 203.88 | 8415 | 120.64 | 8815 | 36.78 |
| 8030 | 198.70 | 8430 | 115.42 | 8830 | 31.53 |
| 8045 | 193.52 | 8445 | 110.18 | 8845 | 26.27 |
| 8100 | 188.34 | 8500 | 104.95 | 8900 | 21.02 |
| 8115 | 183.15 | 8515 | 99.72 | 8915 | 15.76 |
| 8130 | 177.96 | 8530 | 94.48 | 8930 | 10.51 |
| 8145 | 172.77 | 8545 | 89.25 | 8945 | 5.26 |

Emithbonian Tableg.

Table 28.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF 15' EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-14i.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. | Middle latltude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ} 07^{\prime} 30^{\prime \prime}$ | 297.02 | $6^{\circ} 37^{\prime} 30^{\prime \prime}$ | 295.09 | $13^{\circ} 07^{\prime} 30^{\prime \prime}$ | 289.47 |
| - 1500 | 297.02 | 64500 | 295.02 | 131500 | 289.33 |
| - 2230 | 297.02 | 65230 | 294.95 | 132230 | 289.18 |
| - 3000 | 297.01 | 7000 | 294.87 | 133000 | 289.03 |
| - 3730 | 297.01 | 70730 | 294.79 | 133730 | 288.88 |
| - 4500 | 297.00 | 71500 | 294.71 | 134500 | 288.73 |
| - 5230 | 296.99 | 72230 | 294.63 | 135230 | 288.58 |
| 10000 | 296.98 | 73000 | 294.55 | 140000 | 288.43 |
| 10730 | 296.97 | 73730 | 294.47 | 140730 | 288.28 |
| 11500 | 296.96 | 74500 | 294.39 | 141500 | 288.12 |
| $\begin{array}{ll}1 & 22 \\ 1\end{array} 3^{\circ}$ | 296.94 | 75230 | 294.30 | 142230 | 287.96 |
| 13000 | 296.93 | 80000 | 294.21 | 143000 | 287.81 |
| 13730 | 296.91 | 8 8 0730 | 294.12 | 143730 | 287.65 |
| 14500 | 296.89 | 81500 | 294.03 | 144500 | 287.49 |
| 15230 | 296.87 | 82230 | 293.94 | 145230 | 287.33 |
| 2000 | 296.85 | 83000 | 293.85 | 150000 | 287.17 |
| $\begin{array}{llll}2 & 07 & 30\end{array}$ | 296.82 | 83730 | 293.75 | 150730 | 287.00 |
| 21500 | 296.80 | 84500 | 293.66 | 15 1500 | 286.83 |
| 22230 | 296.77 | 85230 | 293.56 | 152230 | 286.67 |
| 23000 | 296.75 | 90000 | 293.47 | $15 \quad 3000$ | 286.50 |
| 23730 | 296.72 | 90730 | 293.37 | 153730 | 286.33 |
| 2 2 4500 | 296.69 | 91500 | 293.27 | 154500 | 286.16 |
| $\begin{array}{lll}2 & 52 & 30 \\ 3 & 00 & 00\end{array}$ | 296.66 296.63 | $\begin{array}{lll}9 & 22 & 30 \\ 9 & 30 & 00\end{array}$ | 293.16 293.06 | $\begin{array}{llll}15 & 52 & 30 \\ 16 & 00 & 00\end{array}$ | 285.99 |
| 30000 | 296.63 | 93000 | 293.06 | 160000 | 285.82 |
| 30730 | 296.60 | 93730 | 292.95 | 160730 | 285.64 |
| $\begin{array}{llll}3 & 15 & 00 \\ 3 & 22 & 30\end{array}$ | 296.56 | 94500 | 292.85 | 161500 | 285.46 |
| $\begin{array}{lll}3 & 22 & 30 \\ 3 & 30 & 00\end{array}$ | 296.53 296.49 | 9 9 10 | 292.74 | 162230 | 285.28 |
| $33^{\circ} 00$ | 296.49 | 100000 | 292.63 | 163000 | 285.10 |
| $\begin{array}{llll}3 & 37 & 30\end{array}$ | 296.45 | $\begin{array}{llll}10 & 07 & 30\end{array}$ | 292.52 | 163730 | 284.92 |
| $\begin{array}{llll}3 & 45 & 00 \\ 3 & 52\end{array}$ | 296.41 | 101500 | 292.41 | 164500 | 284.74 |
| 35230 | 296.36 | 102230 | 292.30 | 165230 | 284.56 |
| 4000 | 296.32 | 103000 | 292.19 | 170000 | 284.38 |
| 40730 | 296.28 | Io 3730 | 292.07 | 170730 | 284.19 |
| 41500 | 296.23 | 104500 | 291.95 | 171500 | 284.00 |
| 42230 | 296.18 | 105230 | 291.83 | 172230 | 283.81 |
| 43000 | 296.13 | 11000 | 291.71 | $17 \quad 3000$ | 283.62 |
| 43730 | 296.08 | $\begin{array}{llll}11 & 07 & 30\end{array}$ | 291.59 | 173730 | 283.43 |
| $\begin{array}{llll}4 & 45 \\ 4 & 52 & 30\end{array}$ | 296.03 | $\begin{array}{llll}\text { II } & 15 & 00 \\ \text { II } & 22 & 30\end{array}$ | 291.47 | 174500 | 283.24 |
| 45230 | 295.98 | $\begin{array}{llll}11 & 22 & 30\end{array}$ | 291.34 | 175230 | 283.05 |
| 50000 | 295.93 | II 3000 | 291.22 | 180000 | 282.86 |
| 50730 | 295.87 | 113730 | 291.09 | 180730 | 282.66 |
| $\begin{array}{llll}5 & 15 & 00 \\ 5 & 22 & 30\end{array}$ | 295.81 | 114500 | 290.96 | 181500 | 282.46 |
| $\begin{array}{llll}5 & 22 & 30 \\ 5 & 30 & 00\end{array}$ | 295.75 295.69 | $\begin{array}{lll}11 & 52 & 30 \\ \text { I2 } & 00 & 00\end{array}$ | 290.83 | $18 \quad 2230$ | 282.26 |
| 53000 | 295.69 | 120000 | 290.70 | 183000 | 282.06 |
| $\begin{array}{llll}5 & 37 & 30\end{array}$ | 295.63 | 120730 | 290.57 | 183730 | 281. 86 |
| $\begin{array}{llll}5 & 45 & 0 \\ 5 & 52 & 30\end{array}$ | 295.57 | 121500 | 290.44 | 184500 | 281. 66 |
| $\begin{array}{llll}5 & 52 & 30 \\ 6 & 00 & 00\end{array}$ | 295.51 | 122230 | 290.30 | 185230 | 28 I .45 |
| 60000 | 295.44 | 123000 | 290.17 | 19000 | 281.25 |
| $\begin{array}{llll}6 & 07 & 30\end{array}$ | 295.37 | 123730 | 290.03 | 190730 | 281.04 |
| $\begin{array}{llll}6 & 15 & 00 \\ 6 & 22 & 30\end{array}$ | 295.31 | 124500 | 289.89 | 191500 | 280.83 |
| $\begin{array}{llll}6 & 22 & 30 \\ 6 & 30 & 00\end{array}$ | 295.24 | 125230 | 289.75 | 192230 | 280.62 |
| 63000 | 295.17 | 130000 | 289.61 | 193000 | 280.41 |

[Derívation of table explained on pp. 1-lii.]

| Míddle latitude of quadrilateral. | Area in Square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $19^{\circ} 37^{\prime} 30^{\prime \prime}$ | 280.20 | $26^{\circ} 07^{\prime} 30^{\prime \prime}$ | 267.38 | $32^{\circ} 37^{\prime} 30^{\prime \prime}$ | 251.15 |
| 194500 | 279.99 | 26 I 500 | 267.10 | 324500 | 250.80 |
| 195230 | 279.77 | $26 \quad 2230$ | 266.82 | $\begin{array}{llll}32 & 52 & 30\end{array}$ | 250.45 |
| 200000 | 279.55 | $26 \quad 30$ 00 | 266.54 | 33000 | 250.11 |
| 200730 | 279.34 | 263730 | 266.25 | $33 \bigcirc 730$ | 249.76 |
| 201500 | 279.12 | 264500 | 265.97 | 331500 | 249.41 |
| 202230 | 278.90 | $26 \quad 5230$ | 265.68 | 332230 | 249.06 |
| 2030 0 | 278.68 | 2700 | 265.39 | 33300 | 248.71 |
| 203730 | 278.46 | 270730 | 265.10 | 333730 | 248.36 |
| 204500 | 278.23 | 271500 | 264.81 | 334500 | 248.00 |
| 205230 | 278.00 | 272230 | 264.52 | 335230 | 247.65 |
| $2 \mathrm{I} \times 0$ | 277.78 | 273000 | 264.23 | 3400 | 247.29 |
| 210730 | 277.55 | 273730 | 263.93 | 340730 | 246.93 |
| 211500 | 277.32 | 274500 | 263.64 | 341500 | 246.57 |
| 212230 | 277.09 | 275230 | 263.34 | 342230 | 246.21 |
| 213000 | 276.86 | 28000 | 263.04 | 343000 | 245.85 |
| 213730 | 276.63 | 280730 | 262.74 | 343730 | 245.49 |
| 214500 | 276.39 | $\begin{array}{llll}28 & 15 & 00\end{array}$ | 262.44 | 3445 00 | 245.13 |
| 215230 | 276.16 | $28 \quad 2230$ | 262.14 | $\begin{array}{lll}34 & 52 & 30\end{array}$ | 244.76 |
| 22000 | 275.92 | $28 \quad 3000$ | 261.84 | 35 00 00 | 244.40 |
| $\begin{array}{llll}22 & 07 & 30\end{array}$ | 275.68 | $\begin{array}{llll}28 & 37 & 30\end{array}$ | 261.53 | 350730 | 244.03 |
| 221500 | 275.44 | 284500 | 261.23 | 351500 | 243.66 |
| $22 \begin{array}{lll}22 & 30\end{array}$ | 275.20 | 285230 | 260.92 | $35 \quad 2230$ | 243.29 |
| $22 \quad 30 \quad 00$ | 274.96 | 290000 | 260.61 | 353000 | 242.92 |
| $\begin{array}{llll}22 & 37 & 30\end{array}$ | 274.72 | 290730 | 260.30 | 353730 | 242.55 |
| 224500 | 274.47 | 291500 | 259.99 | 354500 | 242.18 |
| $\begin{array}{llll}22 & 52 & 30\end{array}$ | 274.22 | 292230 | 259.68 | 355230 | 241.80 |
| 230000 | 273.98 | 293000 | 259.37 | 360000 | 24 I .43 |
| 230730 | 273.73 | 293730 | 259.05 | 360730 | 241.05 |
| $\begin{array}{llll}23 & 15 & 00 \\ 23 & 22 & 30\end{array}$ | 273.48 | 29 <br> 29 <br> 25 | 258.74 | $\begin{array}{llll}36 & 15 & 00\end{array}$ | 240.67 |
| $\begin{array}{llll}23 & 22 & 30\end{array}$ | 273.23 | 295230 | 258.42 | $36 \quad 2230$ | 240.29 |
| $233^{\circ} 0$ | 272.98 | 30000 | 258.10 | $36 \quad 3000$ | 239.91 |
| 233730 | 272.72 | $\begin{array}{llll}30 & 07 & 30\end{array}$ | 257.78 | $\begin{array}{llll}36 & 37 & 30\end{array}$ | 239.53 |
| $\begin{array}{llll}23 & 45 & 00 \\ 23 & 52 & 30\end{array}$ | 272.47 | $\begin{array}{llll}30 & 15 & 0 \\ 30 & 22\end{array}$ | 257.46 | 364500 | 239.15 |
| $\begin{array}{lll}23 & 52 & 30 \\ 24 & 00 & 00\end{array}$ | 272.21 | 302230 | 257.14 | $36 \quad 5230$ | ${ }^{2} 38.77$ |
| 240000 | 271.95 | $30 \quad 3000$ | 256.82 | 370000 | 238.38 |
| 240730 | 271.69 | 303730 | 256.49 | $\begin{array}{lll}37 & 07 & 30\end{array}$ | 237.99 |
| $\begin{array}{llll}24 & 15 & 50 \\ 24 & 22 & 30\end{array}$ | 271.44 | 304500 | 256.17 | $\begin{array}{lll}37 & 1500\end{array}$ | 237.61 |
| $24 \quad 2230$ | 271.17 | 305230 | 255.84 | 372230 | 237.22 |
| 243000 | 270.91 | 310000 | 255.52 | $37 \quad 3000$ | 236.83 |
| 243730 | 270.65 | 31 O7 30 | 255.19 | $\begin{array}{lll}37 & 37 & 30\end{array}$ | 236.44 |
| 244500 | 270.38 | 31.1500 | 254.86 | 374500 | 236.05 |
| 245230 | 270.1 I | 312230 | 254.53 | 375230 | 235.66 |
| $25 \infty 0$ | 269.85 | 313000 | 254.19 | $38 \quad 00$ | 235.26 |
| 250730 | 269.58 | 313730 | 253.86 | 380730 | 234.87 |
| 251500 | 269.31 | 314500 | 253.53 | 38 | 234.47 |
| $\begin{array}{llll}25 & 22 & 30 \\ 25 & 30 & 00\end{array}$ | 269.04 268.76 | $\begin{array}{llll}31 & 52 & 30 \\ 32 & 00 & 0\end{array}$ | 253.19 252.85 | $\begin{array}{lll}38 & 22 & 30 \\ 38 & 30\end{array}$ | 234.07 |
| 253000 | 268.76 | 320000 | 252.85 | $38 \quad 3000$ | 233.68 |
| 253730 | 268.49 | 320730 | 252.51 | 383730 | 233.28 |
| 254500 | 268.21 | $\begin{array}{llll}32 & 15 & 00 \\ 32 & \end{array}$ | 252.17 | 384500 | 232.88 |
| $\begin{array}{llll}25 & 52 & 30 \\ 26 & 00 & 00\end{array}$ | 267.94 | $\begin{array}{llll}32 & 22 & 30\end{array}$ | 251.83 | $\begin{array}{llll}38 & 52 & 30 \\ 30 & 0\end{array}$ | 232.48 |
| 260000 | 267.66 | 323000 | 251.49 | 39000 | 232.07 |

[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $39^{\circ} 07^{\prime} 30^{\prime \prime}$ | 231.67 | $45^{\circ} 37^{\prime} 30^{\prime \prime}$ | 209.17 | $52^{\circ} 07^{\prime} 30^{\prime \prime}$ | 183.90 |
| 39 I 500 | 231.27 | 454500 | 208.71 | 521500 | 183.39 |
| 392230 | 230.86 | 455230 | 208.25 | $\begin{array}{llll}52 & 22 & 30\end{array}$ | 182.88 |
| 393000 | 230.45 | 460000 | 207.78 | 523000 | 182.37 |
| 393730 | 230.04 | $46 \quad 0730$ | 207.32 | 523730 | 18 I .85 |
| 394500 | 229.63 | 46 I 500 | 206.86 | 524500 | 181.34 |
| 395230 | 229.22 | 462230 | 206.39 | 525230 | 180.82 |
| 400000 | 228.81 | $46 \quad 3000$ | 205.92 | 530000 | 180.31 |
| 400730 | 228.40 | 463730 | 205.45 | 530730 | 179.79 |
| 401500 | 227.99 | 464500 | 204.99 | 531500 | 179.27 |
| 402230 | $227 \cdot 57$ | $46 \quad 5230$ | 204.52 | $\begin{array}{lll}53 & 22 & 30\end{array}$ | 178.75 |
| 403000 | 227.15 | 470000 | 204.05 | 533000 | 178.23 |
| 403730 | 226.73 | $\begin{array}{llll}47 & 07 & 30\end{array}$ | 203.57 | 533730 | 177.71 |
| 404500 | 226.32 | 471500 | 203.10 | 53.4500 | 177.19 |
| $40 \quad 5230$ | 225.90 | 472230 | 202.63 | $535^{2} \quad 30$ | 176.67 |
| 410000 | 225.48 | 473000 | 202.15 | 540000 | 176.14 |
| 410730 | 225.06 | 473730 | 201.67 | 540730 | 175.62 |
| 411500 | 224.64 | 47450 | 201.20 | 541500 | 175.10 |
| 412230 | 224.21 | $47 \quad 5230$ | 200.72 | 542230 | 174.57 |
| 413000 | 223.79 | 480000 | 200.24 | 543000 | 174.04 |
| 413730 | 223.36 | 480730 | 199.76 | $54 \quad 3730$ | 173.51 |
| 414500 | 222.93 | 48 15 00 | 199.28 | 544500 | 172.99 |
| $4 \mathrm{4} \quad 5230$ | 222.50 | 482230 | 198.80 | 545230 | 172.46 |
| 420000 | 222.08 | $48 \quad 3000$ | 198.32 | 550000 | 171.93 |
| 420730 | 221.65 | 483730 | 197.83 | 550730 | 171.39 |
| 42 I 500 | 221.21 | 484500 | 197.35 | 551500 | 170.86 |
| 422230 | 220.78 | $48 \quad 5230$ | 196.86 |  | $170.33$ |
| 423000 | 220.35 | 490000 | 196.38 | 553000 | $169.79$ |
| 423730 | 219.91 | 490730 | 195.89 | 553730 | 169.26 |
| 424500 | 219.48 | 491500 | 195.40 | 554500 | 168.72 |
| 425230 | 219.04 | 492230 | 194.91 | $555^{2} 30$ | 168.19 |
| 430000 | 218.60 | 493000 | 194.42 | 560000 | 167.65 |
| 430730 | 218.16 | 4937.30 | 193.93 | 560730 | 167.11 |
| 431500 | 217.73 | 494500 | 193.44 | 561500 | 166.57 |
| 432230 | 217.28 | 495230 | 192.94 | 562230 | 166.03 |
| 433000 | 216.84 | 500000 | 192.45 | 563000 | 165.49 |
| $\begin{array}{lll}43 & 37 & 30 \\ 43 & 45 & 00\end{array}$ | 216.40 | $\begin{array}{llll}50 & 07 & 30\end{array}$ | 191.95 | 563730 | 164.95 |
| 43 <br> 43 <br> 43 <br> 42 | 215.96 | $\begin{array}{llll}50 & 15 & 50 \\ 50 & 22 & 30\end{array}$ | 191.46 | 56.4500 | 164.45 |
| $\begin{array}{llll}43 & 52 & 30 \\ 44 & 00 & 00\end{array}$ | 215.51 215.06 | 502230 | 190.96 | 56 | 163.87 |
| 44 - | 215.6 | 503000 | 190.46 | 57000 | 163.32 |
| 440730 | 214.61 | 503730 | 189.96 | 570730 | 162.78 |
| $\begin{array}{llll}44 & 15 & 00 \\ 44 & 22 & 30\end{array}$ | 214.17 | 504500 | 189.46 | $\begin{array}{llll}57 & 15 & 00\end{array}$ | 162.23 |
| $\begin{array}{llll}44 & 22 & 30 \\ 44 & 30 & 00\end{array}$ | 213.72 | 505230 | 188.96 | $\begin{array}{llll}57 & 22 & 30\end{array}$ | 161.68 |
| 443000 | 213.27 | 510000 | 188.46 | $57 \quad 3000$ | 161.14 |
| 443730 | 212.82 | 510730 | 187.96 | 573730 | 160.59 |
| 444500 | 212.37 | 51 I5 00 | 187.46 | 574500 | 160.04 |
| $\begin{array}{llll}44 & 52 & 30 \\ 45 & \infty & 00\end{array}$ | 211.91 211.46 | $\begin{array}{llll}51 & 22 & 30 \\ 51 & 30\end{array}$ | 186.95 | 575820 | 159.49 |
| 45000 | 211.46 | 513000 | 186.45 | 580000 | 158.94 |
| 450730 | 211.00 | 513730 | 185.94 | 580730 |  |
| $\begin{array}{llll}45 & 15 & 00 \\ 45 & 22 & 30\end{array}$ | 210.55 210.09 | $\begin{array}{ll}51 & 4500 \\ 51 & 52\end{array}$ | 185.43 | $\begin{array}{llll}58 & 15 & 00 \\ 58 & 22 & \end{array}$ | 157.84 |
| 45 45 45 | 210.09 209.63 | $\begin{array}{lll}51 & 52 & 30 \\ 52 & 00 & 0\end{array}$ | 184.92 | 58.2230 | 157.29 |
| 453000 | 209.63 | 520000 | 184.41 | $58 \quad 3000$ | 156.73 |

[Derivation of table explained on pp. l-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $58^{\circ} 37^{\prime} 30^{\prime \prime}$ | 156.18 | $65^{\circ} 07^{\prime} 30^{\prime \prime}$ | 126.34 | $7 \mathrm{I}^{\circ} 37^{\prime} 30^{\prime \prime}$ | 94.78 |
| 584500 | 155.62 | 651500 | 125.75 | 71450 | 94.16 |
| $\begin{array}{llll}58 & 52 \\ 59 & 30\end{array}$ | 155.07 154.51 | 65 65 65 | 125.16 124.57 | 715230 | 93.54 |
| 590000 | 154.51 | 653000 | 124.57 | 72000 | 92.92 |
| 590730 | ${ }^{1} 53.96$ | 653730 | 123.97 | 720730 | 92.30 |
| 591500 | 153.40 | 65450 | 123.38 | $\begin{array}{llll}72 & 15 & 0 \\ 72\end{array}$ | 91.68 |
| 59 59 2230 | $\begin{array}{r}152.84 \\ \hline 152.28\end{array}$ | 655230 66000 | 122.78 $\mathbf{1 2 2 . 1 9}$ | 72 722 720 | 91.05 90.43 |
| $5930 \times 0$ | 152.28 |  | 122.19 | 723000 | 90.43 |
| 593730 | 151.72 | 660730 | 121.59 | $\begin{array}{llll}72 & 37 & 30\end{array}$ | 89.80 |
| 594500 | 151.16 | 66 66 66 | 120.99 | 7245 720 | 88.18 |
| 59 60 60 | 150.60 150.03 | $\begin{array}{lll}66 & 22 & 30 \\ 66 & 30 & 00\end{array}$ | 120.40 119.80 | $\begin{array}{llll}72 & 52 & 30 \\ 73 & 00 & 00\end{array}$ | 88.55 87.93 |
|  |  |  |  |  |  |
| 600730 | 149.47 | 663730 | 119.20 | 73 0730 <br> 15  | 87.30 |
| 601500 | 148.91 | $6645 \%$ | 118.60 | 731500 | 86.67 |
| $\begin{array}{llll}60 & 22 & 30 \\ 60 & 30 & 00\end{array}$ | 148.34 147.77 | $\begin{array}{llll}66 & 52 & 30 \\ 67 & 00 & 00\end{array}$ | 118.00 117.40 | 73 <br> 7322 <br> 73 <br> 30 | 86.05 85.42 |
| 6030 | 147.7 |  |  |  |  |
| 603730 | 147.21 | 670730 | 116.80 | 73 37 30 | 84.79 |
| 6045 \% | 146.64 | 671500 | 116.20 | 734500 | 84.16 |
| $\begin{array}{llll}60 & 52 & 30 \\ 61 & 00 & 00\end{array}$ | 146.07 145.50 | $\begin{array}{llll}67 & 22 & 30 \\ 67 & 30\end{array}$ | 115.59 114.99 | $\begin{array}{llll}73 & 52 \\ 74 & 30 \\ 00\end{array}$ | 83.53 82.91 |
|  | 145.50 |  |  |  |  |
| 610730 | 144.93 | 673730 | 114.39 | 740730 | 82.28 |
| 611500 | 144.36 | 674500 | 113.78 | 74 15 0 <br> 74   | 81.65 |
| $\begin{array}{llll}61 & 22 & 30\end{array}$ | 143.79 | $\begin{array}{lll}67 & 52 \\ 68 & 30\end{array}$ | 113.18 | 742230 74 | $8 \mathrm{8r.01}$ |
| 613000 | 143.22 | 68 00 0 | 112.57 | 743000 | 80.38 |
| 613730 | 142.65 | 68 07 <br> 68  <br> 8  <br> 15  | 111.97 | 743730 | 79.75 |
| 614500 | 142.08 | 68 <br> 68 <br> 68 <br> 15 | 111.36 | 74 | 79.12 |
| $\begin{array}{lll}61 & 52 \\ 62 & 30 \\ & 00\end{array}$ | 141.50 | $\begin{array}{llll}68 & 22 & 30 \\ 68 & 30\end{array}$ | 110.76 10.15 | $\begin{array}{llll}74 & 52 \\ 75 \\ \\ 00\end{array}$ | 78.49 |
| 62000 | 140.93 | 683000 | 110.15 | 750000 | 77.86 |
| 620730 | 140.35 | 683730 | 109.54 | 750730 | 77.22 |
| 621500 | 139.78 | 68 685 | 108.93 | $\begin{array}{llll}75 & 15 \\ 75 & 0 \\ 75\end{array}$ | 76.59 |
| 622230 | 139.20 | $\begin{array}{llll}68 & 52 & 30 \\ 69 & 00 & 00\end{array}$ | 108.32 107.71 | 75 750 | 75.95 |
| 623000 | 138.62 | 690000 | 107.71 | 753000 | 75.32 |
| 623730 | 138.04 | 690730 | 107.10 | 753730 | 74.69 |
| 624500 | 137.47 | $\begin{array}{lll}69 & 15 & 0 \\ 69 & 22\end{array}$ | 106.49 | 75 75 75 50 | 74.05 |
| 62 62 63 | 136.89 $\mathbf{1 3 6 . 3 1}$ | $\begin{array}{lll}69 & 22 & 30 \\ 69 & 30 & 00\end{array}$ | 105.88 105.27 | $\begin{array}{llll}75 & 52 \\ 76 \\ 760 & 30\end{array}$ | 73.42 72.78 |
| 630000 | 136.31 | 693000 | 105.27 | 76 - |  |
| 630730 | 135.73 | 693730 | $104.65^{\circ}$ | 760730 | 72.14 |
| ${ }_{63} 12500$ | 135.15 | 694500 69 | 104.04 | $\begin{array}{llll}76 & 15 & 00 \\ 76 & 22\end{array}$ | 71.51 |
| 63 63 63 30 | 134.56 133.98 | $\begin{array}{lll}69 & 52 & 30 \\ 70 & 00 & 00\end{array}$ | 103.43 102.81 | $\begin{array}{llll}76 & 22 & 30 \\ 76 & 30 & 00\end{array}$ | 70.87 70.24 |
| 633000 | 133.98 | 7000 | 102.81 | 7630 |  |
| 633730 | 133.40 | 70.730 | 102.20 | 763730 | 69.60 |
| 634500 | 132.81 | 701500 | 101.59 | 76 45 <br> 76  <br> 76  <br> 0  | 63.96 |
| $\begin{array}{llll}63 & 52 \\ 64 & 30 \\ 000\end{array}$ | 132.23 $13^{1.64}$ | $\begin{array}{llll}70 & 22 & 30 \\ 70 & 30 & 00\end{array}$ | 100.97 100.35 | $\begin{array}{llll}76 & 52 \\ 77 & 00 \\ & 30\end{array}$ | 18.32 67.68 |
|  |  |  |  |  | 67.04 |
| $\begin{array}{llll}64 & 07 & 30 \\ 64 & 15 & 00\end{array}$ | 131.06 130.47 | 70 <br> 70 <br> 70 | 99.74 9.12 | 771500 77 | 66.41 |
| 642230 | 129.88 | 705230 | 98.50 | $\begin{array}{ll}77 & 22 \\ 77 & 30\end{array}$ | 65.77 |
| 6430 oo | 129.29 | 71000 | 97.88 | 773000 | 65.13 |
| 643730 | 128.70 | 710730 | 97.26 | 773730 | 64.49 |
| 644500 | 128.12 | 71 15 <br> 7150  <br> 150  | 96.65 | $\begin{array}{ll}77 & 45 \\ 77 \\ 50\end{array}$ | 63.85 63.20 |
| $\begin{array}{llll}64 & 52 & 30 \\ 65 & 00 & 00\end{array}$ | 127.53 126.94 | $\begin{array}{llll}71 & 22 & 30 \\ 71 & 30 & 00\end{array}$ | 96.03 95.41 | $\begin{array}{llll}77 & 52 & 30 \\ 78 & 0 & 0\end{array}$ | 63.20 62.56 |
| 650000 | 126.94 | 7130 |  |  |  |

Table 28.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF 15' EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $78^{\circ} 07^{\prime} 30^{\prime \prime \prime}$ | 61.92 | $82^{\circ} 07^{\prime} 30^{\prime \prime}$ | 41.24 | $86^{\circ} 07^{\prime} 30^{\prime \prime}$ | 20.35 |
| 78150 | 61.28 | 821500 | 40.59 | 861500 | 19.69 |
| $\begin{array}{llll}78 & 22 & 30\end{array}$ | 60.64 | 822230 | 39.94 | 862230 | 19.04 |
| $78 \quad 3000$ | 60.00 | 823000 | 39.29 | 863000 | 18.38 |
| $78 \quad 3730$ | 59.35 | 823730 | 38.64 | 863730 | 17.72 |
| 784500 | 58.71 | 824500 | 37.99 | 864500 | 17.07 |
| 785230 | 58.06 | 825230 | 37.34 | 865230 | 16.41 |
| 790000 | 57.42 | 830000 | 36.69 | 870000 | 15.76 |
| $79073^{\circ}$ | 56.78 | 830730 | 36.03 | 870730 | 15.10 |
| 791500 | 56.13 | 831500 | $35 \cdot 38$ | 871500 | 14.44 |
| 792230 | 55.49 | 83 8 8 2230 | 34.73 | 872230 | 13.79 |
| 793000 | 54.84 | 833000 | 34.08 | 873000 | 13.13 |
| 793730 | 54.20 | 833730 | 33.42 | 873730 | 12.48 |
| 794500 | 53.55 | 834500 | 32.77 | 874500 | 11.82 |
| 79 89 $5^{32}$ | 52.91 | 83 8 8 $5^{2} 300$ | 32.12 | 875230 8800 | 11.16 |
| 800000 | 52.26 | 840000 | 31.47 | 880000 | 10.51 |
| 800730 | 51.62 | 840730 | 30.81 | 880730 | 9.85 |
| 8015 | 50.97 | 841500 | 30.16 | 881500 | 9.20 |
| 80 80 80 80 | 50.32 49.68 | 84 84 84 8 | 29.51 28.86 | 88 88 88 30 | 8.54 7.88 |
| 803000 | 49.68 | 843000 | 28.86 | 88300 | 7.88 |
| $\begin{array}{lll}80 & 37 & 30 \\ 80\end{array}$ | 49.03 | 84 84 84 4 | 28.20 | 88 88 88 $3^{30}$ | 7.22 |
| $\begin{array}{llll}80 & 45 & 00 \\ 80 & 52 & 30\end{array}$ | 48.38 47.73 | 84 84 84 84 85 | 2.54 26.89 | 88 88 88 58 | 6.57 5.91 |
| 8 r 000 | 47.08 | 850000 | 26.24 | 890000 | 5.26 |
| 8 81 0730 | 46.44 | 850730 | 25.58 | $89 \bigcirc 730$ | 4.60 |
| 81 81500 81 | 45.79 | 85 85 85 | 24.93 | 891500 | 3.94 |
| 81 <br> 81 <br> 81 <br> 81 | 45.14 44.49 | 85.2230 85 80 | 24.27 | 89 89 89 $3^{30}$ | 3.28 |
| 813000 | 44.49 | 853000 | 23.62 | 893000 | 2.63 |
| 81 <br> 81 <br> 81 <br> 37 | 43.84 | 853730 | 22.97 | 893730 | 1.97 |
| $\begin{array}{lll}81 & 45 & 00 \\ 81 & 52 & 30\end{array}$ | 43.19 42.54 | 854500 85 85 | 22.31 21.66 | 89 89 89 | 1.31 0.66 |
| 81 82 800 | 42.54 4. | - 850030 | 21.00 |  |  |

Smithsonian Tables.
[Derivation of table explained on pp. 1-iii.]

| Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ} 05^{\prime}$ | 132.01 | $8^{\circ} 45^{\prime}$ | 130.51 | $17^{\circ} 25^{\prime}$ | 126.11 |
| - 15 | 132.01 | 855 | 130.46 | 1735 | 126.00 |
| $\bigcirc 25$ | 132.01 | 905 | 130.40 | 1745 | 125.88 |
| - 35 | 132.00 | 915 | 130.34 | 1755 | 125.77 |
| - 45 | 132.00 | 925 | 130.28 | 1805 | 125.65 |
| - 55 | 131.99 | 935 | 130.22 | 1815 | 125.54 |
| 105 | 131.99 | 945 | 130.15 | 1825 | 125.42 |
| 115 | 131.98 | 955 | 130.09 | 1835 | 125.30 |
| 125 | 131.97 | 10.5 | 130.02 | 1845 | 125.18 |
| 135 | 131.96 | 1015 | 129.96 | 1855 | 125.06 |
| 145 | 131.95 | 1025 | 129.89 | 1905 | 124.94 |
| 155 | 131.94 | 1035 | 129.82 | 1915 | 124.81 |
| 205 | 131.93 | 1045 | 129.76 | 1925 | 124.69 |
| 215 | 131.91 | 1055 | 129.68 | 1935 | 124.56 |
| 225 235 | 131.90 | 1105 | 129.61 | 1945 | 124.44 |
| 235 | 131.88 | 1115 | 129.54 | 1955 | 124.31 |
| 245 | 131.86 | 1125 | 129.47 | 20.5 | 124.18 |
| 255 | ${ }_{131.84}$ | 1135 | 129.39 | 2015 | 124.05 |
| $\begin{array}{ll}3 & 05 \\ 3 & 15\end{array}$ | 131.82 | 1145 | 129.32 | 2025 | 123.92 |
| 315 | 131.80 | 1155 | 129.24 | 2035 | 123.79 |
| 325 | 131.78 | 1205 | 129.16 | 2045 | 123.66 |
| 335 | 131.76 | 1215 | 129.08 | 2055 | 123.52 |
| $\begin{array}{ll}3 & 45 \\ 3 & 55\end{array}$ | 131.74 131.71 | $\begin{array}{ll}12 & 25 \\ 12 & 35\end{array}$ | 129.00 128.92 | $\begin{array}{ll}21 & 05 \\ 21\end{array}$ | 123.39 123.25 |
| 405 | 131.68 | 1245 | 128.84 |  |  |
| 415 | 131.66 | 1255 | 128.76 | 2135 | 122.98 |
| 425 | 131.63 | 1305 | 128.67 | 2145 | 122.84 |
| 435 | 137.60 | 1315 | 128.59 | 2155 | 12270 |
| 445 | 131.57 | 1325 | 128.50 | 2205 | 122.56 |
| 455 | 131.54 <br> 131.50 <br> 135 | 13 135 13 | 128.41 | 2215 | 122.42 |
| 505 515 | 131.50 131.47 | 1345 1355 | 128.33 128.24 | 2225 2235 | 122.28 122.13 |
| 525 | 131.44 | 1405 | 128.14 | 2245 | 121.99 |
| 535 | 131.40 | 1415 | 128.05 | 2255 | 121.84 |
| 545 | 131.36 | 1425 | 127.96 | 2305 | 121.69 |
| 555 | 131.33 | 1435 | 127.87 | 2315 | 121.55 |
|  | 131.29 | 1445 | 127.77 | 2325 | 121.40 |
| 6115 625 | 131.25 131.21 13120 | 1455 | 127.67 | 2335 | 121.25 |
| 6125 635 | 131.21 131.16 | $\begin{array}{ll}15 & 05 \\ 15 & 15\end{array}$ | 127.58 127.48 | 2345 2355 | 121.10 |
| 645 | 131.12 | 1525 | 127.38 |  |  |
| 655 | 131.07 | 1535 | 127.28 | $\begin{array}{lll}24 & 5 \\ 24 & 15\end{array}$ | 120.76 |
| 705 | 131.03 | 1545 | 127.18 | 2425 | 120.48 |
| 715 | 130.98 | 1555 | 127.08 | 2435 | 120.33 |
| 725 | 130.93 | 1605 | 126.98 | 2445 | 120.17 |
| 735 745 | 130.88 130.84 130.78 | 1615 | 126.87 | 2455 | 120.01 |
| 745 755 | 130.84 130.79 | 1625 1635 | 126.77 126.66 | 25 25 25 | 119.85 119.69 |
| 8 -5 | 130.73 | 1645 |  |  |  |
| $\begin{array}{ll}8 & 15 \\ 8 & 25\end{array}$ | 130.68 | 1655 | 126.44 | 25 25 25 | 119.53 119.37 |
| 8 8 8 | 130.63 130.57 | 1705 | 126.33 | 2545 | 119.21 |
| 835 | 130.57 | 1715 | 126.22 | 2555 | 119.04 |

Table 29.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF $10^{\prime}$ EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. | Middle latitude of quadrilateral | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $26^{\circ} 05^{\prime}$ | 118.87 | $34^{\circ} 45^{\prime}$ | 108.94 | $43^{\circ} 25^{\prime}$ | 96.50 |
| 2615 | 188.71 118.54 | 3455 | 108.73 | 4335 | 96.24 |
| 2625 2635 | 118.54 118.37 | 3505 3515 | 108.51 108.29 | 4345 43 | 95.98 |
|  |  |  |  |  |  |
| 2645 | 188.21 | 3525 | 108.07 | 4405 | 95.45 |
| 26 27 27 | 118.04 117.87 117.69 | 35 35 35 | 107.85 | 4415 | 95.19 |
| 27 <br> 27 <br> 75 | 117.87 117.69 | 3545 3555 | 107.63 107.45 | 4425 | 94.92 |
|  | 117.69 |  |  | 4435 | 94.65 |
| 2725 | 117.52 | 3605 | 107.19 | 4445 | 94.38 |
| 27 27 275 | 117.35 117.17 | 3615 <br> 36 <br> 15 | 106.96 | 4455 | 94.11 |
| 2745 27 | 117.17 116.99 | 3625 36 | 106.74 106.51 | 4505 | 93.84 |
| 2755 |  |  | 106.51 |  |  |
| 2805 | 116.82 | 3645 | 106.29 |  | 93.30 |
| 2815 | 116.64 | 3655 | 106.06 | 4535 | 93.03 |
| 28 28 28 | 116.46 | 3705 | 105.83 | 4545 | 92.76 |
| 2835 | 116.28 | 3715 | 105.60 | 4555 | 92.48 |
| 2845 | 116.10 | 3725 | 105.37 | $46 \mathrm{o5}$ | 92.21 |
| 2855 | 115.92 | 3735 | 105.14 | 4615 | 91.94 |
| 29.5 | 115.73 | 3745 | 104.98 | 4625 | 91.66 |
| 29 I5 | 115.55 | 3755 | 104.68 | 4635 | 9 T .38 |
| 2925 | 115.37 |  | 104.44 | 4645 | 9 9 .10 |
| 2935 | 115.18 | 38 <br> 38 <br> 8 <br> 15 | 104.21 | 4655 | 90.82 |
| 2945 2955 | ${ }^{114.99}{ }^{114.81}$ | 3825 38 | 103.97 103.74 | 47 47 47 | 90.55 90.27 |
| 3005 | 114.62 | $3^{88} 45$ | 103.50 | 4725 | 89.99 |
| 3015 | 114.43 | $3^{8} 55$ | 103.26 | 4735 | 89.70 |
| 3025 | 114.24 | 3905 | 103.02 | 4745 | 89.42 |
| 3035 | 114.04 | 3915 | 102.78 | 4755 | 89.14 |
| 3045 | 113.85 | 3925 | 102.54 | 4805 | 88.85 |
| 3055 | 113.66 | 3935 | 102.30 | 4815 | 88.57 |
| 31.05 | 113.47 | 3945 | 102.06 | 4825 | 88.28 |
| 3115 | 113.27 | 3955 | 101.82 | 4835 | 88.00 |
| 3125 | 113.07 | 4005 | 101.57 | 4845 | 87.71 |
| 3135 | 112.88 | 4015 | 101. 33 | 4855 | 87.42 |
| 3145 | 112.68 | 4025 | 101.08 | 49.5 | 87.13 |
| 3155 | 112.48 | 4035 | 100.83 | 49 I 5 | 86.84 |
| 3205 | 112.28 | 4045 | 100.59 | 4925 | 86.55 |
| 3215 | 112.08 | 4055 | 100.34 | 4935 | 86.26 |
| 3225 | 111.87 | 415 | 100.09 | 4945 | 85.97 |
| $3^{2} 35$ | 111.67 | 4115 | 99.84 | 4955 | 85.68 |
| 3245 | 111.47 | 4125 | 99.59 | 50.5 | 85.39 |
| 3255 | 111.26 | 4135 | 99.33 | 5015 | 85.09 |
| 3305 3315 | 111.06 110.85 | 41 415 45 | 99.08 98.83 | 5025 50 | 84.80 84.50 |
|  | 110.64 |  | 98.57 |  | 84.21 |
| 3335 | 110.43 | 4215 | 98.32 | 5055 | 83.91 |
| 3345 | 110.22 | 4225 | 98.06 | 5105 | 83.61 |
| 3355 | 110.01 | 4235 | 97.80 | 515 | 83.31 |
| 3405 | 109.80 |  | 97.55 |  | 83.01 |
| 3415 | 109.59 | 4255 | 97.29 | 5135 | 83.71 |
| 3425 | 109.37 | 4305 | 97.03 96.77 | 51 515 59 | 82.41 82.11 |
| 3435 | 109.16. | 4315 | 96.77 | 5155 | 82.11 |

table 29.
AREAS OF QUADRILATERALS OF EARTH'S SURFACE OF 10' EXTENT IN LATITUDE AND LONGITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $52^{\circ} 05^{\prime}$ | 8 I .8 s | $60^{\circ} 45^{\prime}$ | 65.17 | $69^{\circ} 25^{\prime}$ | 46.97 |
| 5215 | 8 I .51 | 6055 | 64.84 | 6935 | 46.60 |
| 5225 | $8 \mathrm{8r} 20$ | 6105 | 64.50 | 6945 | 46.24 |
| 5235 | 80.90 | 6115 | 64.16 | 6955 | 45.88 |
| 5245 | 80.60 | 6I 25 | 63.82 | 7005 | 45.51 |
| 5255 | 80.29 | 6I 35 | 63.48 | 70.15 | 45.15 |
| 5305 | 79.98 | 6145 | 63.14 | $70 \quad 25$ | 44.78 |
| 5315 | 79.68 | 6155 | 62.80 | 7035 | 44.42 |
| 5325 | 79.37 | 6205 | 62.46 | 7045 | 44.05 |
| 5335 | 79.06 | 6215 | 62.12 | 7055 | 43.69 |
| 5345 | 78.75 | 6225 | 61.78 | 7105 | 43.32 |
| 5355 | 78.44 | 6235 | 61.44 | 7115 | 42.95 |
| 5405 | 78.13 | 6245 | 6 I .10 | 7125 | 42.58 |
| 5415 | 77.82 | 6255 | 60.75 | 7135 | 42.22 |
| $\begin{array}{lll}54 & 25 \\ 54 & 35\end{array}$ | 77.51 | 63 63 | 60.41 | 7145 | 41.85 |
| 5435 | 77.19 | 6315 | 60.06 | 7155 | 41.48 |
| 5445 | 76.88 | 6325 | 59.72 | 72 '05 | 41.1 I |
| 5455 | 76.57 | 6335 | 59.37 | 7215 | 40.74 |
| 55 55 | 76.25 | 6345 | 59.03 | 7225 | 40.37 |
| 5515 | 75.94 | 6355 | 58.68 | 7235 | 40.00 |
| 5525 | 75.62 | 6405 | 58.33 | 7245 | 39.63 |
| 55 <br> 55 <br> 55 | 75.30 | $\begin{array}{lll}64 & 15 \\ 64 & 25\end{array}$ | 57.99 | 7255 | 39.26 |
| 5545 | 74.99 | 6425 64 | 57.64 | 7305 | 38.89 |
| 5555 | 74.67 | 6435 | 57.29 | 7315 | 38.52 |
| 5605 | 74.35 |  | 56.94 | 7325 | $3^{8.15}$ |
| $\begin{array}{lll}56 & 15 \\ 56 & 25\end{array}$ | 74.03 73.71 | 64 65 | 56. 59 | 7335 | 37.78 |
| 5625 | 73.71 | 6505 | 56.24 | 7345 | 37.41 |
| 5635 | 73.39 | 6515 | 55.89 | 7355 | 37.03 |
| 5645 | 73.07 | $65 \quad 25$ | 55.54 | 7405 | 36.66 |
| 5655 | 72.75 | 6535 | 55.19 | 7415 | 36.29 |
| 57 57 | 72.43 | 6545 | 54.83 | 74125 | 35.91 |
| 5715 | 72.10 | 6555 | 54.48 | 7435 | 35.54 |
| 5725 | 71.78 | 6605 | 54.13 | 7445 | 35.17 |
| 5735 | 71.46 | 6615 | 53.78 | 7455 | 34.79 |
| 5745 | 71.13 | 6625 | 53.42 | 7505 | 34.42 |
| 5755 | 70.80 | 6635 | 53.06 | 7515 | 34.04 |
| 5805 | 70.48 | 6645 | 52.71 | 7525 | 33.66 |
| 5815 | 70.15 | 6655 | 52.35 | 7535 | 33.29 |
| 58 58 58 | 69.82 60.49 | 6705 | 52.00 | 7545 | 32.91 |
| 5035 | 69.49 | 6715 | 5 I .64 | 7555 | 32.53 |
| 5845 | 69.17 | 6725 | 51.28 | 7605 | 32.16 |
| 5855 | 68.84 | 6735 | 50.93 | 7615 | 3 I .78 |
| 59 <br> 59 <br> 15 | 68.51 68.18 | 6745 6755 | 50.57 | 7625 | 31.40 |
| 5915 | 68.18 | 6755 | 50.21 | 7635 | 31.03 |
| 5925 | 67.84 | 6805 | 49.85 | 7645 | 30.65 |
| 5935 | 67.51 | 6815 | 49.49 | 7655 | 30.27 |
| 5945 59 | 67.18 | 6825 | 49.13 | 7705 | 29.89 |
| 5955 | 66.85 | 6835 | 48.77 | 7715 | 29.51 |
| 6005 | 66.51 | 6845 | 48.41 | 7725 |  |
| 6015 | 66.18 | 6855 | 48.05 | 7735 | 28.76 |
| 60 60 | 65.84 | 6905 | 47.69 | 7745 | 28.37 |
| 6035 | 65.51 | 6915 | $47 \cdot 33$ | 7755 | 27.99 |

LATITUDE AND LONCITUDE.
[Derivation of table explained on pp. 1-lii.]

| Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. | Middle latitude of quadrilateral. | Area in square miles. |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $78^{\circ} 05^{\prime}$ | 27.62 | $82^{\circ} 05^{\prime}$ | 18.43 | $86^{\circ} 05^{\prime}$ | 9.14 |
| $78 \times$ | 27.24 | 82 I 5 | 18.04 | 8615 | 8.75 |
| 7825 | 26.85 | 8225 | 17.65 | 8625 | 8.36 |
| 7835 | 26.47 | 8235 | 17.27 | 8635 | 7.97 |
| 7845 | 26.09 | 8245 | 16.88 | 8645 | 7.59 |
| $78 \quad 55$ | 25.71 | 8255 | 16.50 | 8655 | 7.20 |
| 7905 | 25.33 | 8305 | 16.11 | 8705 | 6.81 |
| 7915 | 24.95 | 8315 | 15.73 | 8715 | 6.42 |
| 7925 | 24.57 | 8325 | 15.34 | 8725 | 6.03 |
| 7935 | 24.18 | 8335 | 14.95 | 8735 | 5.64 |
| 7945 | 23.80 | 8345 | 14.57 | 8745 | 5.25 |
| 7955 | 23.42 | 8355 | 14.18 | 8755 | 4.86 |
|  | 23.04 |  | 13.79 | 8805 | 4.47 |
| 8015 | 22.65 | 8415 | 13.40 | 88 15 | 4.09 |
| 8025 | 22.27 | 8425 | I 3.02 | 8825 | 3.70 |
| 8035 | 21.89 | 8435 | 12.63 | 8835 | $3 \cdot 31$ |
|  | 21.50 |  | 12.24 | 8845 | 2.92 |
| 8055 | 21.12 | 8455 | 11.86 | 8855 | 2.53 |
| 8105 815 | 20.73 | 8505 | 1 I .47 | 895 | 2.14 |
| 8115 | 20.35 | 8515 | 11.08 | 8915 | 1.75 |
| 8125 | 19.97 | 8525 | 10.69 | 8925 | 1.36 |
| 8135 | 19.58 | 8535 | 10.30 | 8935 | 0.97 |
| 81 815 815 | 19.20 18.81 | 85 8555 | 9.92 9.53 | 89 89 | 0.58 |
| 81 55 | 18.81 | 8555 | 9.53 | 8955 | 0.19 |

Smithsonian Tables.

TAbLE 30.
DETERMINATION OF HEICHTS BY THE BAROMETER.
Formula of Babinet.

$$
\begin{gathered}
Z=C \frac{B_{0}-B}{B_{0}+B} \\
C(\text { in feet })=52494\left[1+\frac{t_{0}+t-64}{900}\right]-\text { English Measures. } \\
C(\text { in metres })=16000\left[1+\frac{2\left(t_{0}+t\right)}{1000}\right]-\text { Metric Measurès. }
\end{gathered}
$$

In which $Z=$ Difference of height of two stations in feet or metres.
$\boldsymbol{B}_{\mathrm{o}}, \boldsymbol{B}=$ Barometric readings at the lower and upper stations respectively, corrected for all sources of instrumental error.
$t_{0}, t=$ Air temperatures at the lower and upper stations respectively.
Values of $\mathbf{C}$.
ENGLISH MEASURES.

| $\frac{1}{2}\left(t_{0}+t\right)$. | $\log C$. | $c$. |
| :---: | :---: | :---: |
| F. |  | Feet. |
| $10^{\circ}$ | 4.69834 | 49928 |
| 15 | .70339 | 50511 |
| 20 | . 70837 | 51094 |
| 25 | .71330 | 51677 |
| 30 | .71818 | 52261 |
| 35 | 4.72300 | 52844 |
| 40 | . 72777 | 53428 |
| 45 | .73248 | 54011 |
| 50 | .73715 | 54595 |
| 55 | .74177 | 55178 |
| 60 | 4.74633 | 55761 |
| 65 | .75085 | 56344 |
| 70 | .75532 | 56927 |
| 75 | . 75975 | 57511 |
| 80 | .76413 | 58094 |
| 85 | 4.76847 | 58677 |
| 90 | .77276 | 59260 |
| 95 | .77702 | 59844 |
| 100 | .78123 | 60427 |

METRIC MEASURES.

| $\frac{1}{2}\left(t_{0}+t\right)$. | $\log C$. | C. |
| :---: | :---: | :---: |
| C. |  | Metres. |
| $-10^{\circ}$ | 4.18639 | 15360 |
| -8 | . 19000 | 15488 |
| - 6 | . 19357 | 15616 |
| $-4$ | .19712 | 15744 |
| $-2$ | . 20063 | 15872 |
| 0 | 4.20412 | 16000 |
| +2 | . 20758 | 16128 |
| 4 | .21101 | 16256 |
| 6 8 | .21442 .21780 | 16384 16512 |
| 8 | .21780 |  |
| 10 | 4.22115 | 16640 |
| 12 | . 22448 | 16768 |
| 14 | . 22778 | 16896 |
| 16 | . 23106 | 17024 |
| 18 | . 23431 | 17152 |
| 20 | 4.23754 | 17280 |
| 22 | . 24075 | 17408 |
|  | . 24393 | 17536 |
| 26 | .24709 | 17664 |
| 28 | . 25022 | 17792 |
| 30 | 4.25334 | 17920 |
| 32 | . 25643 | 18048 |
| 34 | . 25950 | 18176 |
| 36 | . 26255 | 18304 |

Gmithsonian Tables.

MEAN REFRACTION．

|  | Refraction． |  |  | Refrac | ction． |  | Refra | tion． |  | Refra | ction． | 苞第苞 | Refra | tion． |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ，＂ | ＂ |  |  |  |  |  | ${ }_{5.3}^{\prime \prime}$ |  |  | $\begin{array}{\|c\|} \hline 1 \\ x .5 \\ \hline \end{array}$ | $\left[\begin{array}{l} 42 \\ 43 \end{array}\right.$ |  | $\begin{aligned} & 111 \\ & 2.2 \\ & 2.1 \end{aligned}$ |
| $\bigcirc$ | 34 54．I |  | 70 | 719.7 |  | 14. | 347.4 |  | 280 |  |  |  |  |  |
| 10 | 3249.2 | 124.9 16.9 | 10 | 710.5 | $\begin{aligned} & 9.2 \\ & 8.8 \end{aligned}$ | 20 | $\frac{342.1}{}$ |  | 20 | 146．2 |  |  | 6 t .8 |  |
| 20 | 3052.3 | 116.9 108.8 | 20 | 71.7 |  | 40 | $\underline{337.0}$ | $\begin{aligned} & 5 \cdot 3 \\ & 5 \cdot 1 \end{aligned}$ | 40 | 145.3 | $\text { I. } 4$ | $\frac{43}{44}$ | 59.7 |  |
| 30 | 293.5 | 108.8 <br> roc． | 30 | 653.3 | 8.4 | 150 | 332.1 | 4.9 4.7 | 290 | 1 |  | 45 | 57.7 | 2.02.050 |
| 40 50 | 2722.7 2549 | 92.9 | 40 | 645.1 637.2 | 7.9 | 20 | 327.4 | 4.5 | 20 | 142.4 | $\begin{aligned} & 1.5 \\ & x .4 \end{aligned}$ |  | 55.7 |  |
| 10 | $\frac{2549.6}{24}$ | 85.2 | 8 50 | $\frac{637.2}{629.6}$ | 7.6 | 40 | 322.9 |  | 40 | 141.0 | $\begin{aligned} & \mathrm{x} .4 \\ & \mathrm{x} .3 \end{aligned}$ | $\frac{46}{47}$ | 53.8 |  |
| 10 | $\frac{246.7}{23}$ | 77.9 | 10 | $\frac{629.6}{622.3}$ | 7.3 | 16 o | 318.6 | 4.3 | 300 | 139.7 | $\begin{array}{l\|l} \mathrm{x} \cdot 3 & 48 \end{array}$ |  | 51.9 <br> 50.2 | 1.9 <br> 1.7 <br> r <br> 1 |
| 20 | ${ }_{21} 55.6$ | 71.1 | 20 | 622.3 615.2 | 7.1 | 20 | 314.5 | 4.0 | 20 | 138.4 | $\begin{aligned} & x .3 \\ & x .3 \end{aligned}$ | 49 | $\frac{50.2}{48.4}$ | 1.81.71.7 |
| 30 | 2050.9 | 64.7 | 30 | 68.4 | 6.8 | 40 | 310.5 |  | 40 | 137.1 |  |  |  |  |
| 40 | 1951.9 | 59.0 | 40 | 6 I .8 | 6.6 | 170 | 36.6 | 3.9 3.7 | 310 | 135.8 | 1.3 | 52 | 46.7 | $\begin{array}{r}1.7 \\ \\ \hline\end{array}$ |
| 50 | 1858.0 | 53.9 | 50 | 555.4 | 6.4 | 20 | $\begin{array}{ll}3 & 2.9\end{array}$ | 3.6 | 20 | 134.5 | I． 3 | $\begin{aligned} & 52 \\ & 53 \end{aligned}$ | 43.5 | 1.6 |
| 20 | 188.6 | 49.4 | 90 | 549.3 | 6.1 | 40 | 259.3 |  | 40 | 133.3 | 1.2 |  |  |  |
| 10 | 1723.0 |  | 10 | 543.3 | 6.0 | 180 | 255.8 |  | 320 | 132.1 |  |  | 4 | 1.5 |
| 20 | 1640.7 | 42.3 39.8 | 20 | 537.6 | 5.7 | 20 | 252.5 | 3.3 | 20 | 130.9 | $1.2$ | 55 | 48.4 38.9 | 1.5 <br> 1.4 <br> 1.4 |
| 30 | 160.9 | 39.8 37.5 | 30 | 532.0 |  | 40 | 249.3 | ${ }^{3.2}$ | 40 | 129.8 | I． 1 | 57 | 37.5 |  |
| 40 | 1523.4 | 37.5 35.6 | 40 | 526.5 | 5.5 | 190 | 246.1 |  | 330 | 128.7 |  |  | 36.1 | 1.4 |
| 50 | 1447.8 | 35.6 | 50 | 521.3 | 5.2 | 20 | 243.1 | 2.9 | 20 | 127.6 | ． 1 | 59 | 34.7 | 1.4 |
| 30 | 1414.6 | 33.2 30.9 | 10. | 516.2 | 5.0 | 40 | 240.2 |  | 40 | I 26.5 | i． | 60 | 33．3 |  |
| 10 | 1343.7 | 28.7 | 10 | 5 II． 2 | 4.8 | 200 | 237.3 | 2.8 | 340 | 125.4 |  | $\begin{aligned} & 6 \mathrm{I} \\ & 62 \end{aligned}$ | 32.0 | 1.3 |
| 20 | 1315.0 | 26.7 | 20 | 56.4 | 4.7 | 20 | 234.5 |  | 20 | 124.3 |  |  | 30.7 | 1.3 1.3 2 |
| 30 | 1248.3 | 24.6 | 30 | 51.7 | 4.5 | 40 | 231.9 |  | 40 | 123.3 |  | 63 | 29.4 | 1.3 <br> r． <br> 2 |
| 40 50 | $\begin{array}{lll}12 & 23.7 \\ 12 & 0.7 \\ 12 & \end{array}$ | 23.0 | 40 50 | 457.2 452.8 | $4 \cdot 4$ | 210 | 229.3 | 2.6 <br> 2.5 | 350 | 122.3 |  | 64 | 28.2 | 1.3 1.3 1.3 1.2 |
| 40 | 1138.9 | 2 L .8 | － | 448.5 | 4.3 |  | 226.8 | 2.52.42.4 | 20 | 121.3 | 1.0 | $\begin{aligned} & 65 \\ & 65 \end{aligned}$ | 26.9 | 1.2 |
| 10 | 1118.3 | 19.7 | 10 | $444 \cdot 3$ | 3.9 | $\underline{42}$ | $\frac{224.3}{221.9}$ |  | 40 | 120.3 | 1．0 | 67 <br> 68 | ． 5 | 1.2 <br> 1.2 <br> 1.2 <br> 1.2 |
| 20 | 1058.6 | 19．0 | 20 | 440.2 |  | $\frac{20}{20}$ |  | 2.3 |  | $\frac{119.3}{118.3}$ |  |  | 24.5 23.3 | 1.2 |
| 30 | 1039.6 | 18.4 | 30 | 436.3 432.4 | 3.9 | $40$ | $217.4$ | 2.2 2.2 | 20 40 | 118.3 117.4 | ${ }^{1.0}$ | 69 | 22.2 | ${ }^{1.2}$ |
| 50 | 103.3 | 16.8 | 50 | 428．7 | 3.7 <br> 3.7 | 230 | 215.2 |  | 370 | $\underline{16.5}$ | 0.9 | 70 | 21.0 | R． x |
| 50 | 946.5 | 16.8 15.6 | 120 | 425.0 |  | 20 | 213.0 | $2.1$ | 20 | 115.6 | 0.90.9 | $\begin{aligned} & 71 \\ & 72 \end{aligned}$ | 19.8 |  |
| 10 | 930.9 | 15.6 <br> 14.9 | 10 | 421.4 | 3.6 | 40 | 210.9 |  | 40 | 114.7 |  |  | 18.8 | ． 1 |
| 20 | 916.0 | 14.9 | 20 | 418.0 | 3.4 <br> 3.4 | 240 | 28.9 |  | 380 | 113.8 | ${ }_{0}^{0.9}$ | 73 | 17.7 |  |
| 30 | 91.9 | 13.5 | 30 | 414.6 |  | 20 | 27.0 |  | 20 | 112.9 | $\begin{aligned} & 0.9 \\ & 0.9 \\ & 0.8 \end{aligned}$ | $\begin{aligned} & 74 \\ & 75 \end{aligned}$ | 16.6 | 1.1 |
| 40 | 848.4 | 13.5 | 40 | 411.3 | 3.3 3.2 | 40 | 2 5．1 |  | 42 | 112.0 |  |  | 15.5 | \％ |
| 50 | 835.6 | ${ }^{2} 2.8$ | 50 | 48.1 |  | 250 | 23.2 | 1.9 x． 8 | 390 | III． |  |  | $\begin{aligned} & 13.4 \\ & 123 \\ & 11.2 \end{aligned}$ | 1． 1 |
| 60 | 823.3 |  | 130 | 44.9 |  | 20 | 21.4 |  |  | 110.3 | $0.9$ | $\begin{aligned} & 77 \\ & 78 \\ & 79 \end{aligned}$ |  | r．r．1.51 |
| 10 | 811.6 |  | 10 | 41.8 |  | 40 | 159.6 | $1.8$ | 40 | 19.5 | $0.8$ |  |  |  |
| 20 | 80.3 | 10.8 | 20 | 358.8 | 2.9 | 260 | 157.8 |  | $\frac{4}{40}$ | 18.7 |  | $\frac{79}{80}$ | 10.2 | ． 1 |
| 30 | 749.5 | 10.3 | 30 | 355.9 | 2.9 | 20 | 156.1 | 1.7 | 20 |  |  |  |  |  |
| 40 50 | 739.2 | 10．0 | 40 | 353.0 <br> 350.2 | 2.8 | 40 | I 54．4 | 1.7 | 40 | 17.1 |  | 82 | 8.1 |  |
| 70 | 719.7 | 9.5 | 140 | 347．4 | 2.82.8 | 270 | 152.8 |  | 4 I | 16.3 | 0.8 | 86 | 4.1 |  |
|  |  |  |  |  |  |  |  |  |  |  |  | 90 | 0.0 |  |
|  |  |  |  |  |  |  | 149.7 |  |  | 14.7 |  |  |  |  |
|  |  |  |  |  |  | 28 o | 148.2 |  | $\underline{420}$ | 14.0 |  |  |  |  |

Smithsonian Tables．

| - | h. m. | $\bigcirc$ | h. m. | $\bigcirc$ | h. m. | 0 | h. m. | - | h. m. | 0 | h. m. |  | . |  | s. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 O | 60 | 40 | 120 | 8 - | 180 | 120 | 240 | 16 o | 300 | 20 0 | 0 | 00 | 0 | 0.000 |
| 1 | - 4 | 6 I | 44 | 121 | 84 | 181 | 124 | 24I | 164 | 301 | 204 | 1 | - 4 | $\mathbf{I}$ | 0.067 |
| 2 | - 8 | 62 | 48 | 122 | 88 | 182 | 128 | 242 | 168 | 302 | 208 | 2 | - 8 | $2$ | 0.133 |
| 3 | 012 | 63 | 412 | 123 | 812 | 183 | 1212 | 243 | 1612 | 303 | 12 |  | 012 | 3 | 0.200 |
| 4 | - 16 | 64 | 416 | 124 | 816 | 184 | 1216 | 244 | 1616 | 304 | 2016 |  | - 16 |  | 0.267 |
|  | 0 | 65 | 420 | 125 | 820 | 185 | 1220 | 245 | 1620 | 305 | 2020 | 5 | 020 | 5 | 0.333 |
| 6 | 0 | 66 | 424 | 126 | 824 | 186 | 1224 | 246 | 1624 | 306 | 2024 | 6 | - 24 | 6 | 0.400 |
| 7 | - 28 | 67 | 428 | 127 | 82 | 187 | 1228 | 247 | 1628 | 307 | 2 | 7 | $\bigcirc 28$ | 7 | 0.467 |
| 8 | - 32 | 68 | 432 | 128 | 832 | 188 | 1232 | 248 | 1632 | 308 | 2032 | 8 | $\bigcirc{ }^{-} 32$ | $8$ | 0.533 |
| 9 | - 36 | 69 | 436 | 129 | 836 | 189 | 1236 | 249 | 1636 | 309 | 2036 | 9 | O 36 | 9 | 0.600 |
| $\overline{10}$ | 040 | 70 | 440 | 130 | 840 | 190 | 1240 | 250 | 1640 | 310 | 2040 | 10 | 040 | 10 | 0.667 |
| 11 | 044 | 7 I | 444 | 131 | 844 | 191 | 1244 | 251 | 1644 | 311 | 2044 | 1 | 044 | 11 | 33 |
| 12 | 048 | 72 | 448 | 132 | 848 | 192 | 1248 | 252 | 1648 | 312 | 2048 | 2 | 048 | 2 | 0.800 |
| 13 | O 52 | 73 | 452 | 133 | 852 | 193 | 1252 | 253 | 1652 | 313 | 2052 | 13 | 052 | 13 | 0.867 |
| 14 | - 56 | 74 | 456 | 134 | 856 | 194 | 1256 | 254 | 1656 | 314 | 2056 | 14 | 056 | 14 | 0.933 |
| 15 | 10 | 75 | 5 o | 135 | 9 o | 195 | 130 | 255 | 170 | 315 | 210 | 15 | 1 | 15 | 1.000 |
| 16 |  | 76 | 54 | 136 | 94 | 196 | 13 | 256 | $\begin{array}{lll}17 & 4\end{array}$ | 316 | 2 I 4 | 16 | 1 | 6 | 1.067 |
| 17 | 18 | 77 | 58 | 137 | 98 | 197 | 138 | 257 | 17 | 317 | 21 | 17 | 1 | 17. | I.I 33 |
| 18 | 12 | 78 | 512 | 138 | 912 | 198 | 1312 | 258 | 1712 | 318 | 2112 | 18 | 11 | 18 | I. 200 |
| 19 | 116 | 79 | 516 | 139 | 916 | 199 | 1316 | 259 | 1716 | 319 | 2116 | 19 | 1 | 19 | 1.267 |
| 20 | 2 | 80 | 520 | 140 | 920 | 200 | 1320 | 260 | 1720 | 320 | 21 | 20 | 120 | 20 | 1.333 |
| 21 | I 24 | 81 | 524 | 141 | 924 | 201 | 1324 | 261 | 17 | 321 | 2124 | 21 | 124 | 21 | 1.400 |
| 22 | 128 | 82 | 528 | 142 | 928 | 02 | 1328 | 262 | 1728 | 322 | 21 | 22 | 128 | 22 | 1.467 |
| 23 | 132 | 83 | 532 | 143 | 932 | 203 | 1332 | 263 | 1732 | 323 | 2132 | 23 | 132 | 23 | 1.533 |
| 24 | 136 | 84 | 536 | 144 | 936 | 204 | 1336 | 264 | 1736 | 324 | 2136 | 24 | 136 | 24 | 1.600 |
| 25 | 140 | 85 | 540 | 145 | 940 | 205 | 1340 | 265 | 1740 | 325 | 2140 | 25 | 140 | 25 | 1.667 |
| 6 | 44 | 86 | 544 | 146 | 944 | 206 | 1344 | 266 | 1744 | 326 | 2144 | 26 | 144 | 26 | 1.733 |
| 27 | 148 | 87 | 548 | 147 | 948 | 207 | 1348 | 267 | 1748 | 327 | 2148 | 27 | 148 | 27 | 1.800 |
| 28 | $1{ }^{1} 2$ | 88 | 552 | 148 | 952 | 208 | 1352 | 268 | 1752 | 328 | 2152 | 28 | 152 | 28 | 1.867 |
| 29 | 156 | 89 | 556 | 149 | 956 | 209 | 1356 | 269 | 1756 | 329 | 2156 | 29 | 156 | 29 | 1.933 |
| 30 | 2 | 90 | 60 | 150 | 10 | 210 | 140 | 270 | 18 o | 330 | 22 | 30 | 20 | 30 | 2.000 |
| 31 |  | 91 |  | 151 | 10 | 211 |  | 271 | 184 | 33 I | 22 | 3 I |  | 31 | 2.067 |
| 32 | 28 | 92 | 6 | 152 | 108 | 212 | 14 | 272 | 18 | 332 | 22 | 32 |  | 32 | 2.133 |
| 33 | 212 | 93 | 612 | 153 | 1012 | 213 | 1412 | 273 | 181 | 333 | 2212 | 33 |  | 33 | 2.200 |
| 3 | 6 | 9 | 616 | 154 | Io 16 | 214 | 1416 | 274 | 1816 | 3334 | 221 | 34 | 216 | 34 | 2.267. |
| 35 | 220 | 95 | 6 | 155 | IO 20 | 215 | 1420 | 275 | 18 | 335 | 22 | 35 | 220 | 35 | 2.333 |
| 36 | 224 | 96 | 624 | 156 | IO 24 | 216 | 1424 | 276 | 18 | 336 | 222 | 36 | 2 | 36 | 2.400 |
| 37 | 228 | 97 | 628 | 157 | 1028 | 217 | 1428 | 277 | 18 | 337 | 2228 | 37 | 2 | 37 | 2.467 |
| 38 | 232 | 98 | 632 | 158 | IO 32 | 218 | 1432 | 278 | I8 32 | 338 |  | 38 | 232 | 38 | 2.533 |
| 39 | 236 | 99 | 636 | 159 | 1036 | 219 | 1436 | 279 | 1836 | 339 | 2236 | 39 | 236 | 39 | 2.600 |
| 40 | 240 | 100 | 640 | 160 | 1040 | 220 | 1440 | 280 | 1840 | 340 | 2240 | 40 | 240 | 40 | 2.667 |
| 41 | 244 | 101 | 644 | 161 | 1044 | 221 | 1444 | 281 | 1844 | 341 | 2244 | 41 | 244 | 41 | 2.733 |
| 42 | 248 | 102 | 648 | 162 | 1048 | 222 | 1448 | 282 | 1848 | 342 | 2248 | 42 | 248 | 42 | 2.800 |
| 43 | 252 | 103 | 652 | 163 | 1052 | 223 | 1452 | 283 | 1852 | 343 | 2252 | 43 | 252 | 43 | 2.867 |
|  | 256 | 104 | 656 | 164 | 1056 | 224 | 1456 | 284 | 1856 | 344 | 2256 | 44 | 256 | 44 | 2.933 |
| 45 | 3 of | 105 | 70 | 165 | II 0 | 225 | 150 | 285 | 190 | 345 | 230 | 45 | 30 | 45 | 3.000 |
| 46 | $3{ }^{3} 8$ | 106 | $\begin{array}{ll}7 & 4 \\ 7\end{array}$ | 166 | $\begin{array}{ll}\text { II } & 4 \\ \text { II } & 8 \\ \text { 1 }\end{array}$ | 226 | 15 | 286 |  | 346 | ${ }^{23} \begin{array}{ll}23 \\ 23\end{array}$ | 46 | 3 | 46 | 3.067 |
| 47 | 38 | 107 | 78 | 167 | $\begin{array}{ll}11 & 8 \\ \text { I }\end{array}$ | 227 | 158 | 287 | 198 | 347 | 238 | 47 | 3 8 | 47 | 3.133 |
| 48 | 312 | 108 | 712 | 168 | II 12 | 228 | 1512 | 288 | 1912 | 348 | 2312 | 48 | 312 | 48 | 3.200 |
| 49 | 316 | 109 | 716 | 169 | 1116 | 229 | 1516 | 289 | 1916 | 349 | 2316 | 49 | 316 | 49 | 3.267 |
| 50 | 320 | 110 | 720 | 170 | 1120 | 230 | 1520 | 290 | 1920 | 350 | 2320 | 50 | 320 | 50 | 3.333 |
| 51 | 324 | 111 | 724 | 171 | 1124 | 23 J | 1524 | 291 | 1924 | 351 | 2324 | 51 | 324 | 51 | 3.400 |
| 52 | 328 | 112 | 728 | 172 | II 28 | 232 | 1528 | 292. | 1928 | 352 | 2328 | 52 | 328 | 52 | 3.467 |
| 53 | 332 | 113 | 732 | 173 | 1132 | 233 | 1532 | 293 | 1932 | 353 | 2332 | 53 | 332 | 53 | 3.533 |
|  | 336 | 1114 | 736 | 174 | II 36 | 235 | 1536 | 294 | 1936 | 354 | 2336 | 55 | 336 | 54 | 3.600 |
| 55 | 340 | 115 | $74{ }^{\circ}$ | 175 | II 40 | 235 | 1540 | 295 | 1940 | 355 | $234^{\circ}$ | 55 | 340 | 55 | 3.667 |
| 56 | 344 | 116 | 744 | 176 | 1144 | 236 | 1544 | 296 | 1944 | 356 | 2344 | 56 | 344 | 56 | 3.733 |
| 57 | 348 | 117 | 748 | 177 | II 48 | 237 | 1548 | 297 | 1948 | 357 | 2348 | 57 | 348 | 57 | 3.800 |
| 58 | 352 356 | 118 | 752 756 | 178 | II 115 | 238 | 115 5 | 298 | 1952 | 358 | 23 5 <br> 23 5 | 58 | 352 | 58 | 3.867 |
| $\frac{59}{60}$ | 450 | 120 | 7 80 | 180 | 120 | $\underline{240}$ | $\frac{1550}{160}$ | 300 | $\frac{1956}{20}$ | $\underline{350}$ | $\frac{2356}{240}$ | 59 | 356 | 59 | $\frac{3.933}{4000}$ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

FOR CONVERSION OF TIME INTO ARC.

Hours of Time into Arc.

| Time. | Arc. | Time. | Arc. | Time. | Arc. | Time. | Arc. | Time. | Arc. | Time. | Arc. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hrs. | - | hrs. | - | hrs. | 。 | hrs. | - | hrs. | - | hrs. | - |
| 1 | 15 | 5 | 75 | 9 | 135 | 13 | 195 | 17 | 255 | 21 |  |
| 2 3 3 | 30 | 6 | 90 905 | 10 | ${ }^{15}$ | 14 | 210 | 18 | 270 | 22 | 330 |
| 3 4 | 45 60 | 7 8 | 105 120 | 112 | 165 180 | 15 16 | 225 240 | 19 20 | 285 300 | 23 24 | 335 345 360 |

Minutes of Time into Arc.

| m. | - | m. | - | m. | - , |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 015 | 21 | 515 | 41 | 1015 |
| 2 | $\bigcirc 30$ | 22 | 530 | 42 | 1030 |
| 3 | - 45 | 23 24 | 545 | 43 | 1045 |
| 4 |  | 24 |  | 44 | 110 |
| 5 | 115 | 25 | 615 | 45 | II 15 |
| 6 | 130 | 26 | 630 | 46 | 1130 |
| 7 |  |  | 645 | 47 | 1145 |
| 8 | 20 | 28 | 70 | 48 | 120 |
| 9 | 215 | 29 | 715 | 49 | 1215 |
| 10 | 230 | 30 | 730 | 50 | 1230 |
| 11 | 245 | 3 I | 745 | 51 | 1245 |
| 12 | 30 | 32 | 80 | 52 | 130 |
| 13 | 315 | 33 | 815 | 53 | 1315 |
| 14 | 330 | 34 | 830 | 54 | 1330 |
| 15 | 345 | 35 | 845 | 55 | 1345 |
| 16 | 40 | 36 | 90 | 56 | 14 - |
| 17 | 415 | 37 | 915 | 57 | 1415 |
| 18 | 430 | 38 | 930 | 58 | 1430 |
| 19 | 445 | 39 | 945 | 59 | 1445 |
| 20 | 5 - | 40 | 10 о | 60 | 150 |

Seconds of Time into Arc.

| s. | ' " | s. | " | s. | " |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 015 | 21 | 515 | 41 | 1015 |
| 2 | - 30 | 22 | 530 | 42 | 1030 |
| 3 | $\bigcirc$ | 23 | 545 | 43 | 1045 |
| 4 | 10 | 24 | 60 | 44 | II 0 |
| 5 | 115 | 25 | 615 | 45 | ${ }_{11} 15$ |
| 6 | 130 | 26 | 630 | 46 | II 30 |
| 7 | 145 | 27 | 645 | 47 | 1145 |
| 8 | 20 | 28 | 70 | 48 | 120 |
| 9 | 215 | 29 | 715 | 49 | 1215 |
| 10 | 230 | 30 | 730 | 50 | 1230 |
| 11 | 245 | 31 | 745 | 51 | 1245 |
| 12 | 30 | 32 | 8 \% | 52 | 130 |
| ${ }^{1} 3$ | 315 | 33 | 815 | 53 | 1315 |
| 14 | 330 | 34 | 830 | 54 | 1330 |
| 15 | 345 | 35 | 845 | 55 | 1345 |
| 16 | 40 | 36 | 90 | 56 | 140 |
| 17 | 415 | 37 | 915 | 57 | 1415 |
| 18 | 430 | 38 | 930 | 58 | 1430 |
| 19 | 445 | 39 | 945 | 59 | 1445 |
| 20 | 50 | 40 | 10 - | 60 | 150 |

Hundredths of a Second of Time into Arc.

| Hundredths of a Second of Time. | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | " | " | " | " | " | " | " | " | $\%$ | " |
| 0.00 | 0.00 | 0.15 | 0.30 | 0.45 | 0.60 | 0.75 | 0.90 | 1.05 | 1.20 | 1. 35 |
| . 10 | 1.50 | 1. 65 | 1.80 | 1.95 | 2.10 | 2.25 | 2.40 | 2.55 | 2.70 | 2.85 |
| . 20 | 3.00 | 3.15 | $3 \cdot 30$ | 3.45 | 3.60 | 3.75 | 3.90 | 4.05 | 4.20 | 4.35 |
| . 30 | 4.50 | 4.65 | 4.80 | 4.95 | 5.10 | 5.25 | 5.40 | $5 \cdot 55$ | $5 \cdot 70$ | 5.85 |
| .40 | 6.00 | 6.15 | 6.30 | 6.45 | 6.60 | 6.75 | 6.90 | 7.05 | 7.20 | $7 \cdot 35$ |
| 0.50 | 7.50 | 7.65 | 7.80 | 7.95 | 8.10 | 8.25 | 8.40 | 8.55 | 8.70 | 8.85 |
| . 60 | 9.00 | 9.15 | 9.30 | 9.45 | 9.60 | 9.75 | 9.90 | 10.05 | 10.20 | 10.35 |
| . 70 | 10.50 | 10.65 | 10.80 | 10.95 | II.10 | 11.25 | 11.40 | 11.55 | 11.70 | 11.85 |
| . 80 | 12.00 | 12.15 | 12.30 | 12.45 | 12.60 | 12.75 | 12.90 | 13.05 | 13.20 | 13.35 |
| . 90 | 13.50 | 13.65 | 13.80 | 13.95 | 14.10 | 14.25 | 14.40 | 14.55 | 14.70 | 14.85 |

Table 34.
CONVERSION OF MEAN TIME INTO SIDEREAL TIME.

| S | m | $\mathrm{m}_{\mathrm{I}}$ | m 2 | m 3 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\begin{array}{ccc}\text { h } & \text { m } & \text { s } \\ 0 & 0 & 0\end{array}$ | $\begin{array}{lc} \hline \text { h } & \text { m } \\ 6 \quad 515 \end{array}$ | $\begin{array}{lcc} \hline \mathrm{h} & \mathrm{~m} & \mathrm{~s} \\ 12 & 10 & 29 \end{array}$ | $\begin{array}{cc} \mathrm{h} & \mathrm{~m} \\ \mathrm{I} 8 \mathrm{~s} \\ 1544 \end{array}$ | $0.00$ | $\begin{aligned} & \mathrm{mg} \\ & 0 \\ & 0 \end{aligned}$ | $0.50$ | ${ }^{\text {m }}$ |
| 1 | 065 | 61120 | 121634 | 182149 | 0.01 | $\bigcirc$ | 0.51 |  |
| 2 | 01210 | 61725 | 122240 | 182754 | 0.02 | 07 | 0.52 | 310 |
| 3 | - 1816 | 62330 | 122845 | 183359 | 0.03 | $\bigcirc 11$ | 0.53 | 314 |
| 4 | - 2421 | 62936 | 123450 | 18405 | 0.04 | $\bigcirc 15$ | 0.54 | 317 |
| 5 | - 3026 | 63541 | 124055 | 184610 | 0.05 | 0 I8 | 0.55 | 321 |
| 6 | - 3631 | 64 I 46 | 1247 I | 185215 | 0.06 | 022 | 0.56 | 325 |
| 7 | 04237 | 64751 | 12536 | 185820 | 0.07 | 026 | 0.57 | 328 |
| 8 | $\bigcirc{ }^{\circ} 4842$ | 65356 | 125911 | 19426 | 0.08 | - 29 | 0.58 | 332 |
| 9 | 05447 | 702 | 13516 | 191031 | 0.09 | $\bigcirc 33$ | 0.59 | 335 |
| 10 | 1052 | 767 | 131121 | 191636 | 0.10 | 037 | 0.60 | 339 |
| 11 | $165^{8}$ | 71212 | 131727 | 192241 | 0.11 | 040 | 0.61 | 343 |
| 12 | 1133 | 71817 | 132332 | 192847 | 0.12 | 044 | 0.62 | 346 |
| 13 | 1198 | 72423 | 132937 | 193452 | 0.13 | 047 | 0.63 | 350 |
| 14 | 12513 | 73028 | 133542 | 194057 | 0.14 | $\bigcirc 51$ | 0.64 | 354 |
| 15 | 13119 | $73^{6} 33$ | 134148 | 19472 | 0.15 | - 55 | 0.65 | 357 |
| 16 | 1 3724 | 74238 | 134753 | 19537 | 0.16 | - 58 | 0.66 | 41 |
| 17 | 14329 | 74844 | 135358 | 195913 | 0.17 | 12 | 0.67 |  |
| 18 | 1 4934 | 75449 | $14 \bigcirc 3$ | 20518 | 0.18 | I 6 | 0.68 | 48 |
| 19 | 1 5540 | $8 \bigcirc 54$ | 1469 | 201123 | 0.19 | 19 | 0.69 | 412 |
| 20 | 2145 | 8659 | 141214 | 201728 | 0.20 | 113 | 0.70 | 416 |
| 21 | 2750 | 8135 | 141819 | 202334 | 0.21 | 117 | 0.71 | 419 |
| 22 | 21355 | 81910 | 142424 | 202939 | 0.22 | 120 | 0.72 | 423 |
| 23 | 2201 | 82515 | 143030 | 203544 | 0.23 | 124 | 0.73 | 427 |
| 24 | 2266 | 83120 | 143635 | 204149 | 0.24 | 128 | 0.74 | 430 |
| 25 | 23211 | 83726 | 144240 | 204755 | 0.25 | 131 | 0.75 | 434 |
| 26 | 23816 | 84331 | 144845 | 20540 | 0.26 | I 35 | 0.76 | $43^{8}$ |
| 27 | 24422 | 84936 | 145451 | 2105 | 0.27 | 139 | 0.77 | 441 |
| 28 | 25027 | 85541 | 15056 | 21610 | 0.28 | I 42 | 0.78 | 445 |
| 29 | 25632 | 9147 | 1571 | 211216 | 0.29 | 146 | 0.79 | 449 |
| 30 | 3237 | 9.752 | 15136 | 211821 | 0.30 | 150 | 0.80 | $45^{2}$ |
| 3 I | 3843 | 91357 | 151912 | 212426 | 0.31 | 153 | 0.81 | 456 |
| 32 | 31448 | $920 \cdot 2$ | 152517 | 213035 | 0.32 | 157 | 0.82 | 459 |
| 33 | 32053 | 9268 | 153122 | 213637 | 0.33 | 21 | 0.83 | 53 |
| 34 | 32658 | 93213 | 153727 | 214242 | 0.34 | 24 | 0.84 | 57 |
| 35 | 3333 | 93818 | 154333 | 214847 | 0.35 | 28 | 0.85 | 510 |
| 36 | 3399 | 94423 | 154938 | 215452 | 0.36 | 211 | 0.86 | 514 |
| 37 | 34514 | 95028 | 155543 | $\begin{array}{lll}22 & 0 & 58\end{array}$ | 0.37 | 215 | 0.87 | 518 |
| 38 | 35119 | 95634 | 16148 | 2273 | 0.38 | 219 | 0.88 | 521 |
| 39 | 35724 | $10 \quad 239$ | 16754 | 22138 | 0.39 | 222 | 0.89 | 525 |
| 40 | 4330 | 10844 | 161359 | 221913 | 0.40 | 226 | 0.90 | 529 |
| 41 | 4935 | 101449 | 16204 | 222519 | 0.41 |  | 0.91 |  |
| 42 | 41540 | 102055 | 16269 | 223124 | 0.42 | 233 | 0.92 | 536 |
| 43 | 42145 | 10270 | 163214 | 223729 | 0.43 | 237 | 0.93 | 540 |
| 44 | 42751 | 10335 | 163820 | 224334 | 0.44 | 241 | 0.94 | 543 |
| 45 | 43356 | 103910 | 164425 | 224939 | 0.45 | 244 | 0.95 | 547 |
| 46 | 440 I | 104516 | 165030 | 225545 | 0.46 | 248 | 0.96 | 551 |
| 47 | 4466 | 105121 | 165635 |  | 0.47 | 252 | 0.97 | 554 |
| 48 | 45212 | 105726 | $17 \quad 241$ | 23755 | 0.48 | 255 | 0.98 | 558 |
| 49 | 45817 | 11331 | 17846 | 23140 | 0.49 | 259 | 0.99 | 62 |
| 50 | 5422 | 11937 | 171451 | 23.206 | 0.50 | 33 | 1.00 | 65 |
| 51 | 51027 | 111542 | 172056 | 232611 | Example: Let the given mean time be $14^{\mathrm{L}} 57^{\mathrm{m}} 3^{2 \mathrm{~L}} .56$. <br> The table gives <br> first for $14^{\mathrm{h}} 54^{\mathrm{ma}} 51^{\mathrm{D}} \quad 2^{\mathrm{m}} 27^{\mathrm{\prime} \mathrm{\prime}}$ then for <br> The sum $\begin{array}{lll} 2 & 41 & 0.44 \\ 227.44 \end{array}$ <br> $14^{\mathrm{b}} 57^{\mathrm{m}} 3^{\mathrm{m}} \cdot 5^{6}+2^{\mathrm{m}} 27^{\mathrm{B}} \cdot 44=15^{\mathrm{b}} 0^{\mathrm{m}} 0^{\mathrm{d}}$ is the required sidereal time. |  |  |  |
| 52 | 51633 | 112147 | 17272 | 233216 |  |  |  |  |
| 53 | 522 5 5 | 11 2752 <br> 11 73 | 17337 | 233821 |  |  |  |  |
| 54 | 528.43 | II 3358 | 173912 | 234427 |  |  |  |  |
| 55 | 53448 | $\begin{array}{llll}11 & 40 & 3\end{array}$ | 174517 | $235032$ |  |  |  |  |
| 56 57 | 54054 54659 | $\begin{array}{llll}\text { II } & 46 & 8 \\ \text { II } & 52 & 13\end{array}$ | $\begin{array}{lllll}17 & 51 & 23 \\ 17 & 57 \\ 28\end{array}$ | 235637 24 2 |  |  |  |  |
| 57 58 | 54659 5 53 | $\begin{array}{llll}11 & 52 & 13 \\ \text { II } & 5 \\ 1 & 19\end{array}$ | 175728 18 18 | $\begin{aligned} & 24242 \\ & 24848 \end{aligned}$ |  |  |  |  |
| 59 | 559.9 | 12424 | 18938 | 241453 |  |  |  |  |
| 60 | 6515 | 121029 | 18 I5 44 | $\frac{2412058}{24}$ |  |  |  |  |

Smithsonian Tableg.

CONVERSION OF SIDEREAL TIME INTO MEAN TIME.

| S | $\begin{gathered} \mathrm{m} \\ 0 \end{gathered}$ | $\underset{\mathbf{I}}{\mathrm{m}}$ | m 2 | $\begin{gathered} m \\ 3 \end{gathered}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | $\begin{array}{llll} \hline h & m & s \\ o & 0 & 0 \end{array}$ | $\begin{array}{l\|cc} \hline h & m & s \\ 6 & 6 & 15 \end{array}$ | $\begin{array}{ccc} \hline h & m & s \\ 12 & 12 & 29 \end{array}$ | $\begin{array}{ccc} \mathrm{h} & \mathrm{~m} & \mathrm{~s} \\ \mathrm{I} 8 & \mathrm{I} 8 & 44 \end{array}$ | $\stackrel{\mathbf{s}}{0.00}$ | $\begin{array}{cc} \mathrm{m} & \mathrm{~s} \\ 0 & 0 \end{array}$ | $\begin{gathered} \mathrm{s} \\ 0.50 \end{gathered}$ | $\begin{array}{cc}\text { m } & \text { s } \\ 3 & 3\end{array}$ |
| 1 | 066 | 61221 | 121835 | 182450 | 0.01 | - 4 | 0.51 |  |
| 2 | 01212 | 61827 | 122442 | 183056 | 0.02 | $\bigcirc 7$ | 0.52 | 310 |
| 3 | - 1819 | 62433 | 123048 | 18372 | 0.03 | $\bigcirc 11$ | 0.53 | 314 |
| 4 | - 2425 | 63040 | 123654 | 18439 | 0.04 | $\bigcirc 15$ | 0.54 | 318 |
| 5 | 0303 I | 63646 | 12430 | 184915 | 0.05 | 018 | 0.55 | 321 |
| 6 | - 3637 | 64252 | 12497 | 18552 L | 0.06 | 022 | 0.56 | 325 |
| 7 | - 4244 | 64858 | 125513 | 19127 | 0.07 | $\bigcirc 26$ | 0.57 | 329 |
| 8 | - 4850 | 6554 | 13119 | 19734 | 0.08 | - 29 | 0.58 | 332 |
| 9 | $\bigcirc 5456$ | 7111 | 13725 | 191340 | 0.09 | $\bigcirc 33$ | 0.59 | 33 |
| 10 | 1 I 2 | 7117 | 131331 | 191946 | 0.10 | 037 | 0.60 | 340 |
| 11 | $\begin{array}{ll}17 & 7\end{array}$ | 71323 | 131938 | $19255^{2}$ | 0.11 | 040 | 0.61 | 34 |
| 12 | 11315 | 71929 | 132544 | 193159 | 0.12 | $\bigcirc 44$ | 0.62 | 347 |
| 13 | 11921 | 72536 | 133150 | $1938{ }^{8} 5$ | 0.13 | 048 | 0.63 | 351 |
| 14 | 12527 | 73142 | 133756 | 194411 | 0.14 | 051 | 0.64 | 35 |
| 15 | 13134 | 73748 | 13443 | 195017 | 0.15 | $\bigcirc 55$ | 0.65 | 35 |
| 16 | I 3740 | 74354 | 13509 | 195623 | 0.16 | - 59 | 0.66 | 4 |
| 17 | 1 4346 | 7501 | 135615 | 20230 | 0.17 | 12 | 0.67 | 4 |
| 18 | I 4952 | 7567 | $14{ }_{14} 22 \mathrm{II}$ | 20836 | 0.18 | 16 | 0.68 | 4 |
| 19 | I 5559 | 8213 | 14828 | 201442 | 0.19 | 110 | 0.69 | 4 I |
| 20 | 225 | 8819 | 141434 | 202048 | 0.20 | 113 | 0.70 | 416 |
| 21 | 2811 | 81426 | 142040 | 202655 | 0.21 | 117 | 0.71 | 420 |
| 22 | 21417 | 82032 | 142646 | 20331 | 0.22 | 121 | 0.72 | 42 |
| 23 | 22024 | 82638 | 143253 | 20397 | 0.23 | 124 | 0.73 | 427 |
| 24 | 22630 | 83244 | 143859 | 204513 | 0.24 | 128 | 0.74 | 43 |
| 25 | 23236 | $83^{8} 51$ | 14455 | 205120 | 0.25 | 132 | 0.75 | 43 |
| 26 | $23^{8} 42$ | 84457 | 145111 | 205726 | 0.26 | 135 | 0.76 | 438 |
| 27 | 24449 | 8513 | 145718 | 21 3 32 <br> 21   | 0.27 0.28 | 139 143 | 0.77 0.78 | 44 |
| 28 | 25055 | 8579 | $\begin{array}{ll}15 & 324\end{array}$ | 21 9 38 <br> 1 15  | 0.28 0.29 | 143 146 | 0.78 0.79 | 446 |
| 29 | 2571 | 9316 | 15930 | 211545 | 0.29 | 146 | 0.79 | 449 |
| 30 | 337 | 9.922 | 151536 | 212151 | 0.30 | 150 | 0.80 | 45 |
| 3 I | 3914 | 91528 | 152143 | 212757 | 0.31 | 154 | 0.81 | 45 |
| 32 | 31520 | 92134 | 152749 15 | $2134 \begin{array}{llll} \\ 21 & 3\end{array}$ | 0.32 0.33 | 157 | 0.82 0.83 |  |
| 33 | 32126 | 92741 | 153355 | 214010 | 0.33 |  | 0.83 0.84 | 5 |
| 34 | 32732 | 93347 | 15401 | 21 4616 | 0.34 0.35 | 25 | 0.83 0.85 | 51 |
| 35 | 33338 | 93953 | 15468 15 15 |  | 0.35 0.36 | 212 | 0.86 | 51 |
| 36 | 33945 | 9 9 5259 | 155214 155820 | $\begin{array}{llll}21 & 58 \\ 22 & 48 \\ 22\end{array}$ | 0.36 0.37 | 216 | 0.87 | 519 |
| 37 38 | 34551 <br> 35157 | 9 52 <br> 9 58 | 155820 16426 | 22 1041 | 0.38 | 219 | 0.88 | 52 |
| 39 | 358 | 10 418 | 161033 | 221647 | 0.39 | 223 | 0.89 | 5 |
| 40 | 4410 | 101024 | 161639 | 222253 | 0.40 | 226 | 0.90 | 53 |
| 41 | 41016 | 101630 | 162245 | 22290 | 0.41 | 230 | 0.91 | 5 |
| 42 | 41622 | 102237 | 162851 | 22356 | 0.42 | 234 | 0.92 | 53 |
| 43 | 42228 | 10 2843 | 163457 | 224112 | 0.43 | 237 | 0.93 | 54 |
| 44 | 42835 | I0 3449 | 1641 | 224718 | 0.44 | 2 4I | 0.94 | 54 |
| 45 | 43441 | 104055 | 164710 | 225324 | 0.45 0.46 |  |  | 54 |
| 46 | 44047 | 10472 | 165316 | 225931 | 0.46 | 248 | 0.96 | 55 |
| 47 | 44653 | 10538 | 165922 | 23537 | 0.47 | 252 256 | 0.97 0.98 | 55 |
| 48 | 453 O | 105914 | 17529 | 231143 | 0.48 0.49 | 256 259 | 0.98 0.99 | 55 |
| 49 | 4596 | II 520 | 171135 | 231749 | 0.49 0.50 |  | 1.00 |  |
| 50 | $5 \quad 512$ | II 1127 | 171741 | 232356 | 0.50 |  |  |  |
| 51 | 5 II 18 | 111733 | 172347 | 23308 | Example: Given $15^{\mathrm{h}} 0^{\mathrm{m}} 0^{\mathrm{a}}$. <br> The table gives <br> first for $14^{b^{4}} 57^{\mathrm{m}} \mathrm{I}^{8} \quad 2^{\mathrm{m}} 27^{8}$ then for 2 42  <br> 15 0 0 0.44 <br> The difference $15^{\mathrm{g}} \mathrm{o}^{\mathrm{m}} 0^{\mathrm{B}}-2^{\mathrm{m}} 27^{\mathrm{B}} \cdot 44=14^{\mathrm{L}} 57^{\mathrm{m}} 3^{2^{1} .56}$ is the required mean time. |  |  |  |
| 52 | 5 5 5 5 723125 | 11 2339 <br> II  | 172954 1736 | 23 <br> 23 <br> 23 <br> 42 <br> 14 |  |  |  |  |
| 53 | 52331 | 11 29 <br> 15  | 17360 | 234214 23481 |  |  |  |  |
| 54 | 52937 | II 3552 |  |  |  |  |  |  |
| 55 | 53543 54150 | $\begin{array}{ll}\text { II } & 41 \\ \text { II } & 58 \\ 48\end{array}$ | 174812 175419 | $\begin{aligned} & 235427 \\ & 24 \quad 033 \end{aligned}$ |  |  |  |  |
| 56 | 54150 54756 | $\begin{array}{llll}11 & 48 & 4 \\ \text { II } & 54 \\ \text { IO }\end{array}$ | 17 5419 <br> 18  <br> 185  | $\begin{array}{ll} 24 & 033 \\ 24 & 639 \end{array}$ |  |  |  |  |
| 58 | 5542 | 12017 | 18631 | 241246 |  |  |  |  |
| 59 | 608 | 12623 | 181237 | 241852 |  |  |  |  |
| 60 | 6615 | 121229 | 181844 | $24245^{8}$ |  |  |  |  |

LENGTH OF ONE DEGREE OF THE MERIDIAN AT DIFFERENT LATITUDES.
[Derivation of table explained on pp. xlvi-xiviii.]

| Latitude. | Metres. | Statute Miles. | Geographic Miles. $I^{\prime}$ of the Eq. | Latitude. | Metres. | Statute Miles. | Geographic Miles. $I^{\prime}$ of the Eq. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 110568.5 | 68.703 | 59. 594 | $45^{\circ}$ | 111132.1 | 69.054 | 59.898 |
| 1 | 110568.8 | 68.704 | 59.594 | 46 | 111151.9 | 69.067 | 59.908 |
| 2 | 110569.8 | 68.705 | 59.595 | 47 | 111171.6 | 69.079 | 59.919 |
| 3 | 110571.5 | 68.706 | 59.596 | 48 | IIII91-3 | 69.091 | 59.929 |
| 4 | 110573.9 | 68.707 | 59.597 | 49 | 111210.9 | 69.103 | $59.94{ }^{\circ}$ |
| 5 | 110577.0 | 68.709 | 59.598 | 50 | 111230.5 | 69.115 | 59.95\% |
| 6 | 110580.7 | 68.711 | 59.600 | 51 | I 11249.9 | 69.127 | 59.961 |
| 7 | 110585.1 | 68.714 | 59.603 | 52 | 111269.2 | 69.139 | 59.972 |
| 8 | 110590.2 | 68.717 | 59.606 | 53 | 111288.3 | 69.151 | 59.982 |
| 9 | 110595.9 | 68.721 | 59.609 | 54 | 111307.3 | 69.163 | 59.992 |
| 10 | 110602.3 | 68.725 | 59.612 | 55 | 111326.0 | 69.175 | 60.002 |
| 11 | 110609.3 | 68.729 | 59.616 | 56 | 111344.5 | 69.186 | 60.012 |
| 12 | 110617.0 | 68.734 | 59.620 | 57 | 111362.7 | 69.198 | 60.022 |
| 13 | 110625.3 | 68.739 | 59.625 | 58 | 111380.7 | 69.209 | 60.032 |
| 14 | 110634.2 | 68.745 | 59.629 | 59 | 111398.4 | 69.220 | 60.041 |
| 15 | 110643.7 | 68.751 | 59.634 | 60 | 111415.7 | 69.230 | 60.091 |
| 16 | 110653.8 | 68.757 | 59.640 | 61 | 111432.7 | 69.241 | 60.060 |
| 17 | 110664.5 | 68.763 | 59.646 | 62 | 111449.4 | 69.251 | 60.069 |
| 18 | 110675.7 | 68.770 | 59.652 | 63 | 111465.7 | 69.261 | 60.077 |
| 19 | I 10687.5 | 68.778 | 59.658 | 64 | 111481.5 | 69.271 | 60.086 |
| 20 | 110699.9 | 68.786 | 59.665 | 65 | 111497.0 | 69.281 | 60.094 |
| 21 | 110712.8 | 68.794 | 59.672 | 66 | 111512.0 | 69.290 | 60.102 |
| 22 | 110726.2 | 68.802 | 59.679 | 67 | 111526.5 | 69.299 | 60.110 |
| 23 | 110740.1 | 68.810 | 59.686 | 68 | 111540.5 | 69.308 | 60.118 |
| 24 | 110754.4 | 68.819 | 59.694 | 69 | II 1554.1 | 69.316 | 60.125 |
| 25 | 110769.2 | 68.829 | 59.702 | 70 | 111567.1 | 69.324 |  |
| 26 | 110784.5 | 68.838 | 59.710 | 71 | 111579.7 | 69.332 | 60.139 |
| 27 28 | 110800.2 | 68.848 68.8 | 59.719 59.727 | 72 | 111591.6 | 69.340 | 60.145 |
| 28 29 | 110816.3 110832.8 | 68.858 68.868 | 59.727 59.736 | 73 | 111603.0 | 69.347 | 60.151 |
| 29 | 110832.8 | 68.868 | 59.736 | 74 | III613.9 | 69.354 | 60.157 |
| 30 | 110849.7 | 68.879 | 59.745 | 75 |  |  |  |
| 31 | 110866.9 | 68.889 | 59.755 | 76 | $\text { II } 633.8$ | $69 \cdot 366$ | 60.168 |
| 32 | 110884.4 | 68.900 | 59.764 | 77 | 111642.8 | 69.372 | 60.173 |
| 33 | 110902.3 | 68.911 | 59.774 | 78 | III651.2 | 69.377 | 60.177 |
| 34 | 110920.4 | 68.923 | 59.784 | 79 | II 1659.0 | 69.382 | 60.182 |
| 35 | 110938.8 | 68.934 | 59.794 | 80 | II 1666.2 | 69.386 | 60.186 |
| 36 | 110957.4 | 68.946 | 59.804 | 8 I | 111672.6 | 69.390 | 60.189 |
| 37 | 110976.3 | 68.957 | 59.814 | 82 | I 11678.5 | 69.394 | 60.192 |
| 38 39 | 110995.3 111014.5 | 68.969 68.98 I | 59.824 59.834 | 83 84 | I 11683.6 | 69.397 | 60.195 |
| 39 | 111014.5 | 68.98I | 59.834 | 84 | 1 11688.1 | 69.400 | 60.197 |
| 40 | 111033.9 | 68.993 | 59.845 | 85 | 111691.9 |  |  |
| 41 | 111053.4 | 69.005 | 59.855 | 86 | 111695.0 | 69.404 | 60.201 |
| 42 | 111073.0 | 69.017 | 59.866 | 87 | $111697 \cdot 4$ | 69.405 | 60.202 |
| 43 | 111092.6 IIII 12.4 | 69.029 | 59.876 59.887 | 88 | 111699.2 | 69.407 | 60.203 |
| 44 | IIIII 12.4 | 69.042 | 59.887 | 89 | 1111700.2 | 69.407 | 60.204 |
| 45 | 111132.1 | 69.054 | 59.898 | 90 | 111700.6 | 69.407 | 60.204 |

Smitheonian Tables.

LENGTH OF ONE DEGREE OF THE PARALLEL AT DIFFERENT LATITUDES.
[Derivation of table explained on p. xlix.]

| Latitude. | Metres. | Statute Miles. | Geographic Miles. $\mathbf{I}^{\prime}$ of the Eq. | Latitude. | Metres. | Statute Miles. | Geographic Miles. $\mathrm{I}^{\prime}$ of the Eq. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $0^{\circ}$ | 111321.9 | 69.171 | 60.000 | $45^{\circ}$ | 78850.0 | 48.995 | 42.498 |
| 1 | 111305.2 | 69.162 | 59.991 | 46 | 77466.5 | 48.135 | 4 I .753 |
| 2 | 111254.6 | 69.130 | 59.964 | 47 | 76059.2 | 47.261 | 40.994 |
| 4 | 111170.4 111052.6 | 69.078 69.005 | 59.98 59.855 | 4 | 74628.5 73174.9 | 46.372 45.469 | 40.223 39.440 |
| 5 | 110901.2 | 68.911 | 59.773 | 50 | 71698.9 | 44.552 | 38.644 |
| 6 | 110716.2 | 68.796 | 59.673 | 51 | 70200.8 | 43.621 | 37.837 |
| 8 | 110497.7 | 68.660 | 59.556 | 52 | 6868 I .1 | 42.676 | 37.018 |
| 8 | 110245.8 | 68.503 68.326 | 59.420 59.266 | 53 | 67140.3 65578 | 41.719 | 36.187 |
| 9 | 109960.5 | 68.326 | 59.266 | 54 | 65578.8 | 40.749 | $35 \cdot 346$ |
| 10 | 109641.9 | 68.128 | 59.095 | 55 | 63997.1 | 39.766 | 34.493 |
| 11 | 109290.1 | 67.909 | 58.905 | 56 | $62395 \cdot 7$ | 38.771 | 33.630 |
| 12 | 108905.2 | 67.670 | 58.697 | 57 | 60775.1 | 37.764 | 33.757 |
| 13 | 108487.3 | 67.411 | ${ }_{58}^{8.472}$ | 58 | 59 5 35.7 | 36.745 | 31.873 |
| 14 | 108036.6 | 67.131 | 58.229 | 59 | 57478.1 | 35.715 | 30.979 |
| 15 | 107553.1 | 66.830 | 57.969 | 60 | 55802.8 | 34.674 | 30.076 |
| 16 | 107037.0 | 66.510 | 57.690 | $6 \mathrm{6r}$ | 54110.2 | 33.622 | 29.164 |
| 17 | 106488.5 | 66.169 | 57.395 | 62 | 52400.9 | 32.560 | 28.243 |
| 18 | 105907.7 | 65.808 | 57.082 | 63 | 50675.4 | 31.488 | 27.313 |
| 19 | 105294.7 | 65.427 | 56.751 | 64 | $48934 \cdot 3$ | 30.406 | 26.374 |
| 20 | 104649.8 | 65.026 | 56.404 | 65 | 47178.0 | 29.315 | 25.428 |
| 21 | 103973.2 | 64.606 | 56.039 | 66 | 45407.1 | 28.215 | 24.473 |
| 22 | 103265.0 | 64.166 | 55.657 | 67 | 43622.2 | 27.106 | 23.511 |
| 23 | 102525.4 | 63.706 | 55.259 | 68 | 41823.8 | 25.988 | 22.542 |
| 24 | 101754.6 | 63.227 | 54.843 | 69 | 40012.4 | 24.862 | ${ }^{21.566}$ |
| 25 | 100953.0 | 62.729 | 54.411 | 70 | 38188.6 | 23.729 | 20.583 |
| 26 | 100120.6 | 62.212 | 53.963 | 71 | 36353.0 | 22.589 | 19.593 |
| 27 | 90257.8 | 6 I .676 | 53.498 | 72 | 34506.2 |  | 18.598 |
| 28 29 | 98364.8 97441.9 | 61.121 60.548 | 53.016 52.519 | 73 74 | 32648.6 30780.9 | 20.287 19.126 | 17.597 16.590 |
| 30 | 96489.3 | 59.956 | 52.006 | 75 | 28903.6 | 17.960 | 15.578 |
| 31 | 95597.3 | 59.345 | 51.476 | 76 | 27017.4 | 16.788 | 14.562 |
| 32 | 94496.2 | 58.717 | 50.931 | 77 | 25122.8 | 15.611 | 13.541 |
| 33 | 93456.3 | 58.071 | 50.37 I | 78 | 23220.4 21310.8 | 14.428 | 12.515 11.486 |
| 34 | 92387.9 | 57.407 | 49.795 | 79 | ${ }^{21310.8}$ | 13.242 | 11.486 |
| 35 | 91291.3 | 56.726 | 49.204 | 80 | 19394.6 | 12.051 | ${ }^{10.453}$ |
| 36 | 90166.8 | 56.027 | 48.598 | 81 | 17472.4 | 10. 857 | 8.417 |
| 37 | 89014.8 | 55.31 II | 47.977 | 82 | 15544.7 13622.2 | 8.659 | 8.378 7.337 |
| 38 | 87835.6 86629.6 | 54.578 53.829 | 47.34 I 46.691 | 83 84 | 13612.2 11675.5 | 8.458 7.255 | 7.337 6.293 |
| 39 | 86629.6 | 53.829 | 46.691 | 8 | 1175.5 | 7.255 |  |
| 40 | 85397.0 | 53.063 | 46.027 | 85 | 9735.1 | 6.049 | 5.247 4.200 |
| 41 | 84138.4 828540 | 52.281 51.483 | $45 \cdot 349$ 44.656 | 86 | 7791.7 5845.9 | 3.632 |  |
| 42 | 82854.0 81544.2 | 51.483 50.669 | 44.656 43.950 | 87 88 | 585.9 3898.3 | ${ }_{2.422}^{3.632}$ | 3.151 2.101 |
| 43 | 81544.2 80209.4 | 50.869 49.840 | 43.931 | 89 | 1949.4 | 1.2II | 1.051 |
| 45 | 78850.0 | 48.995 | 42.498 . | 90 | 0.0 | 0.000 | 0.000 |

Emithsonian Tables.

Table 38.
INTERCONVERSION OF NAUTICAL AND STATUTE MILES.
I nautical mile $=6080.27$ feet.

| Nautical Miles. | Statute Miles. | Statute Miles. | Nautical Miles. |
| :---: | :---: | :---: | :---: |
| 1 | 1.1516 | 1 | 0.8684 |
| 2 | 2.3031 | 2 | 1.7368 |
| 3 | 3.4547 4.6062 | 3 4 | 2.6052 3.4736 |
| 4 | 4.6062 | 4 | 3.4736 |
| 5 | 5.7578 | 5 | 4.3420 |
| 6 | 6.9093 8.0609 | 6 | 5.2104 |
| 7 | 8.0609 9.2124 | 8 | 6.9472 |
| 9 | 10.3640 | 9 | 7.8155 |

## Emithsonian Tables.

* As defined by the United States Coast and Geodetic Survey.


## Table 39.

CONTINENTAL MEASURES OF LENGTH WITH THEIR METRIC AND
The asterisk (*) indicates that the measure is obsolete or seldom used.

| Measure. | Metric Equivalent. | English Equivalent. |
| :---: | :---: | :---: |
| El, Netherlands | 1 metre. | 3.2808 feet. |
| Fathom, Swedish $=6$ feet | 1.7814 " | 5.8445 " |
| Foot, A ustrian,* | 0.31608 | I. $037{ }^{\circ}$ " |
| old French * | 0.32484 0.30480 | $\begin{array}{ll} \text { I.0657 ، } \\ \text { I } & \end{array}$ |
| Rheinlandisch or Rhenish (Prussia, | 0.30480 |  |
| Denmark, Norway)* . | $0.313^{8} 5$ | 1.0297 " |
| Swedish* . . . | 0.2969 | 0.974 I " |
| * Spanish * = ${ }^{\frac{1}{3} \text { vara }}$. . | 0.2786 | 0.9140 " |
| *Klafter, Wiener (Vienna) . ${ }^{\text {a }}$ | 1.89648 | 6.2221 " |
| *Line, old French $=1$ 1 14 foot. . . Mile, Austrian post $* 24000$ feet. | 0.22558 cm . | 0.0888 inch. |
| Mile, Austrian post ${ }^{*}=24000$ feet . . |  | 4.714 statute miles. <br> I. 1508 |
| Swedish = 36000 feet . | 10.69 " | 6.642 - " |
| Norwegian $=36000$ feet . . - | 11.2986 | 7.02 " " |
| Netherlands (mijl) ${ }^{\text {d }}$ |  | 0.6214 " " |
| Prussian (law of 1868) | 7.500 | 4.660 " " |
| Danish . . . . . . . . | $7.5324{ }^{\text {" }}$ | 4.6804 " " |
| PRalm, Netherlands | 0.1 metre. | 0.328 I feet. |
| *Rode, Danish - ${ }^{\text {* }}$ - ${ }^{\text {a }}$ - | 3.7662 | 12.356 |
| *Ruthe, Prussian, Norwegian. | 3.7662 " | 12.356 " |
| Sagene, Russian ${ }^{\text {a }}$ - | 2.1336 | 7 |
| *Toise, old French $=6$ feet | 1.9490 | $6.3943{ }^{\prime \prime}$ |
| *Vara, Spanish. - | 0.8359 | 2.7424 " |
| Wexican $\cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot$ | 0.8380 | 2.7293 " |
| Werst, or versta, Russian $=500$ sagene | 1.0668 km . | 3500 " |

Emithsonian Tables. DERIVED FUNCTIONS.
$g=9.77989+0.05221 \sin ^{2} \phi$
$=9.80599-0.026$ ro cos $2 \phi$ metres.*
$\phi=$ geographical latitude.

| $\phi$ | $g$ | $\log g$ | $\log \frac{1}{2 g}$ | $\log \sqrt{2 g}$ | $\frac{g}{\text { g }}^{\dagger}{ }^{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Metres. |  |  |  | Metres. |
| $0^{\circ}$ | 9.7798 | 0.99033 | 8.70864-10 | 0.64568 | 0.99090 |
| 5 | .7803 | 035 | 862 | 569 | 095 |
| 10 | .7814 | 040 | 857 | 572 | 106 |
| 15 | .7834 | 049 | 848 | 576 | 127 |
| 20 | . 7859 | 060 | 837 | 582 | 152 |
| 25 | .7893 | 075 | 822 | 589 | 186 |
| 30 | . 7929 | 091 | 806 | 597 | 222 |
| 35 | . 7969 | 109 | 788 | 606 | 264 |
| 40 | . 8014 | 129 | 768 | 616 | 309 |
| 45 | . 8060 | 149 | 748 | 626 | 355 |
| 50 | .8105 | 169 | 728 | 636 | 401 |
| 55 | .8150 | 189 | 708 | 646 | 447 |
| 60 | .8191 | 207 | 690 | 655 | 488 |
| 65 | . 8227 | 223 | 674 | 663 | 525 |
| 70 | .8261 | 238 | 659 | 670 | 559 |
| 75 | . 8286 | 249 | 648 | 676 | 584 |
| 80 | . 8306 | 258 | 639 | 680 | 605 |
| 85 | . 8317 | 263 | 634 | 683 | 616 |
| 90 | . 8322 | 265 | 632 | 684 | 621 |

Smitnsonian Tableb.

* From The Solar Parallax and its Related Constants, by Wm. Harkness, Professor of Mathematics, U. S. N.; Washington: Government Printing Office, 889 r .
$\dagger$ This is length of seconds pendulum.

LINEAR EXPANSIONS OF PRINCIPAL METALS IN MICRONS PER METRE (OR MILLIONTHS PER UNIT LENGTH).

| Name of metal. | Expansion per degree $C$. | Expansion per degree $F$. |
| :---: | :---: | :---: |
| Aluminum | 20 | 11.1 |
| Brass . . . . . . . . . . . . . . . | 19 | 10.5 |
| Copper . . . . . . . . . . . . . . . . | 17 | 9.4 |
| Glass . . . | 9 | 5.0 |
| Gold . - | 15 | 8.3 |
| Iron, cast . . . . . . . . . . . . . . . . | 11 | 6.1 |
| Iron, wrought . . . . . . . . . . . . . . | 12 | $\begin{array}{r}6.7 \\ \hline\end{array}$ |
| Lead . . . . . . . . . . . . . . . . . | 28 | 15.5 |
| Platinum Platinum-iridium ${ }^{1}$. . . . . . . . . . . . . | 8.7 | 5.0 4.8 |
| Silver . . . . . . . . . . . . . . . . | 19 | 10.5 |
| Steel, hard . . . . . . . . . . . . . . . | 12 | 6.7 |
| Steel, soft . . . . . . . . . . . . . . . . | 11 | 6.1 |
| Tin . . . . | 19 | 10.5 |
| Zinc . . . | 29 | 16.1 |

Smithsonian Tables.
${ }^{1}$ Of International Prototype Metres.

Table 42.

## FRACTIONAL CHANGE IN A NUMBER CORRESPONDING TO A CHANGE IN ITS LOCARITHM. <br> Computed from the formula,

$$
\frac{\Delta N}{N}=\frac{\Delta \log N}{\mu}
$$

$\mu=$ modulus of common logarithms $=0.43429448$.

| $\begin{aligned} & \text { For } \\ & \Delta \log N \\ & =\mathrm{I} \text { unit in } \end{aligned}$ | $\frac{\Delta N}{N}$ | $\begin{gathered} \text { For } \\ =\Delta \log N \\ =4 \text { units in } \end{gathered}$ | $\frac{\frac{\Delta N}{N}}{\text { (in round numbers) }}$ |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { 4th place } \\ & \text { 5th "6 } \\ & \text { 6th " } \\ & \text { 7th ". } \end{aligned}$ |  | 4th place 5th 6th 7th 7th |  |

Smithsonian Tables.

## APPENDIX.

## CONSTANTS.

| Numerical Constants. | Number. | Logarithm. |
| :---: | :---: | :---: |
| Base of natural (Napierian) logarithms, Log $e$, modulus of common logarithms, Circumference of circle in degrees, <br> " "، " in minutes, <br> " " " in seconds, <br> Circumference of circle, diameter unity, | $=2.7182818$ | 4342945 |
|  | $\mu=0.4342945$ | 9.6377843-10 |
|  |  | 2.5563025 |
|  | 21600 | 4.3344538 |
|  | 1296000 | 6.1126050 |
|  | $r=3.1415926$ | 0.4971499 |
| $$ | $2=0.1013212$ | 57003-10 |
|  | $\sqrt{\pi}=1.7724539$ | 0.2485749 |
|  | $\frac{1}{\sqrt{\pi}}=0.5641896$ | .7514251-10 |
|  | $\overline{2}=1.4142136$ | 0.1505150 |
|  | $\sqrt{3}=1.7320508$ | . 2385607 |
| The arc of a circle equal to its radius is <br> in degrees, $\rho^{\circ}=180 / \pi$ <br> in minutes, $\rho^{\prime}=60 \rho^{\circ}$ <br> in seconds, $\rho^{\prime \prime}=60 \rho^{\prime}$ | $\begin{aligned} & =\quad 57.29578^{\circ} \\ & =\quad 3437.7468^{\prime} \\ & =206264.8^{\prime \prime} \end{aligned}$ | $\begin{aligned} & 1.7581226 \\ & 3.5362739 \\ & 5.3144251 \end{aligned}$ |
| For a circle of unit radius, the <br> $\operatorname{arc}$ of $\mathrm{I}^{\circ} \quad=1 / \rho^{\circ}$ <br> arc of $i^{\prime} \quad=1 / p^{\prime}$ <br> arc (or sine) of $\mathrm{I}^{\prime \prime}=1 / \rho^{\prime \prime}$ | $\begin{aligned} & =0.0174533 \\ & =0.0002909 \\ & =0.00000485 \end{aligned}$ | $\begin{aligned} & 8.2418774-10 \\ & 6.4637261-10 \\ & 4.6855749-10 \end{aligned}$ |
| Geodetical Constants. |  |  |
| Dimensions of the earth (Clarke's spheroid, 1866) and derived quantities. |  |  |
| Equatorial semi-axis in feet, in miles, | $\begin{aligned} & =a=20926062 . \\ & =a=\quad 3963 \cdot 3 \end{aligned}$ | $\begin{gathered} 7.3206875 \\ 3.5980536 \end{gathered}$ |
| Polar semi-axis in feet, in miles, | $\begin{aligned} & =b=20855 \mathrm{I} 2 \mathrm{II} . \\ & =b=\quad 3949.8 \end{aligned}$ | $\begin{aligned} & 7.3192127 \\ & 3.5965788 \end{aligned}$ |
| $(\text { Eccentricity })^{2}=\frac{a^{2}-b^{2}}{a^{2}}$ | $\wedge^{2}=0.00676866$ | .8305030-10 |
| $\text { Flattening }=\frac{a-b}{a}$ | $=f=1 / 294.9784$ | .5302098-10 |
| Perimeter of meridian ellipse, Circumference of equator, Area of earth's surface, | $\begin{aligned} & =24859.76 \mathrm{n} \\ & =24901.96 \\ & =196940400 \mathrm{~s} \end{aligned}$ | re miles. |
| Mean density of the earth (Harkness) Surface density " | $\begin{aligned} & =5.576 \pm 0.016 . \\ & =2.56 \pm 0.16 . \end{aligned}$ |  |
| Acceleration of gravity (Harkness) : <br> $g(\mathrm{~cm}$. per second $)=980.60(\mathrm{r}-0.002662 \cos 2 \phi)$ for latitude $\phi$ and sea level. <br> $g$, at equator $=977.99 ; g$, at Washington $=980.07 ; g$, at Paris $=980.94$; <br> $g$, at poles $=983.2 \mathrm{I} ; g$, at Greenwich $=981.17$. <br> Length of the seconds pendulum (Harkness) : <br> $l=39.01254^{\circ}+0.208268 \sin ^{2} \phi$ inches $=0.990910+0.005290 \sin ^{2} \phi$ metres. |  |  |

## APPENDIX.

## Astronomical Constants (Harkness).

Sidereal year $=365.2563578$ mean solar days.
Sidereal day $=23^{h} 5^{6} 6 \mathrm{~m} 4,5100$ mean solar time.
Mean solar day $=24^{h} 3^{m} 56.5546$ sidereal time.
Mean distance of the earth from the sun $=92800000$ miles.

## Physical Constants.

Velocity of light (Harkness) $=186337$ miles per second $=299878 \mathrm{~km}$. per second.
Velocity of sound through dry air $=1090 \sqrt{1+0.00367 t^{\circ}} \mathrm{C}$. feet per second.
Weight of distilled water, free from air, barometer 30 inches :

|  | Weight in grains. |  | Weight io grammes. |  |
| :---: | :---: | :---: | :---: | :---: |
| Volume. | $6_{2}{ }^{\circ} \mathrm{F}$. | $4^{\circ} \mathrm{C}$. | $62^{\circ} \mathrm{F}$. | $4^{\circ} \mathrm{C}$ |
| 1 cubic inch (determination of 1890) | 252.286 | 252.568 | 16.3479 | 16.3662 |
| 1 cubic centimetre (1890) | 15.3953 | 15.4125 | 0.9976 | 0.9987 |
| I cubic foot ( I 890 ) at $62^{\circ} \mathrm{F}$. | 62.2786 |  |  |  |

A standard atmosphere is the pressure of a vertical column of pure mercury whose height is 760 mm . and temperature $0^{\circ} \mathrm{C}$., under standard gravity at latitude $45^{\circ}$ and at sea level.
I standard atmosphere $=1033$ grammes per sq. $\mathrm{cm} .=14.7$ pounds per sq. inch.
Pressure of mercurial column $x$ inch high $=34.5$ grammes per sq. $\mathrm{cm} .=0.49 \mathrm{I}$ pounds per sq. inch.
Weight of dry air (containing 0.0004 of its weight of carbonic acid) :
I cubic centimetre at temperature $32^{\circ} \mathrm{F}$. and pressure 760 mm . and under the standard value of gravity weighs 0.00129305 gramme.
Density of mercury at $0^{\circ} \mathrm{C}$. (compared with water of maximum density under atmospheric pressure) $=13.5956$.
Freezing point of mercury $=-38 .{ }^{\circ} 5$ C. (Regnault, 1862.)
Coefficient of expansion of air (at const. pressure of 760 mm ) for $1^{\circ} \mathrm{C}$. (DO.) : 0.003670 .
Coefficient of expansion of mercury for Centigrade temperatures (BROCH):
$\Delta=\Delta_{0}\left(\mathrm{I}-0.000 \mathrm{I} 8 \mathrm{I} 792 t-0.000000000175 t^{2}-.000000000035116 t^{3}\right)$.
Coefficient of linear expansion of brass for $1^{\circ} C$., $\beta=0.0000174$ to 0.0000190 .
Coefficient of cubical expansion of glass for $1^{\circ} C_{\text {., }} \boldsymbol{\gamma}=0.000021$ to 0.000028 .
Ordinary glass (Recknagel) : at $10^{\circ} C$., $\boldsymbol{\gamma}=0.0000255$; at $100^{\circ}, \gamma=0.0000276$.
Specific heat of dry air compared with an equal weight of water :
at constant pressure, $K_{p}=0.2374$ (from $0^{\circ}$ to $100^{\circ} C$., Regnault).
at constant volume, $K_{v}=0.1689$.
Ratio of the two specific heats of air (RONTGEN): $K_{p} / K_{v}=1.4053$.
Thermal conductivity of air (Graetz) : $k=0.0000484\left(\mathrm{I}+0.00185 t^{\circ}\right.$, C.) $\frac{\text { gramme. }}{\mathrm{cm} . \sec .}$
[The quantity of heat that passes in unit time through unit area of a plate of unit thickoess, when its opposite faces differ io temperature by one degree.]
Latent heat of liquefaction of ice (BuNSEN) $=80.025$ mass degrees, $C$.
Latent heat of vaporization of water $=606.5-0.695 t^{\circ} \mathrm{C}$.
Absolute zero of temperature (Thomson, Heat, Encyc.Brit.) : $-273 .{ }^{\circ} \mathrm{O} \mathrm{C} .=-459 .{ }^{\circ} 4 \mathrm{~F}$. Mechanical equivalent of heat : *
${ }^{1}$ pound-degree, $F$. (the British thermal unit) $=$ about 778 foot-pounds.
${ }^{1}$ pound-degree, $C$. $=1400$ foot-pounds.
I calorie or kilogramme-degree, $C .=3087$ foot-pounds $=426.8$ kilogrammetres $=4187$ joules (for $g=98 \mathrm{Icm}$.).

## SYNOPTIC CONVERSION OF ENGLISH AND METRIC UNITS. English to Metric.

## Units of length.

1 inch.
I foot.
I yard.
I mile.

Matric equivalante.

| 2.54000 | centimetres. |
| :--- | :--- |
| 0.304801 | metre. |
| 0.914402 | _1 |
| 1.60935 | kilometres. |


| 6.45163 | square centimetres. |
| :---: | :--- |
| 929.034 | " |
| 0.83613 I | square metre. |
| 0.404687 | hectares. |
| 2.59000 | square kilometres. |

## Units of area.

I square inch.
I square foot.
I square yard.
I acre.
I square mile.

## Units of volume.

1 cubic inch.
I cubic foot.
16.3872
0.028317
0.764559

1 cubic yard.

Lagarlthms.
$0.404835-10$
$9.484016=10$
$9.961137-10$
0.206650

0.809669
2.968032
$9.922274-10$
$9.607120-10$
0.4 I 3300
2.4 I 3300

Units of capacity.
I gallon (U. S.) $=231$ cubic inches.
1 quart (U. S.).
3.78544 litres.
0.94636 litres.
4.54683 litres.
$\begin{array}{ll}\text { cubic centimetres. } & \mathbf{1 . 2 1 4} 504 \\ \text { cubic metres or steres. } & 8.452047-\text { ro }\end{array}$
cubic metres or steres.
9.88341 I - IO

Imperial gallon (British).
277.463 cubic inches ( 1890 ).

1 bushel (U.S.) $=2150.42$ cubic inches.
I bushel (British).

## Units of mass.

1 grain.
I pound avoirdupois.
I ounce avoirdupois.
1 ounce troy.
I ton ( 2240 lbs .).
I ton ( 2000 lbs .).
64.7990
0.453593
28.3496
31.1035
1.01605
0.907186
milligrammes.
kilogrammes. grammes. grammes. tonnes. tonnes.
0.578116
9.976056 - 10
0.657709
1.547027
I. 560477
т. 8 II 568 9.656666 - 10
1.452546
I. 4928 IO
0.006914
9.957696 - 10

## Units of velocity.

I foot per sec. ( 0.6818 miles per hr.) $=0.30480$ metres per sec. $=1.0973 \mathrm{~km}$. per hr . I mile per hr. ( I .4667 feet per sec.) $=0.44704$ metres per sec. $=1.6093 \mathrm{~km}$. per hr.

## Units of force.

1 poundal.
Weight of I grain (for $g=98 \mathrm{rcm}$.).

| 13825.5 dynes. | 4.140682 |
| ---: | ---: |
| 63.57 dynes. | 1.803237 |
| $4.45 \times 1 \mathbf{I O}^{5}$ dynes. | 5.648335 |

Units of stress-In gravitation measure.
I pound per square inch $=70.307$ grammes per sq. centimetre. $\quad 1.846997$
I pound per square foot $=4.8824$ kilogrammes per sq. metre.
Units of work - in absoluta maasure.

| I foot-poundal. | 421403 ergs. | 5.624698 |
| :--- | :--- | :--- |

- in gravitation meagure.

1 foot-pound (for $g=981 \mathrm{~cm}$.) $=1356.3 \times 10^{4} \mathrm{ergs}=0.138255$ kilogram-metres.
Units of activity (rate of doing work).
I foot-pound per minute (for $g=981 \mathrm{~cm}$.) $=0.022605$ watts.
I horse-power ( 33000 foot-pounds per min.) $=746 \mathrm{wa} \mathrm{s}=1.01387$ force de cheval.

## Units of heat.

I pound-degree, $F$.
I pound-degree, $\boldsymbol{C}$.
$=252$ small calories or gramme-degrees, $C$.
$=1.8$ pound-degrees, $F$.

## SYNOPTIC CONVERSION OF ENGLISH AND METRIC UNITS. Metric to English.

|  | English squivalsnts. |  | Logarithms. |
| :---: | :---: | :---: | :---: |
| Units of length. <br> I metre ( $10^{8}$ microns). | 39.3700 | inches. | 1.595165 |
|  | 3.28083 | feet. | 0.515984 |
| " | 1.09361 | yards. | 0.038863 |
| I kilometre. | 0.62137 | miles. | 9.793350-10 |
| Units of area. |  |  |  |
| I square centimetre. | 0.15500 | square inches. | 9.190 331-10 |
| I square metre. | 10.7639 | square feet. | I.031 968 |
| " " | I. 19599 | square yards. | 0.077726 |
| 1 hectare. | 2.47104 | acres. | 0.392880 |
| I square kilometre. | 0.38610 | square miles. | 9.586701-10 |
| Units of volume. |  |  |  |
| I cubic centimetre. | 0.0610234 | cubic inches. | 8.785496 - 10 |
| I cubic metre or stère. | 35.3145 | cubic feet. | 1.547953 |
| " ${ }^{\text {a }}$ | I.30794 | cubic yards. | 0.116589 |
| Units of capacity. |  |  |  |
| 1 litre (6r.023 cubic inches). | 0.26417 | gallons (U. S.). | 9.421884-10 |
| " (6.023 | 1.05668 | quarts (U. S.). | 0.023944 |
| " | 0.21993 | Imp. gallons (British). | 9.342291 - 10 |
| I hectolitre. | 2.83774 | bushels (U. S.). | 0.452973 |
|  | 2.75121 | bushels (British). | 0.439523 |
| Units of mass. |  |  |  |
| 1 gramme. | 15.4324 | grains. | 1. 888433 |
| I kilogramme. | 2.20462 | pounds avoirdupois. | 0.343334 |
|  | 35.2739 | ounces avoirdupois. | I. 547454 |
|  | 32.1507 | ounces troy. | 1. 507190 |
| I tonne. | 0.9842 I | tons ( 2240 lbs .). | 9.993086-10 |
|  | $1.10231$ | $\text { tons ( } 2000 \mathrm{lbs} \text {.). }$ | 0.042304 |
| Units of velocity. |  |  |  |
| 1 metre per second. <br> " " " | 3.2808 | feet per second. | 0.515984 |
|  | 2.2369 | miles per hour. | 0.349653 |
| 1 km . per hr. ( 0.2778 m. per sec.). | 0.62137 | miles per hour. | $9.793350-10$ |
| Units of force. I dyne (weight of $(981)^{-1}$ grammes, for $g=981 \mathrm{~cm}$.) $=7.2330 \times 10^{-5}$ poundals. |  |  |  |
| Units of stress-in gravitation msasure. |  |  |  |
| I gramme per square centimetre. | 0.014223 | pounds per sq. inch. |  |
| I kilogramme per square metre. I standard atmosphere. | 0.204817 pounds per sq. foot. |  |  |
|  | I standard atmosphere. 14.7 pounds per sq. inch. (See def. p. 172.) |  |  |
| Units of work - in aboolute maazure. |  |  |  |
|  |  |  |  |
|  |  |  |  |
| - In gravitatlon measura. |  |  |  |
| I kilogramme-metre (for $g=98 \mathrm{Icm}.)=98 \mathrm{I} \times 10^{5} \mathrm{ergs}=7.2330$ foot-pounds. |  |  |  |
| Units of activity (rate of doing work). <br> I watt $=\mathrm{I}$ joule per sec. $(=44.2385$ foot-pounds per minute, for $g=98 \mathrm{Icm}$. $)=0.10194$ kilogramme-metre per sec., for $g=98 \mathrm{Icm}$. |  |  |  |
| I force de cheval $=75$ kilogramme-metres per sec. $=735 \frac{3}{4}$ watts $=0.98632$ horse-power. Units of heat. |  |  |  |
| I calorie or kilogramme-degree $=3.968$ pound-degrees, $F:=2.2046$ pound-degrees, $C$. <br> I small calorie or therm, or gramme-degree $=0.001$ calorie or kilogramme-degree. |  |  |  |

# DIMENSIONS OF PHYSICAL QUANTITIES. 

$L=$ length $; M=$ mass $; T=$ time.

| Quantity, | Dlmensione | Quantity. | Dimensions. |
| :---: | :---: | :---: | :---: |
| Area. | [L2] | Momentum. | [ $\mathrm{LM} \mathrm{T}^{-1}$ ] |
| Volume. | [ $L^{s}$ ] | Moment of Inertia. | [ $\mathrm{M} \mathrm{L}^{2}$ ] |
| Mass. | [M] | Force. | [ $\mathrm{LM} \mathrm{T}{ }^{-2}$ ] |
| Density. | [ $\mathrm{M} \mathrm{L}^{-\ell}$ ] | Stress (per unit area). | $\left[L^{-1} \mathrm{M} \mathrm{T}^{-2}\right.$ ] |
| Velocity. | [ $\mathrm{L} \mathrm{T}^{-1}$ ] | Work or Energy. | $\left[L^{2} \mathrm{M} \mathrm{T}^{-2}\right.$ ] |
| Acceleration. | [ $1 . \mathrm{T}^{-2}$ ] | Rate of Working (Power). | [ $\mathrm{L}^{2} \mathrm{M} \mathrm{T} \mathrm{T}^{-8}$ ] |
| Angle. | [0] | Heat. | $\left[L^{2} \mathrm{M} \mathrm{T}^{-2}\right]$ |
| Angular Velocity. | [ $\mathrm{T}^{-1}$ ] | Thermal Conductivity. | $\left[\mathrm{L}^{-1} \mathrm{M} \mathrm{T}^{-1}\right]$ |

## In Electrostatics.

Quantity of Electricity.
Surface Density: quantity per unit area.
Difference of Potential: quantity of work required to move a quantity of electricity ; (work done) $\div$ (quantity moved).
Electric Force, or Electro-motive Intensity : (quantity) $\div$ (distance ${ }^{2}$ ).
Capacity of an accumulator: $e \div E$.
Specific Inductive Capacity.

## In Magnetics.

Quantity of Magnetism, or Strength of Pole.
Strength or Intensity of Field: (quantity) $\div\left(\right.$ distance $\left.^{2}\right)$.
Magnetic Force.
Magnetic Moment: (quantity) $\times$ (length).
Intensity of Magnetization : magnetic moment per unit volume.
Magnetic Potential: work done in moving a quantity of magnetism ; (work done) $\div$ (quantity moved).
Magnetic Inductive Capacity.

## In Electro-magnetics.

Intensity of Current.
Quantity of Electricity conveyed by current: (intensity) $\times$ (time).
Potential, or difference of potential: (work done) $\div$ (quantity of electricity upon which work is done).
Electric Force: the mechanical force acting on electro-magnetic unit of quantity; (mechanical force) - (quantity).
Resistance of a conductor: $E \div i$.
Capacity: quantity of electricity stored up per unit potential-difference produced by it.
Specific Conductivity: the intensity of current passing across unit area under the action of unit electric force.
Specific Resistance: the reciprocal of specific conductivity.

|  | $\mu$ | [0] |
| :---: | :---: | :---: |
| Symbol. | Dlmenslons in electro-magnetic system. | Name of practical unit. |
| $i$ | [ $\mathrm{L}^{\frac{1}{2}} \mathrm{M}^{\frac{1}{2}} \mathrm{~T}^{-1}$ ] | Ampère. |
| $e$ | [ $L^{\frac{1}{4}} \mathrm{M}^{\frac{1}{2}}$ ] | Coulomb. |
| $E$ | [ $L^{3} \mathrm{M}^{\frac{1}{2}} \mathrm{~T}^{-2}$ ] | Volt. |
| E | [ $L^{\frac{1}{2}} \mathrm{M}^{\frac{1}{2}} \mathrm{~T}^{-2}$ ] |  |
| $\boldsymbol{R}$ | [ $\mathrm{L} \mathrm{T}^{-1}$ ] | Ohm. |
| $q$ | $\left[\mathrm{L}^{-1} \mathrm{~T}^{2}\right]$ | Farad. |
|  | [ $\left.\mathrm{L}^{-2} \mathrm{~T}\right]$ |  |
| $2 \cdot$ | [ $L^{2} \mathrm{~T}^{-1}$ ] |  |

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[^0]:    Smithsonian Institution, Washington City, October 30, 1897.

[^1]:    * Bulletin 26, U. S. Coast and Geodetic Survey. Washington : Government Printing Office, 1893. Published here by permission of Dr. T. C. Mendenhall, Superintendent Coast and Geodetic Survey.

[^2]:    * Note. - Reference to the Act of 1866 results in the establishment of the following : -

[^3]:    * The actual error of the relation of the yard to the metre may be as great as $1 / 200000$ th part, and the actual error of the relation of the pound to the kilogramme as great as $1 / 100000 \mathrm{th}^{\text {part. }}$

[^4]:    ${ }^{*} c_{1}, c_{2}, c_{8}$ are obtained from $C_{1}, C_{2}, C_{8}$ respectively by dividing the latter by the number of degrees in the radius, viz: 57.29578 .

[^5]:    * For the solution of very large triangles and for a full treatment of the theory thereof, consult Die Mathematischen und Physikalischen Theorieen der Höheren Geodüsie, von Dr. F. R. Helmert. Leipzig, 1880, 1884.

[^6]:    * The mass of the earth's atmosphere is about one-millionth part of the entire mass, or about $66 \times 10^{14}$ tons.
    $\dagger$ The values of $A$ and $C$ are those given by Harkness, loc. cit., but they are here abridged to three places of decimals.

[^7]:    * The best treatise on the theory and use of this instrument is to be found in Chauvenet's Manual of Spherical and Practical Astronomy, which should be consulted by one desiring to go into the details of the subject.
    $t$ Other equivalent constants may be used, but those given are most commonly employed.

[^8]:    * For details of theory and practice in time work done according to this plan see Bulletin 49. U. S. Geological Survey.

[^9]:    * The best work of this kind is Chaurenet's Manual of Spherical and Practical Astronomy. It should be consulted by all persons desiring a knowledge of the details of practical astronomy.

[^10]:    * Among which Chauvenet's Manual of Spherical and Practical Astronomy is the best.

[^11]:    * In precise work the computed azimuth requires the following correction for daily aberration, namely:-

    $$
    \Delta A=-0 .{ }^{\prime \prime} 32 \frac{\cos \phi}{\sin z} \cos A
    $$

    where $A$ is to be reckoned from the south by way of the west through $360^{\circ}$.

[^12]:    * The reader should observe that the word probable is here used in a specially technical sense. Thus, the probable error is not "the most probable error," nor " the most probable value of the actual error," etc., as commonly interpreted.

[^13]:    * For the theory of the errors of this species of interpolated values see Annals of Mathematics, vol. ii. pp. 54-59.

[^14]:    * Hence the term least squares,

[^15]:    * The middle ground between these extremes has been little explored; indeed, most practical applications fall at one or the other of the extremes.

[^16]:    * Since the probable error is 0.6745 times the mean error the latter only need be considered.

[^17]:    * Berlin: Verlag von Ernst \& Korn. This work is an invaluable one to the engineer, architect, geographer, etc.

[^18]:    * The meridional distances and the abscissas of the points on the developed parallels in Fig. 4 are one twentieth of the true or tabular values. The ordinates of points on the developed parallels are the tabular values.

[^19]:    * It should be noted that $C N$ is not equal to $E V, N$ and $V$ referring here to points on the developed parallels.
    $\dagger$ Comftes Rendus, Paris, 1850, vol. xxv. p. 309.

[^20]:    * It should be observed that the metric values given in these tables depend on Clarke's value of the ratio of the yard to the metre, which is now known to be erroneous by about the $1 / 100000$ th part.

[^21]:    * Washington, Government Printing Office, 1891.

[^22]:    Smithsonian Tables．

[^23]:    Smithsonian Tagleg.

[^24]:    Smithsonian Tables.

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